**Journal of Innovative Applied Mathematics and Computational Sciences**

J. Innov. Appl. Math. Comput. Sci. **2**(3) (2022), 69–85. n2t.net/ark:/49935/jiamcs.v2i3.25

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# Modified projective synchronization of fractional-order hyperchaotic memristor-based Chua's circuit

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Received 5 February 2022, Accepted 15 December 2022, Published 31 December 2022

**Abstract.** This paper investigates the modified projective synchronization (MPS) between two hyperchaotic memristor-based Chua circuits modeled by two nonlinear integer-order and fractional-order systems. First, a hyperchaotic memristor-based Chua circuit is suggested, and its dynamics are explored using different tools, including stability theory, phase portraits, Lyapunov exponents, and bifurcation diagrams. Another interesting property of this circuit was the coexistence of attractors and the appearance of mixed-mode oscillations. It has been shown that one can achieve MPS with integer-order and incommensurate fractional order memristor-based Chua circuits. Finally, examples of numerical simulation are presented, showing that the theoretical results are in good agreement with the numerical ones.

**Keywords:** Memristor; hyperchaotic system; Chua's circuit; Caputo derivative; incommensurate fractional order Hyperchaotic System; modified projective synchronization.
**2020 Mathematics Subject Classification:** 37M05, 37M20, 37M22, 37M25, 93D05

## 1 Introduction

In 1971, the circuit theorist Leon Chua had published a study entitled "Memristor: the missing circuit element". This achievement has attracted a great research attention across a wide range of disciplines, such as programmable logic [14] and electronics [33] as well as neural networks [42]. Because memristors are non-linear components, their application to build chaotic or hyperchaotic systems has received significant attention in recent decades [9,23,30]. For example, the canonical Chua's circuit has been improved by replacing its diode with a memristor whose output is monotone-increasing [8]. Both chaotic and hyperchaotic systems are clearly

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ISSN (electronic): 2773-4196

defined as nonlinear systems that are highly dependent on initial conditions, unpredictable in the long run and non-periodic. The fact that hyper-chaotic systems have at least two positive Lyapunov exponents makes their dynamics more complex. And hence favourable for many applications. Mainly, for encryption and secure communications [12,17,35,36]. Various models of commensurate fractional-order memristor-based systems have been designed [11,13,21]. However, because of the different fractional-order characteristics of each circuit component, it is more important to consider fractional-order circuits or systems with incommensurate fractional order. Meanwhile, synchronization of chaotic and hyperchaotic systems has become a crucial research domain, especially in secure communication [19]. Various techniques have been proposed for the synchronization of chaotic systems, such as Active control [31], adaptive control [4,35], Feedback control, Prediction based feedback control, Sliding mode control and adaptive fuzzy control [2,5,6,10,31,34,38]. Using these methods, many works for the synchronization problem have been extended to the scope, such as phase synchronization, complete synchronization, anti-synchronization, projective synchronization, generalized projective synchronization, inverse hybrid function projective synchronization, generalized synchronization and MPS [4,18,29,31,41,43], but there are few studies on the MPS between integer-order and incommensurate fractional order hyperchaotic systems.

Motivated by the precedent reasons, a hyperchaotic memristor-based Chua's circuit is suggested, and its dynamics are explored using different tools, including stability theory, phase portraits, Lyapunov exponents, and bifurcation diagrams. Then, using an active control strategy, the problem of MPS between integer-order and incommensurate fractional order hyperchaotic memristor-based systems is explored, and synchronization is proved using the Lyapunov stability theory of fractional systems.

The present paper is organized as follows: in section 2, a mathematical model of the memristor is described, and the Caputo fractional derivative is discussed. In section 3, a novel memristor-based hyperchaotic system is introduced and its dynamical behavior is investigated. MPS between integer-order and incommensurate fractional order hyperchaotic systems is applied using the active control method in section 4. To illustrate the theoretical results, numerical simulations are presented using MATLAB programs. Finally, in the last section, this study concludes with a summary of the accomplished results and a conclusion.

## 2 Preliminaries

### 2.1 Basic memristor model

A memristor is a nonlinear resistor with a memory effect that can be either flux-controlled or charge-controlled [8]. It can be defined as a dual-terminal device having the relationship

$$f(\varphi, q) = 0.$$

Equations (2.1) and (2.2) describe a charge-controlled and a flux-controlled memristor, respectively [20,26]

$$M(q) = \frac{d\varphi(q)}{dq}, v = M(q)i, \tag{2.1}$$

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi}, i = W(\varphi)v, \tag{2.2}$$

