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A comparative study on strategic analysis and forecasting on profit maximization and operational efficiency in manufacturing business through differential equations

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Abstract. Differential equations are fundamental mathematical tools with wide-ranging applications in science and economics. This study delves into their role in business, focusing on strategic analysis and forecasting for profit maximization and operational efficiency in manufacturing. It explores various equation types, from ordinary to partial differentials, highlighting their critical role in modeling economic phenomena. Through a comprehensive case study, this research demonstrates the practical application of differential equations in optimizing production, sales, revenue, and profit. The study emphasizes their impact on strategic decision-making and navigating complex market dynamics for sustained growth and profitability.

Keywords: Differential Equations, Production Optimization, Sales Forecasting, Revenue Growth, Profit Maximization, Manufacturing Business Case Study.
2020 Mathematics Subject Classification: 39A10, 67B89. MSC2020

1 Introduction

Differential equations are foundational mathematical tools with extensive applications across scientific and economic domains. They describe relationships between unknown functions and their derivatives, crucial for modeling dynamic systems. The history of differential equations dates back to the advent of calculus by Newton and Leibniz in the 17th century. Isaac Newton outlined three types of differential equations in his work Methodus fluxionum et Serierum Infinitarum (1671), solving them using infinite series and acknowledging their non-uniqueness of solutions.

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In 1695, Jacob Bernoulli introduced the Bernoulli differential equation, further simplified by Leibniz. Subsequent advancements by D'Alembert, Euler, Bernoulli, and Lagrange contributed to understanding vibrating strings and formulating wave equations. Euler and Lagrange's work on the tautochrone problem in the 1750s led to the Euler-Lagrange equation and the development of Lagrangian mechanics.
Fourier's contributions in 1822 with the heat equation marked a significant milestone, integrating Newton's law of cooling and establishing the basis for modern mathematical physics education. This historical evolution reflects collaborative efforts, shaping differential equations as indispensable tools in diverse scientific disciplines.
Differential equations find vital applications in business, economics, physics, biology, and more. They facilitate strategic analysis, forecasting, and optimization in manufacturing businesses, contributing to profit maximization and operational efficiency. This study explores their theoretical foundations, historical context, and practical applications, focusing on their pivotal role in shaping modern business strategies. Empirical applications further illuminate their potency in varied domains. Ma et al. (2019) [8] explore the relationship between enterprise management and financial performance, using fractional differential equations (FDEs) to model these complex dynamics more effectively than traditional differential equations. The study identifies significant factors affecting financial performance within Chinese enterprises. Yusuf et al. (2019) [15] discuss the applications of differential equations in economics, demonstrating their efficacy in modeling growth, economic cycles, inflation, and unemployment. They underscore the crucial role of these equations in econometrics for estimating models and forecasting outcomes.
Rahman et al. (2020) [10]tackle the challenge of developing an inventory model under uncertain market economies and customers' demand. Their use of interval differential equations, center-radius optimization, and particle swarm optimization helps develop robust solutions, aiding decision-making in uncertain environments.
Chalikias et al. (2016) [2]present a differential equation model analyzing the Greek banking sector's performance, identifying factors influencing its dynamics and offering forecasts. Their model accurately predicts and identifies significant factors influencing the sector's performance from 2000 to 2015."
The applications of differential equations extend further with diverse applications across fields, reflecting their versatility in modeling dynamic systems and resolving complex relationships. By understanding differential equations' historical evolution and practical significance, this study aims to provide insights into leveraging these mathematical tools for strategic decision-making and navigating complex business landscapes

2 Types of differential equations and applications

Differential equations can be divided into several types. Apart from describing the properties of the equation itself, these classes of differential equations can help inform the choice of approach to a solution. Commonly used distinctions include whether the equation is ordinary or partial, linear or non-linear, and homogeneous or heterogeneous. This list is far from exhaustive; there are many other properties and subclasses of differential equations which can be very useful in specific contexts.

