Journal of Innovative Applied Mathematics and Computational Sciences

J. Innov. Appl. Math. Comput. Sci. **3**(2) (2023), 121–141. DOI: 10.58205/jiamcs.v3i2.60

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On numerical and analytical solutions of the generalized Burgers-Fisher equation

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Received 09 September 2023, Accepted 25 December 2023, Published 17 January 2024

Abstract. In this paper, the semi-analytic iterative method (SAIM) and modified simple equation method (MSEM) have been implemented to obtain solutions of the generalized Burgers-Fisher equation (GBFE). To demonstrate the accuracy, efficacy as well as reliability of the methods in finding the exact solution of the equation, a selection of numerical examples were given and a comparison was made with other well-known methods from the literature such as variational iteration method, homotopy perturbation method and diagonally implicit Runge-Kutta method. The results have shown that between the proposed methods, the MSEM is much faster, easier, more concise and straightforward for solving nonlinear partial differential equations as it does not require the use of any symbolic computation software such as Maple or Mathematica. Additionally, the iterative procedure of the SAIM has merit in that each solution is an improvement of the previous iterate and as more and more iterations are taken, the solution converges to the exact solution of the equation.

Keywords: Generalized Burgers-Fisher equation, Semi-analytic iterative method, Modified simple equation method.

2020 Mathematics Subject Classification: 35A15, 35C05, 47J25, 47J30 MSC2020

1 Introduction

The generalized Burgers-Fisher equation is a very important nonlinear partial differential equation which has found application in many areas of applied sciences such as heat conduction, fluid mechanics, elasticity, gas dynamics, plasma physics, and number theory. This equation has also been applied in various fields of physics and engineering, for instance, dispersion of pollutants in rivers, chemical kinetics, ion propagation in plasma, solid state physics, optical fibres, shock-wave formation and propagation, traffic flow, turbulence, financial mathematics, sound wave in viscous medium, and in some other applications [4].

The Burgers-Fisher equation in its generalized form arises from a fusion of the Burgers' and Fisher's equations and is therefore of great importance for describing different mechanisms in applied sciences. However, this equation is a prototypical model for describing the interaction between reaction mechanisms, convection effects and diffusion transports [1].

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ISSN (electronic): 2773-4196

The aim of this paper is to obtain solutions of the GBFE using two methods. The first is the semi-analytic iterative method (SAIM) first proposed by Temimi and Ansari [20, 21] and the second is the modified simple equation method (MSEM), applied, for example, in the solution of Burgers, Huxley and Burgers-Huxley equations [5] and nonlinear evolution equations [9, 19]. We also compare our results from these two methods with those from other well-known methods in the literature: the variational iteration method [18], homotopy perturbation method [16, 18] and diagonally implicit Runge-Kutta method [14].

Several other numerical and analytical methods have been used in the literature to solve the GBFE. For example, Mohammadi [13] applied a numerical method based on exponential spline and finite difference approximations in the solution of the GBFE and studied error analysis, stability and convergence properties of the method. Mendoza and Muriel [12] obtained new travelling wave solutions to the GBFE using a method that exploits the existence of a λ-symmetry for the class of equations that can be linearised by a generalized Sundman transformation. Kumar and Saha Ray [10] used the discontinuous Legendre wavelet Galerkin method to obtain the numerical solution of the Burgers-Fisher and generalized Burgers-Fisher equations. Zhong et al. [22] applied the modified high-order Haar wavelet scheme with the third-order Runge-Kutta method to the solution of the GBFE and generalized Burgers-Huxley equation. This method was found to have improved the speed of convergence while ensuring stability. And Ramya et al. [15] deployed the Exp Function (EF) and Exponential Rational Function (ERF) methods to investigate the analytical solutions of the time-fractional GBFE by means of a conformable operator

$$u_t + uu_x + \gamma u_{xx} = 0,$$

is a fundamental nonlinear partial differential equation appearing in various areas of mathematical physics such as those mentioned earlier. The Fisher's equation

$$u_t + \gamma u_{xx} = \beta u(1-u),$$

is a reaction-diffusion equation first proposed by Fisher in the context of population dynamics. In this paper we consider the generalized Burgers-Fisher equation of the form:

$$u_t + \alpha u^\delta u_x + \gamma u_{xx} = \beta u(1-u^\delta),\ 0 \le x \le 1,\ t \ge 0, \tag{1.1}$$

subject to the boundary and initial conditions below, where α, β, γ and δ are arbitrary constants in $\mathbb{R}$ with $\alpha \neq 0$ [12] and $\beta \ge 0$, $\delta > 0$ [7]. Here, the boundary and initial conditions are, respectively,

$$u(0,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{\alpha\delta}{2(\delta+1)}\left(\frac{\alpha}{\delta+1} + \frac{\beta(\delta+1)}{\alpha}\right)t\right)\right]^{\frac{1}{\delta}} = g_1(t), \quad t \ge 0,$$

$$u(1,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(1 - \left(\frac{\alpha}{\delta+1} + \frac{\beta(\delta+1)}{\alpha}\right)t\right)\right)\right]^{\frac{1}{\delta}} = g_2(t), \quad t \ge 0,$$

and

$$u(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}} = g(x). \tag{1.2}$$

The exact solution of Eq. (1.1) when $\gamma = -1$ is of the form:

$$u(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(x - \left(\frac{\alpha}{\delta+1} + \frac{\beta(\delta+1)}{\alpha}\right)t\right)\right)\right]^{\frac{1}{\delta}}. \tag{1.3}$$

The rest of the paper is structured as follows: The proposed methods of solution are reviewed in Section 2, while Section 3 gives illustrative numerical examples. Finally, Section 4 concludes the paper.

2 Review of the Proposed Methods

2.1 Preliminary Concepts of SAIM

The SAIM was proposed and it uses an iterative approach to provide solution to the nonlinear generalized Burgers-Fisher equation. To explain the basic idea of the SAIM, let us consider the general equation

$$L[u(x,t)] + N[u(x,t)] = G(x,t), \tag{2.1}$$

with boundary conditions

$$C\left(u, \frac{\partial u}{\partial t}\right) = 0,$$

where u is the unknown function, L the linear operator, N the nonlinear operator, G is a source term which is a known function of the independent variables x and t and C the boundary operator. The initial approximation is a primary step in the SAIM. Therefore, assuming that the initial guess $u_0(x,t)$ is a solution of the problem, the solution of the equation can be obtained by solving

$$L[u_0(x,t)] = 0 \text{ with } C\left(u_0, \frac{\partial u_0}{\partial t}\right) = 0,$$

from which $u_0(x,t) = u(x,0) = g(x)$ (see Eq. (1.2) above). To find the next iteration, we solve the equation

$$L[u_1(x,t)] = -N[u_0(x,t)] + G(x,t) \text{ with } C\left(u_1, \frac{\partial u_1}{\partial t}\right) = 0.$$

Similarly, the third iteration involves solving the equation

$$L[u_2(x,t)] = -N[u_1(x,t)] + G(x,t) \text{ with } C\left(u_2, \frac{\partial u_2}{\partial t}\right) = 0,$$

and so on. Thus, after several iterations, we obtain the general form of the problem to be solved using the SAIM as

$$L[u_{n+1}(x,t)] = -N[u_n(x,t)] + G(x,t) \text{ with } C\left(u_{n+1}, \frac{\partial u_{n+1}}{\partial t}\right) = 0,$$

which gives the general iterative relation for solving Eq. (2.1) as

$$u_{n+1}(x,t) = u_{n+1}(x,0) + L^{-1}\left\{-N\left[u_n(x,t) + G(x,t)\right]\right\},$$

where $L^{-1} = \int_0^t (\cdot)ds$. It is important to note that each of the $u_n(x,t)$ are standalone solutions to Eq. (2.1) [2]. We therefore believe that this iterative procedure has merit in that each solution is an improvement of the previous iterate and as more and more iterations are taken, the solution converges to the solution of Eq. (2.1). Details of the SAIM can be found in Latif et al. [11], Ibrahim et al. [6], Kasumo [8] and Selamat et al. [17].