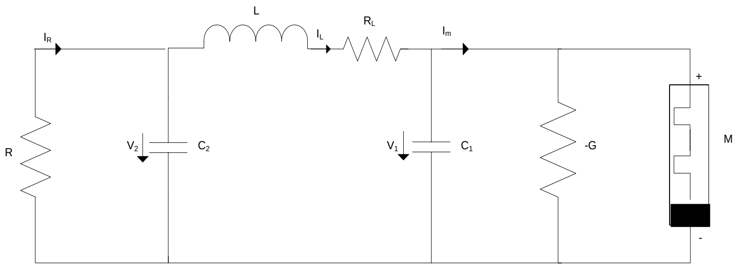


Figure 2.1: Modified memristor-based Chua's circuit

Where $\varphi$ denotes the magnetic flux and $q$ the charge, $W(\varphi)$ and $M(q)$ are called the memductance and memristance respectively.
This study considers a flux-controlled memristor whose characteristics are described by a piecewise quadratic function $q(\varphi)$ given by

$$q(\varphi) = -a\varphi + 0.5b\varphi|\varphi|.$$

With $a$ and $b$ being positive parameters.
Hence, its memductance function is

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi} = -a + b|\varphi|.$$

## 2.2 Caputo fractional derivative

**Definition 2.1.** The Caputo fractional derivative of order $\alpha$ of a continuous function $f : \mathbb{R}^+ \mapsto \mathbb{R}$ is defined by:

$$D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha-m+1}} d\tau, & m-1 < \alpha < m, \\ \frac{d^m}{dt^m} f(t), & \alpha = m, \end{cases}$$

where $m = \lceil \alpha \rceil$, and $\Gamma$ is the $\Gamma$-function defined by

$$\Gamma(z) = \int_0^{+\infty} e^{-t} t^{z-1} dt, \quad \Gamma(z+1) = z\Gamma(z).$$

**Theorem 2.2.** *Consider the incommensurate fractional order system*

$$D^{\alpha_i} x_i = f(x_1, x_2, ..., x_n, t), i = 1, 2, ..., n, \tag{2.3}$$

*Where $\alpha_1 \neq \alpha_2 \neq ... \neq \alpha_n$. Suppose that $m$ is the least common multiple of the denominators $u_i$'s of $\alpha_i$'s, where $\alpha_i = \frac{v_i}{u_i}$, $u_i, v_i \in \mathbb{Z}^+$ for $i = 1, 2, ..., n$. Denote $\gamma = \frac{1}{m}$ and $J$ be the Jacobian matrix $J = \frac{df}{dx}$ evaluated at the equilibrium, where $f = [f_1, f_2, ..., f_n]^T$, $x = [x_1, x_2, ..., x_n]^T$. System (2.3) is asymptotically stable if $|arg(\lambda_i)| > \gamma \frac{\pi}{2}$ is satisfied for all roots $\lambda_i$ of the following equation :*

$$det(diag([\lambda^{m\alpha_1}, \lambda^{m\alpha_2}, ..., \lambda^{m\alpha_n}]) - J) = 0, \tag{2.4}$$

# 3 Building a memristor-based system and its analysis

In this section, an alternative memristor-based Chua's circuit is proposed by replacing the non-linear diode in the original circuit with a negative conductance and a passive flux-controlled memristor described by (2.2) in parallel and changing the inductance's position that becomes between the two capacitances as shown in Figure 2.1.
Kirchhoff Laws allow us to describe the suggested circuit theoretically by the following four-dimensional differential system

$$\begin{cases} \dfrac{dV_1(t)}{dt} = \dfrac{1}{C_1}\left[I_L(t) + GV_1(t) - W(\phi)\, V_1(t)\right], \\ \dfrac{dV_2(t)}{dt} = \dfrac{1}{C_2}\left[\dfrac{V_2(t)}{R} - I_L(t)\right], \\ \dfrac{dI_L(t)}{dt} = \dfrac{1}{L}\left[-V_1(t) + V_2(t) - R_L I_L(t)\right], \\ \dfrac{d\phi(t)}{dt} = V_1(t), \end{cases} \tag{3.1}$$

where $W(\phi)$ is defined by (2.2) and $V_i, i = 1.2$ voltages, $R, R_L$ and $G$ resistances, $C_i, i = 1.2$ capacitances, $I_L$ current, $L$ the inductance and $\phi$ the magnetic flux through the memristor.
By setting $x = V_1,\ y = V_2,\ z = I_L,\ \omega = \phi,\ C_2 = 1, R = 1,\ \alpha = \dfrac{1}{C_1},\ \beta = \dfrac{1}{L},\ \gamma = \dfrac{R_L}{L}$ and $\xi = G$ then (3.1) can be converted into its dimensionless form

$$\begin{cases} \dot{x} = \alpha[z + \xi x - (-a + b|\omega|)x], \\ \dot{y} = y - z, \\ \dot{z} = -\beta(x - y) - \gamma z, \\ \dot{w} = x, \end{cases} \tag{3.2}$$

where $x, y, z$ and $\omega$ are the states and $\alpha, \beta, \gamma, \xi, a$ and $b$ are assumed to be positive constant parameters.

## 3.1 Stability analysis

The equilibrium points of system (3.2) are its solutions, taking each equation of the system equal to zero. Thus, the following equilibrium points are obtained

$$P_e = \left\{(x, y, z, \omega);\ x = 0,\ y = 0,\ z = 0 \text{ and } \omega = \omega_e \in \mathbb{R}\right\}. \tag{3.3}$$

Hence, each point on the $\omega -$ axis is an equilibrium point of (3.2), and (3.3) is called the equilibrium set.
The Jacobian matrix at each equilibrium point $P_e$ is

$$J(P_e) = \begin{bmatrix} \alpha(\xi - W(w_e)) & 0 & \alpha & 0 \\ 0 & 1 & -1 & 0 \\ -\beta & \beta & -\gamma & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{3.4}$$