2.1 Ordinary differential equations (ODEs):

These equations involve an unknown function of a single real or complex variable along with its derivatives and given functions. In the business domain, ODEs find applications in modeling various economic factors over time. They're used to analyze trends in sales, growth rates, investment returns, and interest rates. ODEs help in forecasting market behaviors, optimizing business processes, and managing financial portfolios by predicting changes in variables like market demand or consumer behavior.

2.2 Partial differential equations (PDEs):

PDEs involve unknown multivariable functions and their partial derivatives. In business, PDEs can model complex scenarios such as risk management, pricing of financial derivatives, and analyzing the diffusion of innovation in markets. For instance, they're used in option pricing models, where they evaluate how options' values change concerning various influencing factors like time, market volatility, and underlying asset price changes. PDEs are also applied in predicting how changes in market conditions or consumer preferences impact product or service adoption rates.

2.3 Non-linear differential equations:

These equations aren't linear in the unknown function and its derivatives. They find their use in business contexts where relationships between variables aren't linear, such as in modeling market saturation, competition dynamics, or consumer behavior. They're employed in understanding how changes in advertising expenditure or pricing strategies affect sales revenue, considering complex interactions between variables.

2.4 Equation order and degree:

The order of a differential equation is the highest derivative of the unknown function present in the equation. Understanding the order and degree of differential equations is crucial in business analytics. For instance, first-order equations might represent linear relationships between variables, aiding in analyzing trends. Meanwhile, higher-order equations could model more intricate relationships, useful in predicting scenarios where multiple factors interact to influence outcomes, like in supply chain optimization or resource allocation.
These classifications provide valuable tools for analyzing trends, making predictions, and optimizing business strategies across various domains within the business sector.
Moving forward, as we transition from the conceptual realm of differential equations to their practical application, we encounter TechGena vibrant testament to the real-world impact of these mathematical foundations within the dynamic landscape of business. Reflecting on the core principles elucidated earlierembracing rates of change, modeling dynamic systems, and interrelating variablesTechGen's mission in smartphone manufacturing emerges as a living embodiment of these theories. The forthcoming case study unveils a comprehensive five-year journey undertaken by TechGen, showcasing the profound fusion of mathematical theories with pragmatic business strategies. Here, the astute utilization of differential equations by TechGen not only echoes the theoretical constructs discussed but also illuminates the tangible influence of these mathematical tools in shaping sustainable business models and fostering tangible growth.

3 Thypothetical case study: Techgen - pioneering profitable growth in smartphone manufacturing through differential equations

Scenario- Predicting and optimizing business operations for profitable growth

3.1 Introduction:

TechGen, a forward-thinking startup in the smartphone manufacturing industry, is dedicated to reshaping the market landscape through innovative technology and strategic decision-making. Founded in 2023, TechGen is on a mission to be a trailblazer in the industry by optimizing its production, sales, and overall financial performance using the power of differential equations.

3.1.1 Company overview:

Founding Year: 2023
Vision: To lead the market with cutting-edge smartphone solutions that seamlessly integrate technology and user experience.
Mission: Utilize data-driven insights, particularly through differential equations, to enhance production efficiency, maximize sales, and ensure sustained profitability.

3.1.2 Utilizing differential equations for optimization:

Recognizing the intricacies of smartphone manufacturing, TechGen aims to harness the predictive power of differential equations to optimize production, sales, revenue, and profit. The company is committed to achieving profitability from its inception, emphasizing the importance of a sound and sustainable business model.

3.1.3 Present situation

Business Operations	Initial Values
Production	3000 smartphones
Sales	3000 smartphones
Revenue	$900,000
Profit	$150,000

TechGen's commitment to profitability is reflected in its current financial standing. Producing 3000 smartphones yearly, the company sells all the 3000 units, generating $900,000 in revenue. With a profit of $150,000, TechGen sets a benchmark for startups aiming not only for growth but also for sustained profitability from the outset.

3.1.4 Strategic focus areas:

Technological Enhancement: Continue leveraging technology to improve production efficiency and reduce costs.
Sales Strategy Optimization: Further refine sales strategies, considering advertising impacts and market dynamics to sustain and enhance profitability.