2.2 Application of SAIM to the GBFE

Consider the generalized Burgers-Fisher equation

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1 - u^\delta), \tag{2.2}$$

subject to the initial condition

$$u(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}}.$$

The equation (2.2) can be rearranged as:

$$u_t - u_{xx} + \alpha u^\delta u_x - \beta u(1 - u^\delta) = 0, \tag{2.3}$$

where α, β and δ are constants with $\alpha \neq 0$, $\beta \geq 0$, $\delta > 0$. By applying the SAIM to (2.3) after comparing with (2.1), we have

$$Lu + Nu = 0,$$

with $Lu = u_t$, $Nu = -u_{xx} + \alpha u^\delta u_x - \beta u(1 - u^\delta)$, $G(x,t) = 0$. Thus, the primary problem that needs to be solved, together with its associated initial condition, is

$$L(u_0) = 0, \text{ with } u_0(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}},$$

where

$$\int_0^t u_{0_s}(x,s)ds = 0.$$

Thus, we obtain

$$u_0(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}}.$$

For the second iteration, we need to solve the equation

$$L[u_1(x,t)] = -N[u_0(x,t)] + G(x,t) \text{ with } u_1(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}},$$

after rearranging Eq. (2.2) as

$$u_t = u_{xx} - \alpha u^\delta u_x + \beta u(1 - u^\delta), \tag{2.4}$$

where

$$\int_0^t u_{1_s}(x,s)ds = \int_0^t \left[(u_0)_{xx} - \alpha(u_0)^\delta(u_0)_x + \beta u_0(1 - u_0^\delta)\right] ds.$$

For the third iteration, we solve Eq. (2.4) by solving the equation

$$L[u_2(x,t)] + N[u_1(x,t)] + G(x,t) = 0 \text{ with } u_2(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}},$$

where

$$\int_0^t u_{2_s}(x,s)ds = \int_0^t \left[(u_1)_{xx} - \alpha(u_1)^\delta(u_1)_x + \beta u_1(1 - u_1^\delta)\right] ds,$$

and so on. Thus, by the same iterative steps, the other solutions can be generated from the general iterative formula

$$L[u_{n+1}(x,t)] + N[u_n(x,t)] + G(x,t) = 0 \text{ with } u_{n+1}(x,0) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right]^{\frac{1}{\delta}},$$

where

$$\int_0^t u_{(n+1)_s}(x,s)ds = \int_0^t \left[(u_n)_{xx} - \alpha(u_n)^\delta(u_n)_x + \beta u_n(1-u_n^\delta)\right] ds.$$

2.3 Preliminary Concepts of MSEM

Suppose the nonlinear partial differential equation to be considered is of the form

$$P(u,\ u_t,\ u_x,\ u_{tt},\ u_{xx}, \ldots) = 0, \tag{2.5}$$

where P is a polynomial in $u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. The main steps of the MSEM are outlined in Ayati et al. [5] and are as follows :

Step 1. The travelling wave transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct,$$

where c is the speed of the travelling wave, permits us to reduce Eq. (2.5) into the following ordinary differential equation:

$$F(u,\ u',\ u'', \ldots) = 0, \tag{2.6}$$

where F is a polynomial in $u(\xi)$ and its total derivatives, wherein $u'(\xi) = \dfrac{du}{d\xi}$ [9]. Hence, we use the following changes [19]:

$$\frac{\partial}{\partial t}(\cdot) = -c\frac{\partial}{\partial \xi}(\cdot), \quad \frac{\partial}{\partial x}(\cdot) = \frac{\partial}{\partial \xi}(\cdot), \quad \frac{\partial^2}{\partial x^2}(\cdot) = \frac{\partial^2}{\partial \xi^2}(\cdot), \quad \ldots \tag{2.7}$$

for other derivatives.

Step 2. We suppose that the exact solution of Eq. (2.6) is of the form

$$u(\xi) = \sum_{k=0}^{N} A_k \left[\frac{\Phi'(\xi)}{\Phi(\xi)}\right]^k, \tag{2.8}$$

where A_k $(k = 0,\ 1,\ 2,\ 3, \ldots,\ N)$ are arbitrary constants to be determined, such that $A_N \neq 0$ and $\Phi(\xi)$ is an unknown function to be determined later such that $\Phi'(\xi) \neq 0$.

Step 3. We determine the positive integer N in Eq. (2.8) by considering the homogeneous balance between the highest order derivatives or linear terms and the highest order nonlinear terms occurring in Eq. (2.6).

Step 4. Inserting Eq. (2.8) into Eq. (2.6) and computing the necessary derivatives $u',\ u'', \ldots,$ we then account for the function $\Phi(\xi)$. As a result of this substitution, we get a polynomial in

$\frac{\Phi'(\xi)}{\Phi(\xi)}$ and its derivatives. Equating all the coefficients of like power of this polynomial to zero, we obtain a system of equations which can be solved to find A_k $(k = 0, 1, 2, 3, \ldots)$ and $\Phi(\xi)$. Consequently, we can get the exact solution of Eq. (2.5).