The characteristic polynomial of the system (3.2) is given by

$$\begin{aligned} P(\lambda) &= \lambda\left[\lambda^3 + [\gamma - 1 - \alpha(\xi - W(w_e))]\lambda^2 + [(-\gamma+1)\alpha(\xi - W(w_e)) + (1+\alpha)\beta - \gamma]\lambda \right. \\ &\quad \left. +\alpha[(\gamma-\beta)(\xi - W(w_e)) - \beta]\right] = \lambda Q(\lambda). \end{aligned} \tag{3.5}$$

Setting the system parameters as

$$\alpha = 5,\ \beta = 5,\ \gamma = 0.11,\ \xi = 3,\ a = 1.5,\ b = 1\ \ and\ \ W(\omega_e) = -a + b|\omega_e|. \tag{3.6}$$

Then, the characteristic polynomial (3.5) becomes

$$\begin{aligned} P(\lambda) &= \lambda Q(\lambda) \\ &= \lambda\left[\lambda^3 + (5|w_e| - 23.4)\lambda^2 + (50.15 - 4.5|w_e|)\lambda + 24.5|w_e| - 135.25\right] = 0. \end{aligned} \tag{3.7}$$

In order to find the range $\omega_e$ for which the system (3.2) has a three-dimensional stable manifold (Regardless of the eigenvalue being zero), one applies Routh-Hurwitz stability criterion to $Q(\lambda)$. So, all its roots have negative real parts if and only if the following conditions are satisfied

$$\begin{cases} 5|w_e| - 23.4 > 0, \\ 24.5|w_e| - 135.25 > 0, \\ -22.5|w_e|^2 + 331.55|w_e| - 1038.3 > 0, \end{cases} \tag{3.8}$$

Hence,

$$5.5204 < |w_e| < 10.221,$$

In contrast, chaos has a greater possibility of occurrence if (3.7) has one or more roots with positive real parts, that is

$$|w_e| < 5.5204, \quad \text{or} \quad |w_e| > 10.221. \tag{3.9}$$

According to the above results, we deduce that the initial value of the state variable $\omega(t)$ can affect considerably the dynamical behavior of the system (3.2).

## 3.2 Bifurcation and Lyapunov Exponents spectrum

### 3.2.1 Dynamical behaviors versus the parameter $a$

In this section, the parameters take the following values $\alpha = 5, \beta = 5, \gamma = 0.1, b = 1, \xi = 3$ and let $a$ vary over a certain interval to discuss the complex dynamics of the system (3.2) with the initial condition $(x, y, z, w_0) = (-0.5, 0.1, 0.01, -1)$. The bifurcation diagram of $y$ and the corresponding Lyapunov exponents spectrum for $a$ varying from 0 to 6 with a step size $h = 0.001$ are obtained as depicted in Figure 3.1 and Figure 3.2, respectively, which are in good coincidence.

From these figures it is obvious that system (3.2) displays period 1 orbit for $a \in ]0.02, 1.41[ \cup ]2.04, 3.24[$. For $a \in ]1.41, 2.1[ \cup ]3.24, 6[$ system (3.2) demonstrates chaotic and hyperchaotic behavior.

In particular, for $a = 3$ the Lyapunov exponents are

$$L_1 \approx 0.1417,\ L_2 \approx 0.0942,\ L_3 \approx 0.042,\ L_4 \approx -52.2119. \tag{3.10}$$