Financial Sustainability: Maintain a balanced approach to revenue and cost dynamics to ensure continued profitability.
In a nutshell, TechGen invests heavily to use up-to-date cutting-edge technology (increases purchase and use of multiple modern machines and plants) to improve their production while maintaining efficiency. Likewise, they also invest heavily in refining sales strategies (increase in ads spending, brand awareness etc.) to improve their sales, optimizing overall revenue and thus, maintaining and growing a sustainable profit.
This sets the stage for the exploration of TechGen's journey through the application of differential equations to address these strategic focus areas. The subsequent sections will delve into the mathematical models, simulations, and strategic insights derived from the application of differential equations.

3.2 Mathematical models used:

TechGen used the powerful mathematical tool Differential Equations to predict and optimize its production, sales, and overall financial performance. From their observation, they got the following differential equations as their result for each of their operations:

3.2.1 Production

Let $P(t)$ represent the number of smartphones in production at time t. The differential equation could be something like:
$\frac{dP}{dt}$ = Initial Production Rate + Production Growth $\times P$.

Let's assign values to the parameters for the production phase.
$\frac{dP}{dt} = 3000 + 0.02 \times P$.
Here:
$\frac{dP}{dt}$: This represents the rate of change of the number of smartphones in production (P) with respect to time (t). In simpler terms, it tells us how the production quantity changes over years.
P: This is the variable representing the number of smartphones in production at any given time.
3000: This is the initial production rate, indicating that the company starts by producing 3000 smartphones per year.
$0.02\times$ P: This term introduces a positive growth factor. This signifies technological enhancement, reflecting the company's commitment to leveraging technology to improve production efficiency. The more smartphones are in production (P), the higher the growth rate, indicating increased efficiency and technological advancements over time.
In a nutshell, this equation essentially says that the rate of change of the number of smartphones in production is equal to the initial production rate plus a growth term proportional to the current production quantity.

3.2.2 Sales

Extending the production model, let S(t) represent the number of smartphones sold at time t. The differential equation might be:
$\frac{dS}{dt}$= Initial Sales Rate (S = P) + Sales Growth $\times S$.
Let's assign values to the parameters for the sales phase.

$dS/dt = 3000 + 0.01 \times S$.

Here:
dS/dt: This represents the rate of change of the number of smartphones sold (S) with respect to time (t). In simpler terms, it tells us how the sales quantity changes over years. P: This is the variable representing the number of smartphones in production at any given time. In this context, it also represents the initial sales rate.
Sales Growth $\times S$: This term introduces a positive growth factor. This reflects sales strategy optimization, indicating that the company is refining its sales strategies over time. The more smartphones are sold (S), the higher the growth rate in sales, representing improved sales strategies, effective advertising, and better adaptation to market dynamics.
In a nutshell, this equation essentially says that the rate of change of the number of smartphones sold is equal to the initial production rate (which also serves as the initial sales rate) plus a growth term proportional to the current sales quantity.

3.2.3 Revenue

Integrating the sales model, let R(t) represent the revenue generated at time t. The equation could include factors like pricing and promotions:
$R(t) =$*Unit Price* $\times$ S + *Promotion Impact* $\times R$.
Let's assign values to the parameters for the revenue phase.
$R(t) = 300S + 0.05R$.
Here,

S: This represents the number of smartphones sold.
R(t): This represents the revenue generated at time t.
R: This variable represents the total revenue generated by the company.
Unit Price: Let's set the unit price at $300 per smartphone.
Promotion Impact $\times$ **R.**: This term represents the impact of promotions on revenue. The value 0.05 is a coefficient, indicating a 5pc increase in revenue due to promotional activities introducing a positive growth factor related to promotion impact. This aligns with financial sustainability, indicating that the company is using promotions to positively impact revenue. The more revenue (R) generated, the higher the growth in promotions, reflecting a balanced approach to revenue and cost dynamics.
This equation calculates the revenue generated by the company from the sale of smartphones, accounting for the unit price and the impact of promotions on overall revenue.