2.4 Application of MSEM to the GBFE

In this section, the modified simple equation method was applied to find the exact solutions and then the solitary wave solutions of the generalized Burgers-Fisher equation (GBFE) of the form

$$u_t + \alpha u^\delta u_x - u_{xx} = \beta u(1 - u^\delta). \tag{2.9}$$

Rearranging Eq. (2.9) yields

$$u_t - u_{xx} + \alpha u^\delta u_x - \beta u(1 - u^\delta) = 0, \tag{2.10}$$

where α, δ and β are nonzero constants. Using the travelling wave transformation,

$$u(x,t) = u(\xi), \quad \xi = x - ct,$$

where c is constant, and substituting (2.7) into (2.10), we obtain an ODE

$$-cu' - u'' + \alpha u^\delta u' - \beta u(1 - u^\delta) = 0, \tag{2.11}$$

where in Eq. (2.11) $u \Rightarrow u(\xi)$, and integrating (2.11) with respect to ξ gives

$$-cu - u' + \alpha u^\delta u + \beta(u^\delta - 1) = 0. \tag{2.12}$$

Balancing the highest-order derivative u' and the nonlinear term u^δ, where δ is a positive integer ($\delta \geq 2$). The solution of Eq. (2.9) takes the form

$$u(\xi) = \sum_{k=0}^{N} A_k \left[\frac{\Phi'(\xi)}{\Phi(\xi)}\right]^k.$$

Thus,

$$u(\xi) = A_0 + A_1\left(\frac{\Phi'}{\Phi}\right) + \cdots + A_N\left(\frac{\Phi'}{\Phi}\right). \tag{2.13}$$

where A_k $(k = 0, 1, \ldots, N)$ are arbitrary constants in $\mathbb{R}$ such that $A_N \neq 0$, and $\Phi(\xi)$ is an unknown function to be determined later. Therefore, the needful computations for Eq. (2.12) are as follows:

$$u'(\xi) = A_1\left[\frac{\Phi''}{\Phi} - \left(\frac{\Phi'}{\Phi}\right)^2\right],$$

$$u''(\xi) = A_1\left[\frac{\Phi'''}{\Phi} - 3\frac{\Phi'\Phi''}{\Phi^2} + 2\left(\frac{\Phi'}{\Phi}\right)^3\right].$$

Substituting Eq. (2.13) and u' into Eq. (2.12) yields a polynomial in $\frac{1}{\Phi^j}$ $(j = 0, 1, 2, \ldots)$ and equating the coefficients of Φ^0, Φ^{-1}, Φ^{-2}, $\Phi^{-3}, \ldots$ to zero, gives the values of $A_0, A_1, \ldots, A_N$ and an ODE of the form $F(u, u', u'', \ldots) = 0$. Thus,

$$-cA_0 + \frac{\alpha}{2}A_0^2 - \beta A_0\left(1 - A_0^\delta\right) = 0,$$

and we get

$$u(\xi) = A_0 \left(\frac{\Phi'}{\Phi}\right)^0 + A_1 \left(\frac{\Phi'}{\Phi}\right)^1 + \cdots + A_N \left(\frac{\Phi'}{\Phi}\right)^N.$$

Hence, the exact solution of Eq. (2.10) is obtained by $u(\xi) \mapsto u(x,t)$ as

$$u(x,t) = A_0 + A_1 \left(\frac{\Phi'}{\Phi}\right) + \cdots + A_N \left(\frac{\Phi'}{\Phi}\right)^N = \sum_{k=0}^{N} A_k \left(\frac{\Phi'}{\Phi}\right)^k. \tag{2.14}$$

This is the proof of the formula in (2.8).

3 Numerical Examples

In this section we study a selection of examples illustrating the applicability of both the SAIM and MSEM for solving a nonlinear generalized Burgers-Fisher equation with parameters in $\mathbb{R}$. All the computations associated with these examples were performed using an HP 250 G5 Notebook PC with Intel Celeron CPU N3060 at 1.6 GHz with 2 GB internal memory and 64 bit operating system (Windows 10 Pro, Version 22H2). All the figures in this section were constructed using MATLAB R2023a. The results are presented in tables and figures accompanying the discussion.

Example 3.1. In order to ascertain the capability and reliability of the proposed methods, we consider the Burgers-Fisher equation (2.2) for $\delta = 1$, $\alpha = -1$ and $\beta = 2$ [18], that is,

$$u_t = u_{xx} + uu_x + 2u - 2u^2, \tag{3.1}$$

subject to the initial condition

$$u(x,0) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}} + e^{-\frac{x}{4}}}.$$

SAIM Scheme. By applying the SAIM to (3.1) after comparing with (2.1), we have

$$Lu + Nu = 0,$$

with $Lu = u_t$, $Nu = -u_{xx} - uu_x - 2u + 2u^2$ and $G(x,t) = 0$. Thus, the primary problem to be solved, with its initial condition, is

$$L(u_0) = 0 \text{ with } u_0(x,0) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}} + e^{-\frac{x}{4}}},$$

where

$$\int_0^t u_{0_s}(x,s)ds = 0.$$

Thus, we obtained

$$u_0(x,t) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}} + e^{-\frac{x}{4}}}.$$

For the second iteration, we solve the following equation

$$L[u_1(x,t)] + N[u_0(x,t)] + G(x,t) = 0 \text{ with } u_1(x,0) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}} + e^{-\frac{x}{4}}}.$$

After considering Eq. (3.1) with

$$\int_0^t u_{1_t}(x,s)ds = \int_0^t \left[(u_0)_{xx} - (u_0)(u_0)_x + u_0 - (u_0)^2\right] ds,$$

that is,

$$\int_0^t u_{1_t}(x,s)ds = \int_0^t \left[\left(\frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}}\right)_{xx} - \left(\frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}}\right)\left(\frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}}\right)_x + \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}} - \left(\frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}}\right)^2\right] ds,$$

we obtained

$$u_1(x,t) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}} - \left[\frac{2e^{\frac{x}{2}}}{\left(e^{\frac{x}{4}}+e^{-\frac{x}{4}}\right)^2} - \frac{3e^{\frac{x}{4}}}{\left(e^{\frac{x}{4}}+e^{-\frac{x}{4}}\right)}\right] t. \tag{3.2}$$

Simplifying (3.2), we have

$$u_1(x,t) = \frac{e^{\frac{x}{2}}\left(1+3t+e^{\frac{x}{2}}+te^{\frac{x}{2}}\right)}{\left(e^{\frac{x}{2}}+1\right)^2}.$$

The next iteration to be solved is

$$L[u_2(x,t)] + N[u_1(x,t)] + G(x,t) = 0 \text{ with } u_2(x,0) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}},$$

where

$$\int_0^t u_{2_s}(x,s)ds = \int_0^t \left[(u_1)_{xx} - (u_1)(u_1)_x + u_1 - (u_1)^2\right] ds.$$

From this we obtain

$$u_2(x,t) = \frac{e^{\frac{x}{2}}}{e^{\frac{x}{2}}+1} + \frac{\left(3+e^{\frac{x}{2}}\right)te^{\frac{x}{2}}\left(9t+12e^{\frac{x}{2}}+6e^x+6te^{\frac{x}{2}}-4t^2e^x-12t^2e^{\frac{x}{2}}-3te^x+6\right)}{6\left(e^{\frac{x}{2}}+1\right)^4}.$$

By the same iterative steps, the other solutions can be obtained from solving these problems in the general form

$$L[u_{n+1}(x,t)] + N[u_n(x,t)] + G(x,t) = 0 \text{ with } u_{n+1}(x,0) = \frac{e^{\frac{x}{4}}}{e^{\frac{x}{4}}+e^{-\frac{x}{4}}},$$

where

$$\int_0^t u_{(n+1)_s}(x,s)ds = \int_0^t \left[(u_n)_{xx} - (u_n)(u_n)_x + u_n - (u_n)^2\right] ds,$$

thus achieving the following approximate solution in a series form [3]:

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) = \sum_{n=0}^{\infty} v_n.$$

This series converges to the exact solution

$$u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{1}{4}\left(x+\frac{9t}{2}\right)\right). \tag{3.3}$$

MSEM Scheme. Rearranging Eq. (3.1) gives

$$u_t - u_{xx} - uu_x - 2u + 2u^2 = 0. \tag{3.4}$$

Using the travelling wave transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct,$$

and considering Eq. (2.7), Eq. (3.4) becomes

$$-cu' - u'' - uu' - 2u + 2u^2 = 0, \tag{3.5}$$

where in Eq. (3.5), $u \Rightarrow u(\xi)$ and by integration, Eq. (3.5), gives

$$-cu - u' - \frac{u^2}{2} - 2u + 2u^2 = 0, \tag{3.6}$$

or

$$-cu - \frac{3}{2}u^2 - 2u = u'. \tag{3.7}$$

Balancing the highest-order derivative u' and the nonlinear term u^2 in Eq. (3.7) $(N+1 = 2N)$, we obtain $N = 1$. Therefore, the solution of Eq. (3.1) takes the form

$$u(\xi) = A_0 + A_1 \left(\frac{\Phi'}{\Phi}\right), \tag{3.8}$$

where A_0 and A_1 are arbitrary constants in $\mathbb{R}$ such that $A_1 \neq 0$, and $\Phi(\xi)$ is an unknown function to be determined later. Now, it can be seen that

$$u'(\xi) = A_1 \left[\frac{\Phi''}{\Phi} - \left(\frac{\Phi'}{\Phi}\right)^2\right], \tag{3.9}$$

$$u''(\xi) = A_1 \left[\frac{\Phi'''}{\Phi} - 3\frac{\Phi'\Phi''}{\Phi^2} + 2\left(\frac{\Phi'}{\Phi}\right)^3\right]. \tag{3.10}$$

Substituting Eq. (3.8) and Eq. (3.9) into Eq. (3.7) yields a polynomial and equating coefficients of Φ^{-j} $(j = 0,\ 1,\ 2)$ to zero, we obtain

$$-cA_0 - \frac{3}{2}A_0^2 - 2A_0 = 0, \tag{3.11}$$

as shown in [5]. That is,

$$(-c - 3A_0 - 2)\Phi' - \Phi'' = 0, \tag{3.12}$$

and

$$\left(-\frac{3}{2}A_1^2 + A_1\right)(\Phi')^2 = 0. \tag{3.13}$$

By solving Eqs. (3.11) and (3.13) the following results are obtained:

$$A_0 = 0, \ -\tfrac{2}{3}(c+2) \text{ and } A_1 = \tfrac{2}{3}.$$

Case 1. When $A_0 = 0$, Eq. (3.12) becomes

$$(-c-2)\Phi' - \Phi'' = 0.$$

So

$$\Phi' = Ae^{-(c+2)\xi}, \tag{3.14}$$

where A is an arbitrary constant in $\mathbb{R}$. Integrating (3.14) with respect to ξ, $\Phi(\xi)$ will be obtained as follows:

$$\Phi = -\frac{A}{c+2}e^{-(c+2)\xi} + B,$$

where B is a constant of integration. Now the exact solution of Eq. (3.4) has the form

$$u_1(x,t) = \frac{\frac{2}{3}Ae^{-(c+2)(x-ct)}}{-\frac{A}{c+2}e^{-(c+2)(x-ct)} + B},$$

where $c \neq -2$ and $B \neq 0$.

Case 2. When $A_0 = -\frac{2}{3}(c+2)$, Eq. (3.12) reduces to

$$(c+2)\Phi' - \Phi'' = 0.$$

So

$$\Phi' = Ae^{(c+2)\xi},$$

and, by integration,

$$\Phi = \frac{A}{c+2}e^{(c+2)\xi} + B.$$

Now, the exact solution of Eq. (3.4) has the form

$$u_2(x,t) = \frac{-\frac{2}{3}(c+2)B}{\frac{A}{c+2}e^{(c+2)(x-ct)} + B},$$

or

$$u_2(x,t) = \frac{-\frac{2}{3}(c+2)^2B}{Ae^{(c+2)(x-ct)} + (c+2)B},$$

where $c \neq -2$ and $A, B \neq 0$. Thus,

$$u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{1}{4}\left(x + \frac{9t}{2}\right)\right). \tag{3.15}$$

Results for Example 3.1. Here, we validate the authenticity and efficacy of both SAIM and MSEM in providing solutions to the generalized Burgers-Fisher equation by comparing the exact solution with the solutions obtained from both the SAIM and the MSEM. Furthermore, we have compared solutions of SAIM and MSEM for $\delta = 1$, $\alpha = -1$ and $\beta = 2$. The exact solution of the generalized Burgers-Fisher equation in Eq. (2.2) is given by

$$u(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(x - \left(\frac{\alpha}{\delta+1} + \frac{\beta(\delta+1)}{\alpha}\right)t\right)\right)\right]^{\frac{1}{\delta}}. \tag{3.16}$$

Setting $\delta = 1$, $\alpha = -1$ and $\beta = 2$ gives the exact solution in (3.15).

Comparison of exact solution and approximate solutions from SAIM, MSEM, VIM and HPM. We present the exact and approximate solutions of the generalized Burgers-Fisher equation for $0 \leq x \leq 1$ and $t = 0.1$. The results are shown in Table 3.1 and Figures 3.1 and 3.2 from which it can be seen that SAIM and MSEM both produce exact solutions also obtained using VIM [18] and HPM [16].

Table 3.1: Comparison of exact solution and approximate solutions from SAIM, MSEM, VIM and HPM for Example 3.1 ($\delta = 1$, $\alpha = -1$ and $\beta = 2$)

x	$u_{\text{Exact}}(x,t)$	$u_{\text{SAIM}}(x,t)$	$u_{\text{MSEM}}(x,t)$	$u_{\text{VIM}}(x,t)$	$u_{\text{HPM}}(x,t)$
0	0.556013890544	0.556013890544	0.556013890544	0.556013890544	0.556013890544
0.1	0.568319983478	0.568319983478	0.568319983478	0.568319983478	0.568319983478
0.2	0.580542304820	0.580542304820	0.580542304820	0.580542304820	0.580542304820
0.3	0.592666599954	0.592666599954	0.592666599954	0.592666599954	0.592666599954
0.4	0.604679084714	0.604679084714	0.604679084714	0.604679084714	0.604679084714
0.5	0.616566504521	0.616566504521	0.616566504521	0.616566504521	0.616566504521
0.6	0.628316188295	0.628316188295	0.628316188295	0.628316188295	0.628316188295
0.7	0.639916096738	0.639916096738	0.639916096738	0.639916096738	0.639916096738
0.8	0.651354864666	0.651354864666	0.651354864666	0.651354864666	0.651354864666
0.9	0.662621837170	0.662621837170	0.662621837170	0.662621837170	0.662621837170
1	0.673707099455	0.673707099455	0.673707099455	0.673707099455	0.673707099455