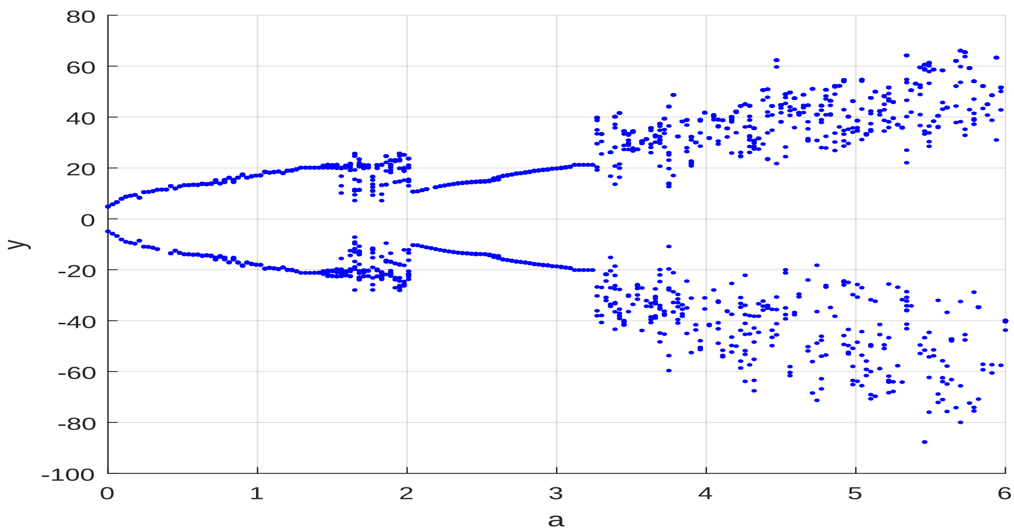


Figure 3.1: Bifurcation diagram with respect to the parameter $a$ for $w_0 = -1$

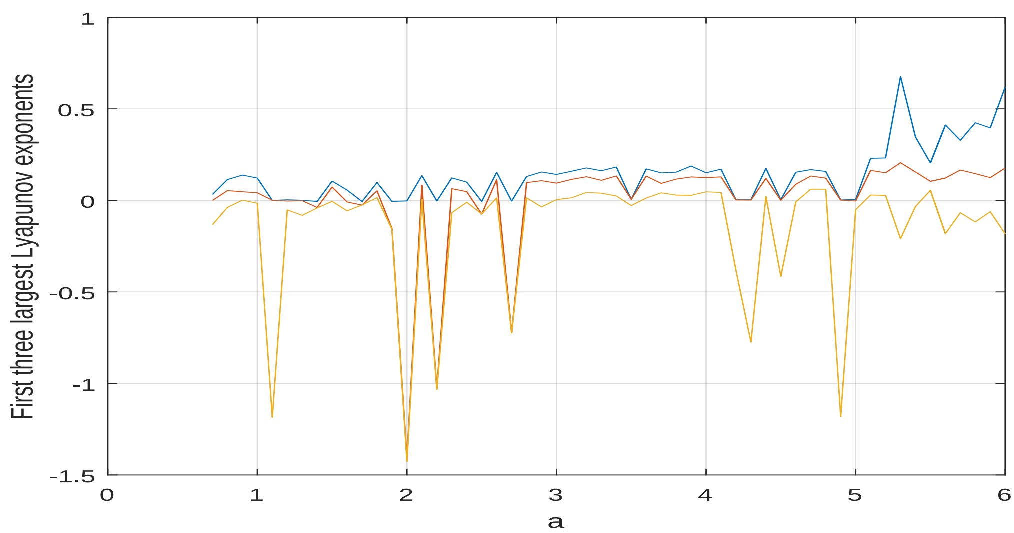


Figure 3.2: The three largest Lyapunov exponents of the system (3.2) versus the parameter $a$ for $w_0 = -1$

Since $L_1 + L_2 + L_3 + L_4 = -51.9719 < 0$, $L_1 > 0, L_2 > 0$, then the system (3.2) is hyperchaotic. The Kaplan-Yorke dimension of its attractor is

$$D_{KY} \approx 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3 + \frac{0.1417 + 0.0942 + 0.042}{51.2119} = 3.0046, \tag{3.11}$$

which is a fractal dimension.

### 3.2.2 Dynamical behaviors versus the initial state $w_0$

In the aim to study the impact of initial condition values on the dynamical behavior of the system (3.2), for the set of parameter values (7), different diagrams are presented to identify chaos.
Considering the initial condition $(x, y, z, w_0) = (-0.5, 0.1, 0.01, w_0)$, the Lyapunov exponents spectrum and the corresponding bifurcation diagram of $y$, for $w_0$ varying from $-15$ to 15 with step 0.01 are obtained as shown in Figure 3.4 and Figure 3.5, respectively. From these diagrams, one observes that when the value of initial state $w_0$ belongs to the following four intervals: $[-15, -11.91]$, $[-5.52, -0.9]$, $[0.9, 5.52]$, $[11.91, 15]$, then system (3.2) exhibits chaos. Furthermore, the two diagrams indicate symmetry versus $w_0 = 0$.

Particularly, for $w_0 = -1$ the Lyapunov exponents are [7]

$$L_1 = 0.1485,\ L_2 = 0.0420,\ L_3 = -0.0154,\ L_4 = -31.7725. \tag{3.12}$$

Since $L_1 + L_2 + L_3 + L_4 = -31.5975 < 0$, $L_1 > 0, L_2 > 0$, then the system (3.2) is hyperchaotic. The Kaplan-Yorke dimension of its attractor is

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3 + \frac{0.1485 + 0.0420 - 0.0154}{31.7725} = 3.0055, \tag{3.13}$$

which is a fractal dimension.
Some phase portraits are depicted in Figure 3.3 for different values of the initial condition $w_0$. In particular, a period-1 orbits are shown in 3.3(b), 3.3(e), and 3.3(h). Moreover, 3.3(c), 3.3(g) represents a stable equilibrium point, and 3.3(a), 3.3(d), 3.3(f) and 3.3(i) displays chaotic attractors.