3.2.4 Profit

For profit $\Pi(t)$, which is the revenue minus costs, we can have a differential equation like:
$\frac{d\pi}{dt} =$ Revenue $-$ Costs.
Let's set the costs at 250 per smartphone. This includes various expenses associated with production, marketing, and other operational aspects.
The profit analysis equation becomes:
$\frac{d\pi}{dt} = R - 250 \times S$.
Here:
R: This represents the revenue generated.
S: This represents the number of smartphones sold.

$250 \times$ S: This represents the costs associated with producing and selling smartphones.
$\frac{d\pi}{dt}$: This represents the rate of change of profit with respect to time.
This equation reflects the rate of change of profit over time, considering revenue and costs. The positive growth in revenue, along with effective cost management, contributes to sustained financial sustainability and profitability.

3.3 Mathematical calculations

Euler's method is used in the following calculations, for simplicity:

3.3.1 Year 1

Production:
Initial Production $P(0)$: 3000 smartphones.
Rate of Change $(dP/dt(0))$: $3000 + 0.02 \times P(0)$.
Time Step (Δt): 1 year.
Calculations:
$P(1) = P(0) + \frac{dp}{dt}(0) \times \Delta t$,
$P(1) = 3000 + (3000 + 0.02 \times 3000) \times 1$,
$P(1) = 6600\ units$.

This implies that the total production for the first year is 6600 smartphones.
Revenue:
Initial Revenue R (0): \$900,000.
Unit Price: \$300 per smartphone.
Promotion Impact: 5.
Calculations:
R (1) = $300 \times$ S (1) + $0.05 \times$ R (0).
R (1) = $300 \times 6300 + 0.05 \times 900000$.
R (1) = \$1,935,000.
This implies that the total revenue for the first year is \$1,935,000.
Profit:
Initial Profit $(\pi(0))$: $\$150,000$.
Rate of Change $(\frac{d\pi}{dt}(0))$: $R(0) - 250 \times S(0)$.
Time Step (Δt): 1 year.
Calculations:
$\pi(1) = \pi(0) + \frac{d\pi}{dt}(0) \times \Delta t$,
$\pi(1) = 150000 + (1935000 - 250 \times 6300) \times 1$,
$\pi(1) = 150000 + (1935000 - 1575000)$,
$\pi(1) = \$510,000$.
This implies that the total profit for the first year is 510,000.
Summary of first year:

Business Operations	**Values**
Production	6600 smartphones
Sales	63,00 smartphones
Revenue	\$1,935,000
Profit	\$510,000

TechGen's strategic focus on technological enhancement, sales strategy optimization, and financial sustainability has significantly impacted its business operations within the first year. The increased production rate, evident in the rise from 3000 to 6600 smartphones, reflects the company's successful technological advancements, leveraging innovation to bolster efficiency. Moreover, the growth in sales from 3000 to 6300 smartphones highlights the effective optimization of sales strategies, demonstrating a refined approach to advertising and adapting to market dynamics. Consequently, this substantial growth in both production and sales has resulted in a remarkable surge in revenue from $900,000 to $1,935,000, showcasing TechGen's sustained focus on financial sustainability. Ultimately, the marked increase in profit from $150,000 to $510,000 underscores the company's holistic commitment to leveraging differential equations for operational optimization, yielding tangible results in profitability and growth.
The values for the business operations of TechGen, including production, sales, revenue, and profit, are determined using the outlined differential equations. The summarized results for subsequent years are presented below.

3.3.2 Year 2

Business Operations	Values
Production	8712 smartphones
Sales	8088 smartphones
Revenue	$2,426,400
Profit	$675,600

In the second year, TechGen continues its trajectory of growth and optimization. The production of smartphones increases significantly from 6600 to 8712, showcasing the company's sustained technological enhancements and improved production efficiency. Correspondingly, sales grow from 6300 to 8088 smartphones, indicating the continued success in sales strategy optimization and adapting to market dynamics. This considerable growth in both production and sales results in a noteworthy rise in revenue from $1,935,000 to $2,426,400, demonstrating TechGen's persistent focus on maintaining financial sustainability. As a direct consequence, the profit also experiences a substantial surge from $510,000 to $675,600, highlighting the efficacy of TechGen's differential equation-based strategies in fostering consistent profitability and operational growth.