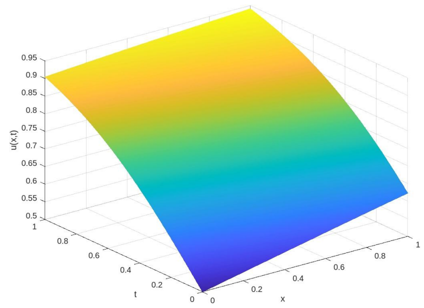

Figure 3.1: Surface plot for Example 3.1

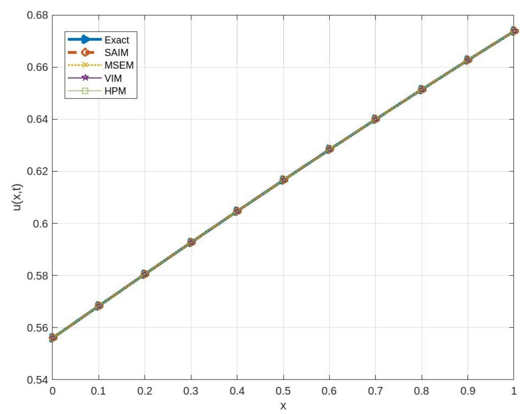

Figure 3.2: Comparison of exact solution and approximate solutions from SAIM, MSEM, VIM and HPM for $0 \leq x \leq 1$ for a fixed $t = 0.1$

Example 3.2. We consider the GBFE (2.2) for $\delta = 1$, $\alpha = 1$ and $\beta = 1$ as shown below [14]:

$$u_t = u_{xx} - uu_x + u - u^2, \tag{3.17}$$

subject to the initial condition

$$u(x,0) = \frac{e^{-\frac{x}{2}}}{e^{-\frac{x}{2}} + 1}.$$

SAIM Scheme. By applying the SAIM to Eq. (3.17), after comparing with (2.1), we have $Lu = u_t$, $Nu = -u_{xx} + uu_x - u + u^2$, $G(x,t) = 0$. Thus, the primary problem that needs to be solved, together with its initial condition, is

$$L(u_0) = 0 \text{ with } u_0(x,0) = \frac{e^{-\frac{x}{2}}}{e^{-\frac{x}{2}} + 1},$$

where

$$\int_0^t u_{0_t}(x,t)ds = 0,$$

from which we obtained

$$u_0(x,t) = \frac{e^{-\frac{x}{2}}}{e^{-\frac{x}{2}}+1}.$$

Applying the SAIM idea successively, we obtain the following:

$$u_1(x,t) = \frac{e^{-\frac{x}{2}}+(t+1)e^{-x}}{(e^{-\frac{x}{2}}+1)^2},$$

$$u_2(x,t) = \frac{e^{-\frac{x}{2}}+(t+1)e^{-x}}{(e^{-\frac{x}{2}}+1)^2} + \frac{t(3t+6e^{\frac{x}{2}}+3e^{x}+3te^{\frac{x}{2}}+t^2+3)}{3(e^{\frac{x}{2}}+1)^4},$$

and so on. Thus, the series converges to the exact solution

$$u(x,t) = \frac{1}{2}+\frac{1}{2}\tanh\left(\frac{5t}{8}-\frac{x}{4}\right). \tag{3.18}$$

MSEM Scheme. Eq. (3.17) can be rewritten as

$$u_t - u_{xx} + uu_x - u + u^2 = 0. \tag{3.19}$$

We use the travelling wave transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct.$$

Considering Eq. (2.7), Eq. (3.19) becomes

$$-cu' - u'' + uu' - u + u^2 = 0, \tag{3.20}$$

where in Eq. (3.20), $u \Rightarrow u(\xi)$ and by integration, Eq. (3.20), gives

$$-cu - u' + \frac{u^2}{2} - u + u^2 = 0,$$

or

$$-cu + \frac{3}{2}u^2 - u = u'. \tag{3.21}$$

Balancing the highest-order derivative u' and the nonlinear term u^2 in Eq. (3.21) $(N+1=2N)$, we obtain $N=1$. Therefore, the solution of Eq. (3.19) takes the following form

$$u(\xi) = A_0 + A_1\left(\frac{\Phi'}{\Phi}\right), \tag{3.22}$$

where A_0 and A_1 are arbitrary constants in $\mathbb{R}$ such that $A_1 \neq 0$, and $\Phi(\xi)$ is an unknown function to be determined later. Substituting Eq. (3.22) and Eq. (3.9) into Eq. (3.21) yields a polynomial and equating coefficients of Φ^{-j} $(j = 0,\ 1,\ 2)$ to zero, we obtain

$$-cA_0 + \frac{3}{2}A_0^2 - A_0 = 0, \tag{3.23}$$

as shown by [5]. That is,

$$(-c+3A_0-1)\Phi' - \Phi'' = 0, \tag{3.24}$$

and

$$\left(\frac{3}{2}A_1^2 + A_1\right)(\Phi')^2 = 0. \tag{3.25}$$

By solving Eqs. (3.23) and (3.25) we obtain the following results:

$$A_0 = 0,\ \tfrac{2}{3}(c+1) \text{ and } A_1 = -\tfrac{2}{3}.$$

Case 1. When $A_0 = 0$, Eq. (3.23) becomes

$$(-c-1)\Phi' - \Phi'' = 0.$$

So

$$\Phi' = Ae^{-(c+1)\xi}, \tag{3.26}$$

where A is an arbitrary constant in $\mathbb{R}$. Integrating (3.26) with respect to ξ, $\Phi(\xi)$ will be obtained as follows

$$\Phi = -\frac{A}{c+1}e^{-(c+1)\xi} + B,$$

where B is a constant of integration. Now the exact solution of Eq. (3.19) has the form

$$u_1(x,t) = \frac{-\frac{2}{3}Ae^{-(c+1)(x-ct)}}{-\frac{A}{c+1}e^{-(c+1)(x-ct)} + B},$$

where $c \neq -1$ and $B \neq 0$.

Case 2. When $A_0 = \frac{2}{3}(c+1)$, Eq. (3.24) becomes

$$(c+1)\Phi' - \Phi'' = 0.$$

So

$$\Phi' = Ae^{(c+1)\xi},$$

and, by integration,

$$\Phi = \frac{A}{c+1}e^{(c+1)\xi} + B.$$

Now, the exact solution of Eq. (3.19) has the form

$$u_2(x,t) = \frac{\frac{2}{3}(c+1)B}{\frac{A}{c+1}e^{(c+1)(x-ct)} + B},$$

or

$$u_2(x,t) = \frac{\frac{2}{3}(c+1)^2B}{Ae^{(c+1)(x-ct)} + (c+1)B},$$

where $c \neq -1$ and $A, B \neq 0$. Thus,

$$u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{5t}{8} - \frac{x}{4}\right). \tag{3.27}$$