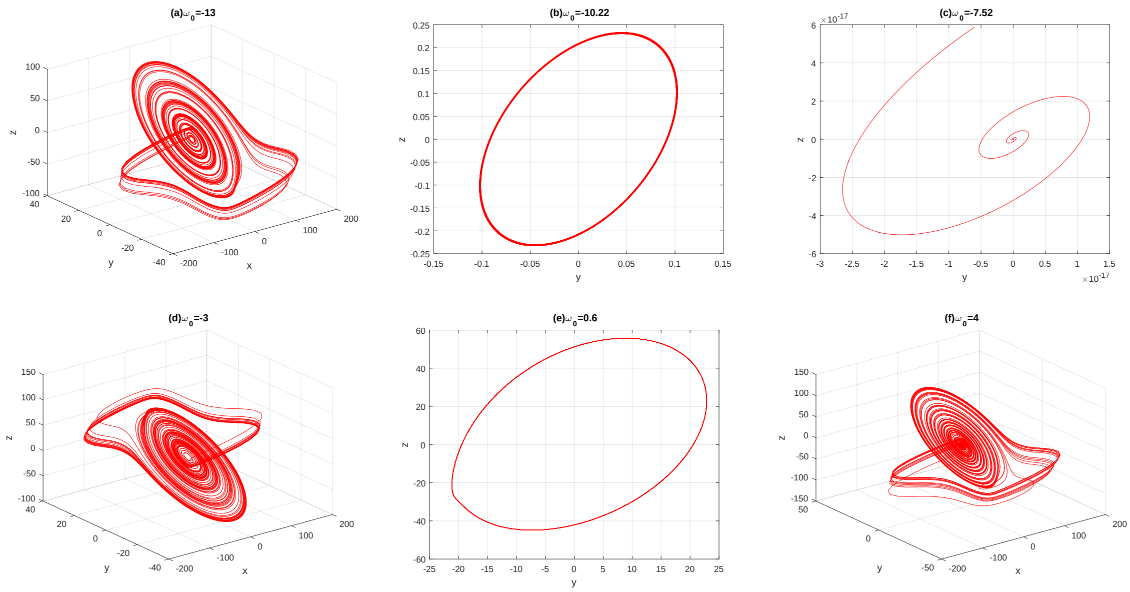


Figure 3.3: Some attractors for different values of initial condition $w_0$ : (a) $w_0 = -13$, (b) $w_0 = -10.22$, (c) $w_0 = -7.52$, (d) $w_0 = -3$, (e) $w_0 = 0.6$, (f) $w_0 = 4$, (g)

# 4 Modified projective synchronization between integer-order and incommensurate fractional order hyperchaotic systems

This section presents a theoretical analysis of the modified projective synchronization between integer-order and incommensurate fractional order hyperchaotic systems by applying the active control method based on the stability theorem of fractional-order linear systems.

## 4.1 Theoretical analysis

Giving two hyperchaotic systems: master and slave described respectively by :

$$\dot{X} = F(X), \tag{4.1}$$

$$D^{\alpha} Y = G(Y), \tag{4.2}$$

in order to make the study easier, (4.2) is rewritten as:

$$D^{\alpha} Y = AY + g(Y) + U, \tag{4.3}$$

where $X(t) = (x_1, x_2, ..., x_n)$, $Y(t) = (y_1, y_2, ..., y_n)$ are states of the master and the slave systems, respectively, $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ where $0 < \alpha_i < 1$ is the fractional-order, $A \in \mathbb{R}^{n \times n}$, $g$ are the linear part and the nonlinear part of the system (4.3), respectively, and $U = (u_1, u_2, ..., u_n)$ is a control input vector.
The error state is defined as:

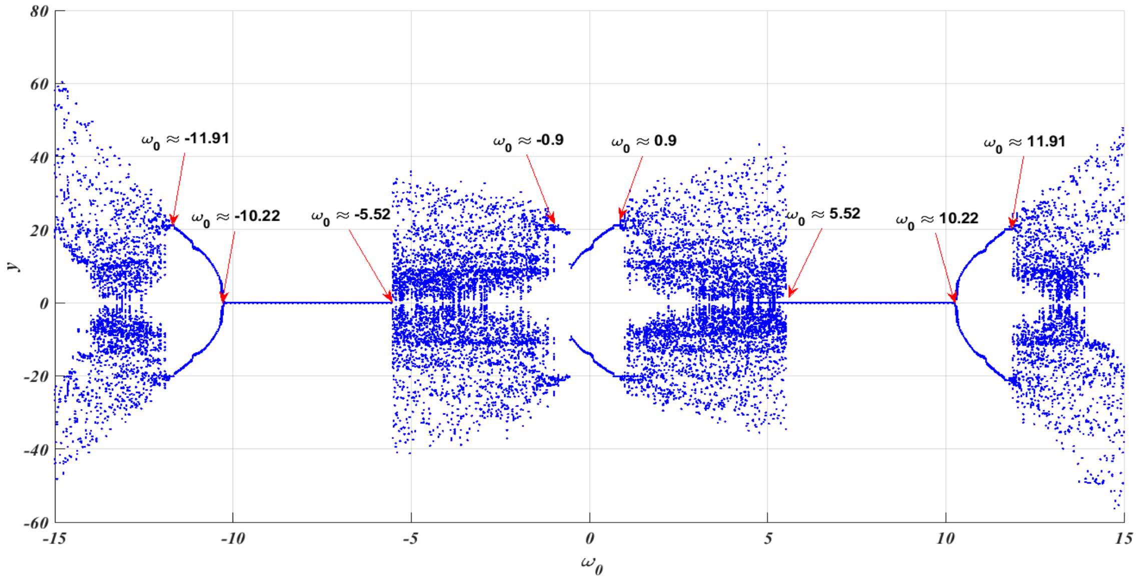


Figure 3.4: Bifurcation diagram with respect to the fourth coordinate $w_0$ of initial condition for $b = 1$