3.3.3 Year 3

Business Operations	Values
Production	10551 smartphones
Sales	9828 smartphones
Revenue	$2,948,400
Profit	$818,400

Entering its third year, TechGen maintains its upward trajectory in business operations. The production of smartphones escalates from 8712 to 10551, emphasizing the sustained technological advancements and continual improvement in production efficiency. Sales also demonstrate steady growth, increasing from 8088 to 9828 smartphones, signifying the continued success of sales strategies and adept navigation of market dynamics. This substantial growth in production and sales results in a notable uptick in revenue, rising from $2,426,400 to $2,948,400, showcasing TechGen's unwavering commitment to financial sustainability. Consequently, the profit also experiences significant growth, elevating from $675,600 to $818,400, underscoring the effectiveness of TechGen's differential equation-based strategies in fostering consistent profitability and operational expansion.

3.3.4 Year 4

Business Operations	**Values**
Production	12661 smartphones
Sales	11814 smartphones
Revenue	$3,544,200
Profit	$984,200

In its fourth year, TechGen sustains its impressive growth and operational optimization. The production of smartphones continues to rise significantly from 10551 to 12661, highlighting the ongoing technological enhancements and the company's commitment to improving production efficiency. Correspondingly, sales see a steady increase from 9828 to 11814 smartphones, showcasing the continued success of refined sales strategies and adept responses to evolving market dynamics. This substantial growth in both production and sales contributes to a noteworthy increase in revenue, surging from $2,948,400 to $3,544,200, underscoring TechGen's persistent focus on maintaining financial sustainability. Consequently, the profit also experiences significant growth, escalating from $818,400 to $984,200, reaffirming the efficacy of TechGen's differential equation-based strategies in fostering consistent profitability and operational expansion.

3.4 Simulating results:

TechGen's performance over five years portrays a progressive growth trajectory in its smartphone manufacturing operations:

Business Operations	**Production**	**Sales**	**Revenue (in Dol.)**	**Profit(in Dol.)**
First Year	6600	6300	1,935,000	510,000
Second Year	8712	8088	2,426,400	675,600
Third Year	10551	9828	2,948,400	818,400
Fourth Year	12661	11814	3,544,200	984,200
Fifth Year	15194	14105	4,231,500	1,175,500

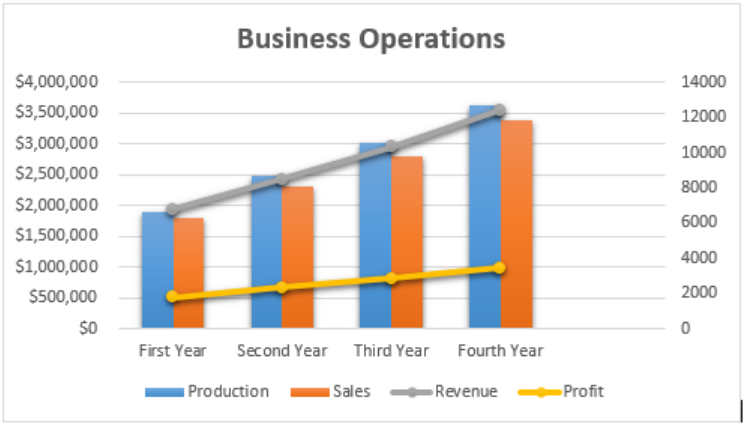

Figure 3.1: Business operation

3.4.1 Production and sales:

The production of smartphones steadily increased from an initial 6600 units in the first year to 15194 units by the fifth year. Correspondingly, sales experienced a similar upward trend, starting at 6300 units in the first year and reaching 14105 units by the fifth year. This consistent growth in production and sales demonstrates TechGen's ability to expand its manufacturing capabilities and effectively penetrate the market.