Results for Example 3.2. Here, we validate the authenticity and efficacy of both the SAIM and the MSEM in providing solutions to the generalized Burgers-Fisher equation by comparing the exact solution with the solutions from both methods. Furthermore, we have compared solutions from the SAIM and MSEM for $\delta = 1$, $\alpha = 1$ and $\beta = 1$. The exact solution of the generalized Burgers-Fisher equation in Eq. (2.2) is given by

$$u(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(x - \left(\frac{\alpha}{\delta+1} + \frac{\beta(\delta+1)}{\alpha}\right)t\right)\right)\right]^{\frac{1}{\delta}}.$$

Table 3.2: Comparison of exact solution and approximate solutions from SAIM, MSEM and DIRK for Example 3.2 for $0 \le x \le 1$ for a fixed $t = 0.001$

x	$u_{\text{Exact}}(x,t)$	$u_{\text{SAIM}}(x,t)$	$u_{\text{MSEM}}(x,t)$	$u_{\text{DIRK}}(x,t)$	$E = \lvert u_{\text{DIRK}} - u_{\text{Exact}}\rvert$
0	0.500312500	0.500312500	0.500312500	0.500296875	0.000015625
0.2	0.475332542	0.475332542	0.475332542	0.475696774	0.000364232
0.4	0.450475418	0.450475418	0.450475418	0.450807926	0.000332508
0.6	0.425863084	0.425863084	0.425863084	0.426164317	0.000300477
0.8	0.401612703	0.401612703	0.401612703	0.401884622	0.000271919
1	0.377834468	0.377834468	0.377834468	0.377819776	0.000014692

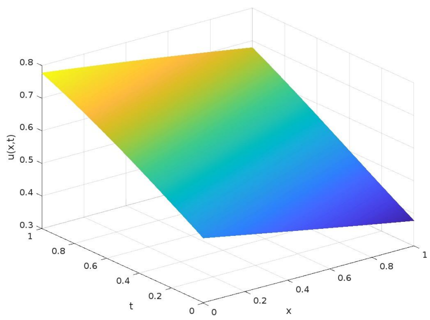

Figure 3.3: Surface plot for Example 3.2

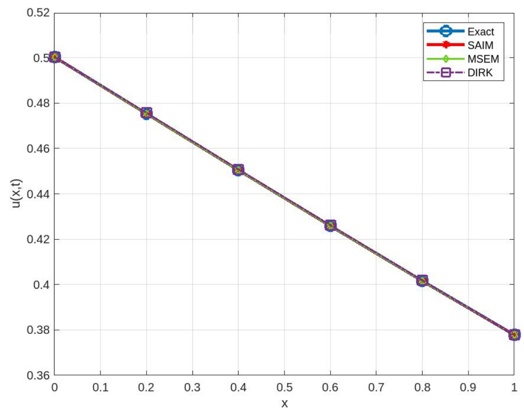

Figure 3.4: Comparison of exact solutions and approximate solutions from SAIM, MSEM and DIRK for $0 \le x \le 1$ for a fixed $t = 0.001$

Thus, for $\delta = 1$, $\alpha = 1$ and $\beta = 1$, the exact solution is given by Eq. (3.27).

Comparison of exact solutions and approximate solutions from SAIM, MSEM and DIRK. We present the exact and approximate results of generalized Burgers-Fisher equation for $0 \le x \le 1$ and $t = 0.001$. The results are shown in Table 3.2 and Figures 3.3 and 3.4. Here also, SAIM and MSEM perform better than DIRK [14].

Example 3.3. Consider the GBFE (2.2) for $\delta = 3$, $\alpha = 50$ and $\beta = 0.01$. Then we have [14]:

$$u_t = u_{xx} - 50u^3 u_x + 0.01u - 0.01u^4 \tag{3.28}$$

subject to the initial condition

$$u(x,0) = \left(\frac{1}{2} + \frac{e^{-\frac{75x}{2}} - 1}{2(e^{-\frac{75x}{2}} + 1)}\right)^{\frac{1}{3}}.$$

SAIM Scheme. By applying the SAIM to (3.28) after comparing with (2.1), we have

$$Lu + Nu = 0,$$

with $Nu = -u_{xx} + 50u^3u_x - 0.01u + 0.01u^4$ and $G(x,t) = 0$. Thus, the primary problem to be solved is

$$L(u_0) = 0 \text{ with } u_0(x,0) = \left(\frac{1}{2} + \frac{e^{-\frac{75x}{2}} - 1}{2(e^{-\frac{75x}{2}} + 1)}\right)^{\frac{1}{3}},$$

where

$$\int_0^t u_{0_t}(x,t)ds = 0.$$

From this, we obtain

$$u_0(x,t) = \left(\frac{1}{2} + \frac{e^{-\frac{75x}{2}} - 1}{2(e^{-\frac{75x}{2}} + 1)}\right)^{\frac{1}{3}}.$$

For the next iteration, we obtain the following

$$u_1(x,t) = \left(\frac{1}{2} + \frac{e^{-\frac{75x}{2}} - 1}{2(e^{-\frac{75x}{2}} + 1)}\right)^{\frac{1}{3}} - \left[50\left(\frac{1}{2} + \frac{e^{-\frac{75x}{2}} - 1}{2(e^{-\frac{75x}{2}} + 2)}\right)^{\frac{1}{3}} + \frac{\left(\frac{1}{2} + \frac{e^{-\frac{75x}{2}} - 1}{2(e^{-\frac{75x}{2}} + 2)}\right)^{\frac{1}{4}}}{100} - \frac{e^{-\frac{75x}{2}} - 1}{100(e^{-\frac{75x}{2}} + 2)} - \frac{1}{200}\right],$$

and so on. Continuing in this way, we obtain the exact solution

$$u(x,t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{23439t}{100} - \frac{75x}{4}\right)\right)^{\frac{1}{3}}. \tag{3.29}$$

MSEM Scheme. Considering Eq. (3.28), we rearrange it to obtain

$$u_t - u_{xx} + 50u^3u_x - 0.01u + 0.01u^4 = 0. \tag{3.30}$$

We use the travelling wave transformation

$$u(x,t) = u(\xi), \quad \xi = x - ct.$$

Considering Eq. (2.7), Eq. (3.30) becomes

$$-cu' - u'' + 50u^3u' - 0.01u + 0.01u^4 = 0, \tag{3.31}$$

where in Eq. (3.31), $u \Rightarrow u(\xi)$ and by integration, Eq. (3.31) gives

$$-cu - u' - 0.01u + 12.51u^4 = 0.$$

Thus,

$$-cu - 0.01u + 12.51u^4 = u'. \tag{3.32}$$

Therefore, the solution of Eq. (3.30) takes the form

$$u(\xi) = A_0 + A_1\left(\frac{\Phi'}{\Phi}\right), \tag{3.33}$$

where A_0 and A_1 are arbitrary constants in $\mathbb{R}$ such that $A_1 \neq 0$, and $\Phi(\xi)$ is an unknown function to be determined later. Substituting Eq. (3.33) and Eq. (3.9) into Eq. (3.32) yields a polynomial and equating coefficients of Φ^{-j} $(j = 0,\ 1,\ 2)$ to zero, we obtain