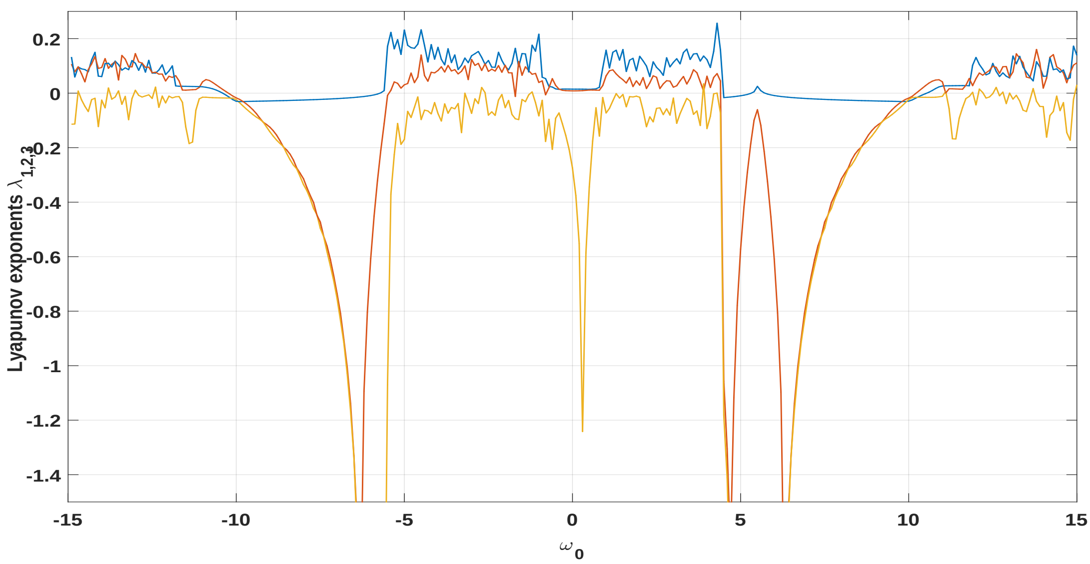


Figure 3.5: The three largest Lyapunov exponents of the system (3.2) versus the parameter $w_0$ for $b = 1$

$$e(t) = CY - X. \tag{4.4}$$

Where $C = diag(c_1, c_2, ..., c_n)$ denotes a scaling matrix. The objective of our work is to achieve synchronization between the two hyperchaotic systems (4.1) and (4.2) which could be achieved using the MPS technique when:

$$\lim_{t \to +\infty} e(t) = \lim_{t \to +\infty} \|CY(t) - X(t)\| = 0. \tag{4.5}$$

Hence the error system from equations (4.1) and (4.3) is as follows:

$$\begin{aligned} D^\alpha e &= CD^\alpha Y - D^\alpha X, && (4.6) \\ &= CAY + Cg(Y) + CU - D^\alpha X. && (4.7) \end{aligned}$$

In order to realize the MPS between integer order and incommensurate fractional order hyperchaotic systems, an active control $U$ is chosen whereas the error system (4.4) asymptotically converges to zero. To achieve the stability of the system, we take the active control $U = (u_1, u_2, ..., u_n)^T$, such that:

$$U = C^{-1}((A + M)e - CAY - Cg(Y) + D^\alpha X), \tag{4.8}$$

where $M \in \mathbb{R}^{n \times n}$ is a gain matrix to be determined.
Substituting (4.8) into (4.7)yields :

$$D^\alpha e \quad = \quad (A + M)e. \tag{4.9}$$

**Proposition 4.1.** *If the matrix $M$ is selected such that all roots $\lambda_i$ of the characteristic equation:*

$$det(diag([\lambda^{m\alpha_1}, \lambda^{m\alpha_2}, ..., \lambda^{m\alpha_n}]) - (A + M)) \quad = \quad 0,$$

*satisfy $|arg(\lambda_i)| > \frac{\pi}{2m}$, $i = 1, 2, ..., n$, where $m$ is the least common multiple of the denominators of $\alpha_i$, then the master system (4.1) and slave system (4.3) can be synchronized under the controller (4.8).*

*Proof.* Immediately, using **theorem** 2.2. □

### 4.2 Numerical example and simulation results

To confirm the theoretical results obtained in the above sections, we perform numerical simulation by adopting the novel hyperchaotic system as a master system and its incommensurate fractional order version as a slave system.
The master system is defined as

$$\begin{cases} \dot{x}_1 = \alpha[x_3 + \xi x_1 - (-a + b|\omega|)x_1], \\ \dot{x}_2 = x_2 - x_3, \\ \dot{x}_3 = -\beta(x_1 - x_2) - \gamma x_3, \\ \dot{x}_4 = x_1, \end{cases} \tag{4.10}$$

The slave system is expressed by

$$\begin{cases} D^{\alpha_1}y_1 = \alpha[y_3 + \xi y_1 - (-a + b|\omega|)y_1] + u_1, \\ D^{\alpha_2}y_2 = y_2 - y_3 + u_2, \\ D^{\alpha_3}y_3 = -\beta(y_1 - y_2) - \gamma y_3 + u_3, \\ D^{\alpha_4}y_4 = y_1 + u_4, \end{cases} \tag{4.11}$$

where $u_1, u_2, ..., u_4$ are the active control functions, and $\alpha$ is a rational number between 0 and 1. The linear part of the system (4.3) is given by

$$A = \begin{bmatrix} \alpha(a+\xi) & 0 & \alpha & 0 \\ 0 & 1 & -1 & 0 \\ -\beta & \beta & -\gamma & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{4.12}$$