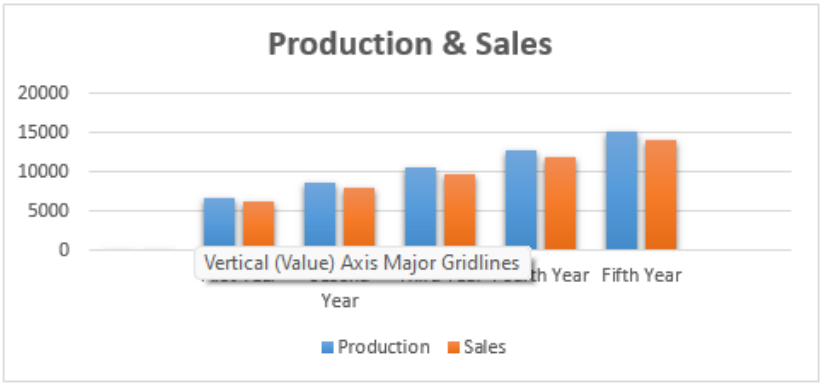

Figure 3.2: Production & Sales

3.4.2 Revenue and profit:

As production and sales escalated, TechGen experienced a substantial surge in revenue and profit. Revenue climbed from 1,935,000 in the first year to an impressive 4,231,500 by the fifth year. This growth in revenue directly impacted profit, rising from 510,000 initially to 1,175,500 by the fifth year. The consistent increase in revenue and profit indicates the company's successful strategies in managing costs, optimizing sales, and maintaining financial sustainability. This condensed representation allows us to quickly grasp the upward trend in TechGen's performance. It showcases the company's capacity for consistent growth in production, sales, revenue, and profit, providing a concise yet comprehensive overview of its evolving success story.

Figure 3.3: Revenue and Profit

3.5 Scenario analysis:

TechGen's journey unfolds impressively over the five-year period. The application of differential equation represents the progressive growth and financial performance of TechGen over five years in its smartphone manufacturing operations.

3.5.1 Market dynamics:

External factors such as evolving market competition, technological advancements, and changing consumer preferences played a role in influencing TechGen's growth trajectory. These factors influenced the company's strategies and responses, contributing to its progressive success.

3.5.2 Sales strategies and market expansion:

While TechGen saw substantial growth, a more detailed exploration of sales strategies, like targeted advertising, strategic partnerships, and international market penetration, could fortify future market expansion.

3.5.3 Risk management:

Diversification strategies serve not only to sustain growth but also act as risk mitigation measures. Considering new product lines or emerging markets could shield against risks associated with reliance on a single product or market segment.
Diversification strategies serve not only to sustain growth but also act as risk mitigation measures. Considering new product lines or emerging markets could shield against risks associated with reliance on a single product or market segment.

3.5.4 Long-term sustainability:

Addressing operational efficiency in the context of long-term sustainability is crucial. Emphasize how efficiency improvements should align with TechGen's long-term strategic goals and adaptability to market changes.
In summary, TechGen's commendable growth in production, sales, revenue, and profit over

five years is evident. Concentrating on operational efficiency, diversification, and aligning strategies with market dynamics will fortify its market presence and financial stability.

3.6 Strategic insights:

This pivotal analysis serves as a fundamental pillar in delineating actionable recommendations from the exhaustive evaluation of TechGen's performance across the last five years. These insights encapsulate strategic directions and actionable plans that can steer TechGen towards sustained growth, market leadership, and heightened competitiveness within the dynamic smartphone manufacturing landscape.

3.6.1 Operational efficiency enhancement:

TechGen can optimize internal processes by embracing innovative technologies and automation in manufacturing. Implementing advanced machinery, streamlining assembly lines, and adopting AI-driven production scheduling can significantly boost production efficiency.

3.6.2 Market expansion strategies:

Expanding into untapped markets or emerging economies presents immense growth opportunities. TechGen could strategically enter new geographic locations or target niche demographics through localized marketing campaigns, catering specifically to diverse consumer preferences.

3.6.3 Product diversification:

Diversifying product lines beyond smartphones, such as introducing smart accessories or exploring adjacent tech sectors like wearables, could bolster TechGen's resilience against market volatility and cater to diverse consumer needs.