$$-cA_0 - 0.01A_0^2 + 12.51A_0^4 = 0, \tag{3.34}$$

(see Ayati et al. [5]). Thus, we have

$$(-c - 0.01 + 50.04A_0^3)\Phi' - \Phi'' = 0, \tag{3.35}$$

and

$$(75.06A_0^2A_1^2 + A_1)(\Phi')^2 = 0. \tag{3.36}$$

By solving Eqs. (3.34) and (3.36) we obtain the following results:

$$A_0 = 0,\ -100c \text{ and } A_1 = -\tfrac{1}{750600c^2}.$$

Case 1. When $A_0 = 0$, Eq. (3.35) becomes

$$(-c - 0.01)\Phi' - \Phi'' = 0.$$

So

$$\Phi' = Ae^{-(c+0.01)\xi}, \tag{3.37}$$

where A is an arbitrary constant in $\mathbb{R}$. Integrating Eq. (3.37) with respect to ξ gives $\Phi(\xi)$ as

$$\Phi = -\frac{A}{c+0.01}e^{-(c+0.01)\xi} + B,$$

where B is a constant of integration. Now the exact solution of Eq. (3.30) has the form

$$u_1(x,t) = \frac{-\frac{1}{750600c^2}Ae^{-(c+0.01)(x-ct)}}{-\frac{A}{c+0.01}e^{-(c+0.01)(x-ct)} + B},$$

where $c \neq -0.01$ and $A, B \neq 0$.

Case 2. When $A_0 = -100c$, Eq. (3.35) becomes

$$(-c - 0.01 - 50040000c^3)\Phi' - \Phi'' = 0.$$

So

$$\Phi' = Ae^{-(c+0.01+50040000c^3)\xi},$$

and, by integration,

$$\Phi = -\frac{A}{c+0.01+50040000c^3}e^{-(c+0.01+50040000c^3)\xi} + B.$$

Now, the exact solution of Eq. (3.30) has the form

$$u_2(x,t) = \frac{-(c+0.01+50040000c^3)B}{-\frac{A}{c+0.01+50040000c^3}e^{-(c+0.01+50040000c^3)(x-ct)} + B},$$

or

$$u_2(x,t) = -\frac{100c^2B}{Ae^{-(c+0.01+50040000c^3)(x-ct)} + cB},$$

where $A, B, c \neq 0$. Thus,

$$u(x,t) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{23439t}{100} - \frac{75x}{4}\right)\right)^{\frac{1}{3}}. \quad (3.38)$$

Results for Example 3.3. Here, we validate the authenticity and efficacy of both SAIM and MSEM in providing solutions to the generalized Burgers-Fisher equation by comparing the exact solution with results from both the SAIM and the MSEM. In particular, we have compared solutions of SAIM and MSEM for $\delta = 3$, $\alpha = 50$ and $\beta = 0.01$. The exact solution of the generalized Burgers-Fisher equation in Eq. (2.2) is given by

$$u(x,t) = \left[\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(x - \left(\frac{\alpha}{\delta+1} + \frac{\beta(\delta+1)}{\alpha}\right)t\right)\right)\right]^{\frac{1}{\delta}}.$$

Thus, just like the SAIM, the MSEM yields the exact solution (3.38) for $\delta = 3$, $\alpha = 50$ and $\beta = 0.01$.

Comparison of exact solutions and approximate solutions for SAIM, MSEM and DIRK for a fixed $t = 0.001$. We present the exact and approximate solutions of the generalized Burgers-Fisher equation for $\delta = 3$, $\alpha = 50$ and $\beta = 0.01$ for $0 \leq x \leq 1$ and $t = 0.001$. The results are shown in Table 3.3 and Figures 3.5 and 3.6. As in the previous examples, both SAIM and MSEM perform better than DIRK.

Table 3.3: Comparison of exact solutions and approximate solutions from SAIM, MSEM and DIRK for Example 3.3 ($\delta = 3$, $\alpha = 50$, $\beta = 0.01$ and $t = 0.001$)

x	$u_{\text{Exact}}(x,t)$	$u_{\text{SAIM}}(x,t)$	$u_{\text{MSEM}}(x,t)$	$u_{\text{DIRK}}(x,t)$	$E = \lvert u_{\text{DIRK}} - u_{\text{Exact}}\rvert$
0	0.850447265480	0.850447265480	0.850447265480	0.847875175000	0.002572090480
0.2	0.095939788493	0.095939788493	0.095939788493	0.082152476100	0.013787312393
0.4	0.007877535617	0.007877535617	0.007877535617	0.006747621900	0.001129913717
0.6	0.000646627593	0.000646627593	0.000646627593	0.000553878871	0.000092748722
0.8	0.000053079393	0.000053079393	0.000053079393	0.000045467670	0.000007611723
1	0.000003814697	0.000003814697	0.000003814697	0.000003814697	0.000000000000

From the findings above, SAIM and MSEM both perform better than DIRK, as can be seen from the absolute errors in Figure 3.7. However, MSEM obtains the exact solutions much faster than SAIM and is more reliable and effective. Also, for SAIM each of the u_i are standalone solutions and the iterative procedure has merit in that each solution is an improvement of the previous iterate and as more and more iterations are taken, the solution converges to the exact solution of the equation.

4 Conclusion

In this work, the semi-analytic iterative method and modified simple equation method have been successfully implemented to obtain the solution for the generalized Burgers-Fisher equation. A comparison of the two methods shows that the modified simple equation method is effective and much simpler than the semi-analytic iterative method and other methods from

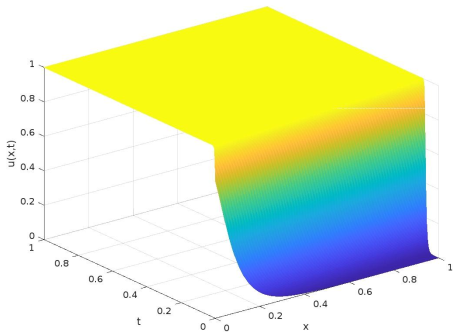

Figure 3.5: Surface plot for Example 3.3

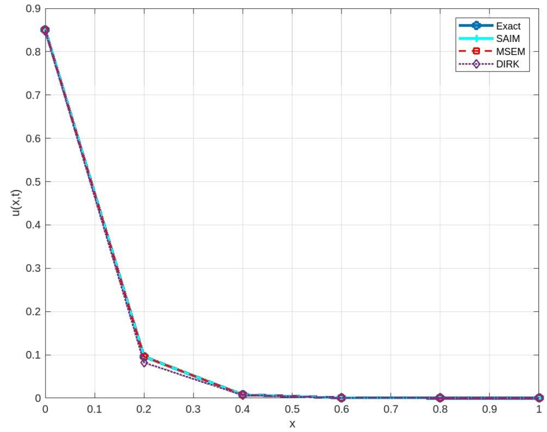

Figure 3.6: Comparison of exact solution and approximate solutions from SAIM, MSEM and DIRK for $0 \leq x \leq 1$ for a fixed $t = 0.001$