The matrix $C$ is picked out in agreement with the MPS control technique proposed in equation (4.4) then

$$C = diag(5, 10, 0.1, 12), \tag{4.13}$$

and the gain matrix $M$ is chosen as

$$M = \begin{bmatrix} -\alpha\xi - 2\alpha a & 0 & 1-\alpha & 0 \\ 0 & -2 & 1 & 0 \\ \beta & -\beta & -\gamma & 0 \\ -1 & 0 & 0 & -1 \end{bmatrix} \tag{4.14}$$

With the values given in (4.8) and (4.14), the error system becomes

$$\begin{bmatrix} D^{\alpha_1}e_1 \\ D^{\alpha_2}e_2 \\ D^{\alpha_3}e_3 \\ D^{\alpha_4}e_4 \end{bmatrix} = \begin{bmatrix} -\alpha a & 0 & 1 & 0 \\ 0 & -1 & 1 & 0.11 \\ 0 & 0 & -\gamma & 0 \\ -1.5 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{4.15}$$

and the characteristic equation:

$$det(diag([\lambda^{m\alpha_1}, \lambda^{m\alpha_2}, \lambda^{m\alpha_3}, \lambda^{m\alpha_4}]) - (A + M)) = 0, \tag{4.16}$$

it can be transformed to:

$$(\lambda^{m\alpha_1} + 7.5)(\lambda^{m\alpha_2} + 1)(\lambda^{m\alpha_3} + 0.11)(\lambda^{m\alpha_4} + 1) = 0, \tag{4.17}$$

Where $m$ is the least common multiple of the denominators of $\alpha_i$, for $i = 1, 2, 3$ and 4, the master system (4.10) and the slave system (4.11) are synchronized if all roots $\lambda$ of (4.17) satisfy

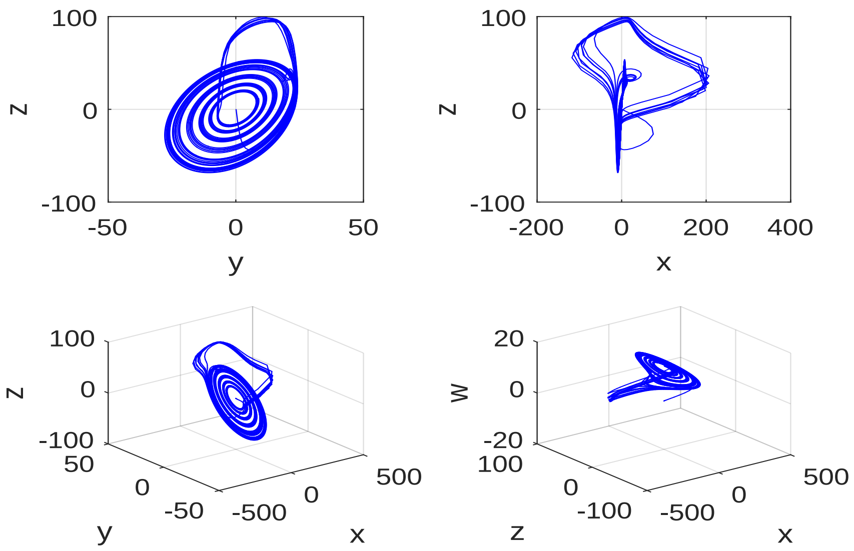


Figure 4.1: Some chaotic attractors of novel incommensurate fractional order system (4.11)

$|arg(\lambda_i)| > \frac{\pi}{2m}$.

Let us take $(\alpha, \beta, \xi, a, \gamma) = (5, 5, 3, 1.5, 0.11)$ and $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.95, 1, 1, 1)$, substituting in (4.17) yields:

$$(\lambda^{19} + 7.5)(\lambda^{20} + 1)(\lambda^{20} + 0.11)(\lambda^{20} + 1) \quad = \quad 0, \tag{4.18}$$

Obviously, all roots $\lambda_i$ of (4.18) must satisfy the condition $|arg(\lambda_i)| > \frac{\pi}{40}$, consequently the master system (4.10) and the slave system (4.11) are synchronized, under the controller (4.8).

Finally, for numerical simulation, the Adams method [16] is used to solve the systems with time step size $h = 0.02$, the error system has the initial values:

$$e_1(0) = 0.1, e_2(0) = 0.2, e_3(0) = 0.1, e_4(0) = -1.$$

The parameter values of the hyperchaotic systems are taken as in the hyperchaotic case (**??**) and the different fractional-orders are taken as:

$$(\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4) = (0.95, 1, 1, 1).$$

Figure 4.1 illustrates the attractors of the novel incommensurate fractional order system (4.11).
Figure 4.2 illustrates the synchronization errors between integer-order and incommensurate fractional order systems.
Figure 4.3 illustrates the error functions evolution (4.15).
From Figure 4.3, for the given parameters, numerical results clearly show that errors converge to zero, and so the MPS is effectively implemented under the controller (4.8).