3.6.4 Financial sustainability planning:

Balancing growth aspirations with financial stability is crucial. TechGen should maintain a prudent approach to cost management, ensuring investments in growth initiatives don't compromise the company's long-term financial health.

3.6.5 Technological advancements:

Staying at the forefront of technological advancements in smartphone manufacturing is imperative. Exploring innovations like AI integration for user experience enhancement or investing in sustainable production practices can solidify TechGen's position as an industry leader.

3.6.6 Customer-centric approach:

Prioritizing customer feedback and needs is paramount. Developing personalized user experiences, offering after-sales support, and actively seeking consumer insights through surveys or focus groups can foster brand loyalty and attract new customers.

3.6.7 Risk management:

TechGen should proactively assess and mitigate potential risks. Strategies such as diversifying suppliers, conducting market scenario planning, and establishing contingency plans for unforeseen disruptions will fortify TechGen against market uncertainties.
By implementing these strategies, TechGen can proactively position itself for sustained growth, market resilience, and continued success in the competitive smartphone industry.

3.7 Result summary:

TechGen's journey, guided by differential equations, exemplifies a remarkable rise in the smartphone manufacturing domain. Analyzing five years of growth reveals how strategic planning and technology integration powered TechGen's ascent since its 2023 inception.
Throughout this journey, the use of differential equations proved instrumental, fueling steady increases in production, sales, revenue, and profit. Yet, there's room for more progress. To sustain this growth, TechGen should focus on refining operations, exploring new avenues, and staying innovative. This hypothetical case study showcases the potency of data-driven strategies. It illuminates how blending mathematical insights with business acumen can pave the way for budding enterprises, offering a roadmap for sustainable success in competitive industries.

4 Conclusion

In this comprehensive exploration of differential equations within the realm of business and economic domains, we've delved into the fundamental concepts, historical evolution, and practical applications that form the backbone of this mathematical discipline. Beginning with an insightful overview, we established the critical role of differential equations as essential tools transcending traditional scientific domains, emerging as pivotal models in understanding dynamic systems and relationships between variables. The theoretical foundation laid the groundwork, showcasing the versatility and applicability of differential equations in diverse business contexts. From ordinary differential equations (ODEs) predicting economic trends to partial differential equations (PDEs) optimizing market strategies, each equation type found its niche in modeling various aspects of business operations, be it sales forecasting, revenue optimization, or profit sustainability.
The hypothetical case study of TechGen served as a real-world application, demonstrating how differential equations become indispensable in strategizing, optimizing, and ensuring sustained profitability in the smartphone manufacturing industry. Through simulated scenarios and mathematical calculations, we witnessed the tangible impact of differential equations on TechGen's growth trajectory, underscoring their instrumental role in predicting, strategizing, and driving operational success. Simulated results of TechGen's performance over five years showcase a notable upward trajectory across key business metrics, reflecting the successful application of differential equations in strategic decision-making.
The production of smartphones steadily increased from 6600 units in the first year to 15194 units by the fifth year, mirroring the upward trend in sales from 6300 units initially to 14105 units over the same period. This consistent growth underscores TechGen's ability to scale manufacturing operations and effectively capture market demand. The corresponding rise in revenue from $1,935,000 to $4,231,500 and profit from $510,000 to $1,175,500 reflects the com-

pany's adeptness in optimizing sales, managing costs, and driving financial sustainability. These results validate the strategic foresight enabled by differential equations, which played a pivotal role in predicting and optimizing production, sales, revenue, and profit for TechGen. The demonstrated success highlights the practical significance of differential equations in guiding operational strategies and ensuring sustained profitability in the manufacturing industry. TechGen's progressive growth trajectory, as evidenced by the simulated results, underscores the transformative impact of leveraging mathematical models in business decision-making. Differential equations, in this context, serve not only as predictive tools but as strategic enablers, empowering businesses to navigate complex market dynamics and achieve long-term success.

Declarations

Availability of data and materials

Data sharing is not applicable to this paper

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Conflict of interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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