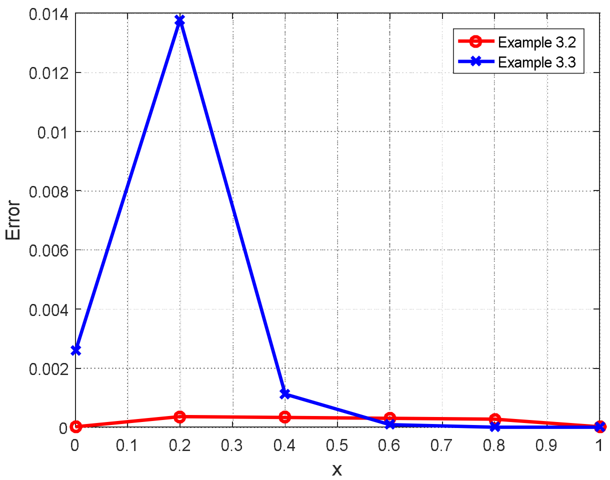

Figure 3.7: Absolute errors involving DIRK for Examples 3.2 and 3.3

the literature like variational iteration method and homotopy perturbation method. The SAIM and the MSEM can both be applied in the solution of many other nonlinear partial differential equations. Therefore, possible future work might include the use of these methods for solving initial and boundary value ODEs and many other linear and nonlinear PDEs for cases where δ is a negative integer and α and β are complex numbers.

Declarations

Availability of data and materials

Data sharing is not applicable to this paper.

Funding

Not applicable.

Authors' contributions

JL wrote the manuscript, contributed to modelling and conducted initial data analyses. CK conceived the research and the modelling, as well as contributed to writing. Both authors read and approved the final manuscript.

Conflict of interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgements

The authors are grateful for the support of Mulungushi University, in particular the Department of Mathematics and Statistics in the School of Natural and Applied Sciences, for providing a conducive environment for the implementation of this research. The Editors and reviewers also deserve special thanks for their useful comments that resulted in significant improvements in the paper.

References

[1] T. K. Akinfe and A. C. Loyinmi, *A solitary wave solution to the generalized Burgers-Fisher's equation using an improved differential transform method: A hybrid scheme approach*, Heliyon, **7**(5) (2021), 1–16. DOI

[2] M. A. Al-Jawary and A. S. Mohammed, *A semi-analytical iterative method for solving linear and nonlinear partial differential equations*, International Journal of Science and Research, **6**(5) (2015), 978–982. DOI

[3] M. A. Al-Jawary, M. M. Azeez and G. H. Radhi, *Analytical and numerical solutions for the nonlinear Burgers and advection-diffusion equations by using a semi-analytical iterative method*, Computers & Mathematics with Applications, **76**(1) (2018), 155–171. DOI

[4] S. Arora, R. Jain and V. K. Kukreja, *A robust Hermite spline collocation technique to study generalized Burgers-Huxley equation, generalized Burgers-Fisher equation and modified Burgers' equation*, Journal of Ocean Engineering and Science (2022), DOI

[5] Z. Ayati, M. Moradi and M. Mirzazadeh, *Application of modified simple equation method to Burgers, Huxley and Burgers-Huxley equations*, Iranian Journal of Numerical Analysis and Optimization, **5**(2) (2015), 59–73. DOI

[6] N. I. Ibrahim and O. H. Mohammed, *Semi-analytic technique for solving fractional partial differential equations with conformable derivatives*, Al-Nahrain Journal of Science, **24**(1) (2021), 39–44. DOI

[7] M. Javidi, *Modified pseudospectral method for generalized Burgers-Fisher equation*, International Mathematical Forum, **1**(32) (2006), 1555–1564.

[8] C. Kasumo, *On exact solutions of Klein-Gordon equations using the semi analytic iterative method*, International Journal of Advances in Applied Mathematics and Mechanics, **8**(2) (2020), 54–63.

[9] K. Khan, M. A. Akbar and N. H. M. Ali, *The modified simple equation method for exact and solitary wave solutions of nonlinear evolution equation: the GZK-BBM equation and right-handed noncommutative Burgers equations*, ISRN Mathematical Physics, **2013** (2013), Article ID 146704, 5 pages. DOI

[10] S. Kumar and S. Saha Ray, *Numerical treatment for Burgers-Fisher and generalized Burgers-Fisher equations*, Mathematical Sciences, **15** (2021), 21–28. DOI

[11] B. Latif, M. S. Selamat, A. N. Rosli, A. I. Yusoff and N. M. Hasan, *The semi analytics iterative method for solving Newell-Whitehead-Segel equation*, Mathematics and Statistics, **8**(2) (2020), 89–94. DOI

[12] J. Mendoza and C. Muriel, *New exact solutions for a generalised Burgers-Fisher equation*, Chaos, Solitons and Fractals, **152** (2021), 111360. DOI

[13] R. Mohammadi, *Spline solution of the generalized Burgers-Fisher equation*, Applicable Analysis, **91**(12) (2011), 2189–2215. DOI

[14] M. A. Pirdawood and Y. A. Sabawi, *High-order solution of generalized Burgers-Fisher equation using compact finite difference and DIRK methods*, Journal of Physics: Conference Series, **1999** (2021), 012088. DOI

[15] S. Ramya, K. Krishnakumar and R. Ilangovane, *Exact solutions of time fractional generalized Burgers-Fisher equation using exp and exponential rational function methods*, International Journal of Dynamics and Control, (2023), 1-11. DOI

[16] M. M. Rashidi, D. D. Ganji and S. Dinarvand, *Explicit analytical solutions of the generalized Burger and Burger-Fisher equations by homotopy perturbation method*, Numerial Methods for Partial Differential Equations, **25**(2) (2009), 409–417. DOI

[17] M. S. Selamat, A. Muhidir, N. N. Abd Razak and Z.A. Hassanuddin, *Semi analytical iterative method for solving Klein-Gordon equation*, GADING Journal of Science and Technology, **3**(1) (2020), 10–18.

[18] M. Soori, *The variational iteration method and the homotopy perturbation method to the exact solution of the generalized Burgers-Fisher equation*, Calculus of Variations and Partial Differential Equations, **5**(8) (2018), 19–26.

[19] N. Taghizadeh, M. Mirzazadeh, A. S. Paghaleh and J. Vahidi, *Exact solutions of nonlinear evolution equations by using the modified simple equation method*, Ain Shams Engineering Journal, **3**(3) (2012), 321–325. DOI

[20] H. Temimi and A. R. Ansari, *A semi analytical iterative technique for solving nonlinear problems*, Computers & Mathematics with Applications, **61**(2) (2011), 203–210. DOI

[21] H. Temimi and A. R. Ansari, *A computational iteration method for solving nonlinear ordinary differential equations*, LMS Journal of Computation and Mathematics, **18**(1) (2015), 730–753. DOI

[22] M. Zhong, Q. J. Yang and S. F. Tian, *The modified high-order Haar wavelet scheme with Runge-Kutta method in the generalized Burgers-Huxley equation*, Modern Physics Letters B, **35**(24) (2021), 2150419. DOI