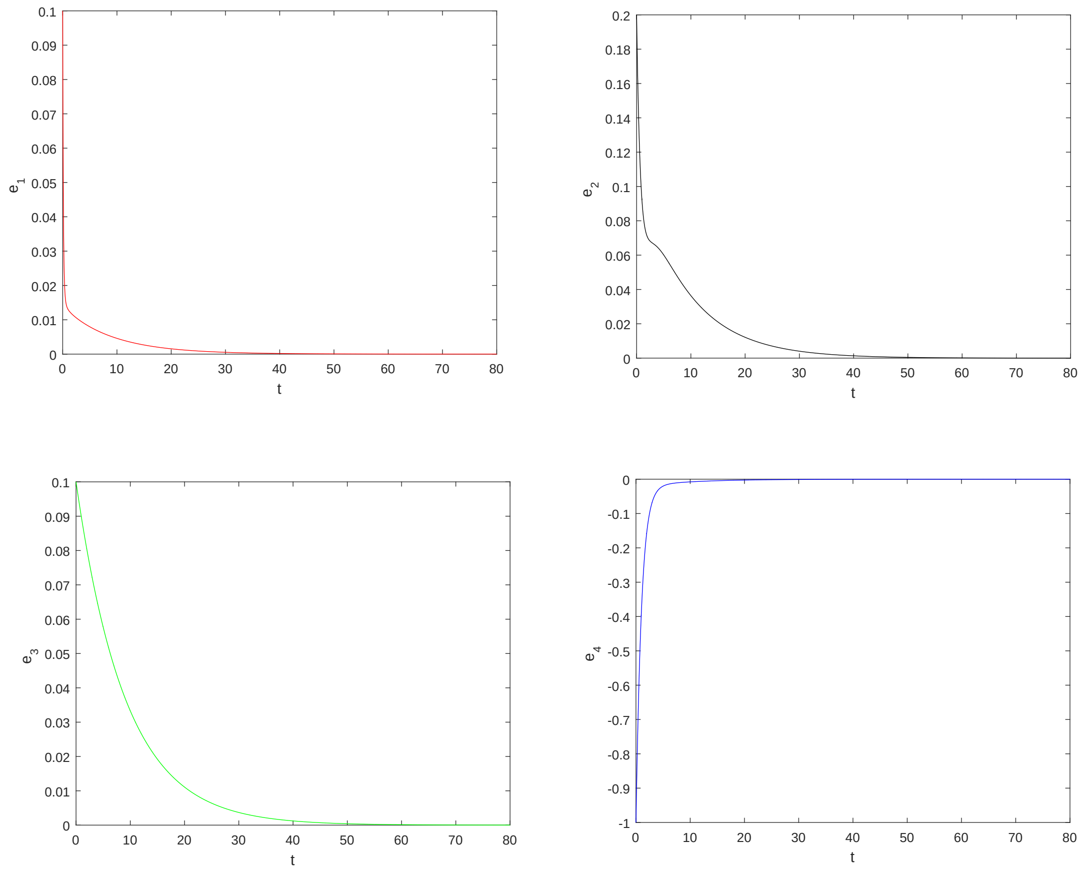


Figure 4.2: Synchronization errors between integer order and incommensurate fractional order systems

# 5 Conclusion

The synchronization between integer-order and fractional-order versions of a new memristor-based circuit with hyperchaotic dynamics was examined in this study. In order to derive the dynamical analysis, the stability theorems for fractional-order systems were applied, and the findings show that the variation of the fractional-order derivative significantly affects the proposed model's dynamical behavior. An MPS controller for synchronizing two hyperchaotic systems with integer and incommensurate fractional orders has been developed. Some numerical simulations have been provided to illustrate the theoretical results. We will use the proposed memristor-based hyperchaotic circuit for secure communication in the future by modulating the original signals into the chaotic sequences generated by the master circuit and transferring the combined signals to the receiver over a communication channel. Signals are received, and the MPS controller decodes them using the slave memristor-based circuit. Therefore, the relevant research is still in its early stages, and our next articles will discuss circuit implementations.

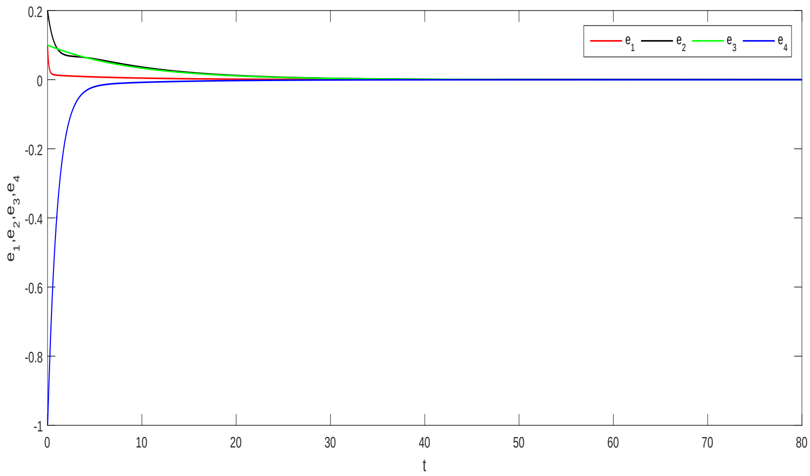


Figure 4.3: The synchronization errors of (4.10) and (4.11)

# Declarations

## Availability of data and materials

Data sharing not applicable to this article.

## Funding

Not applicable.

## Conflict of Interest

The authors have no conflicts of interest to declare.

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