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Galerkin method for the numerical solution of some class of differential equations by utilizing Gegenbauer wavelets

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Abstract. Many differential equations that emerge from modeling physical phenomena do not always possess well-known analytical solutions. Additionally, wavelets have attracted considerable attention from both theoretical and applied researchers in recent decades. In this study, we introduce the Galerkin method for numerically solving a specific class of differential equations by employing Gegenbauer wavelets (GWGM). In this approach, Gegenbauer wavelets serve as weight functions and are treated as basis elements, enabling us to derive the numerical solution. The numerical solutions obtained through this method are compared with several existing methods and the exact solution. Various examples are presented to demonstrate the effectiveness and applicability of the proposed technique.

Keywords: Gegenbauer wavelets, Function approximation, Galerkin method, Differential equations.

2020 Mathematics Subject Classification: 34A40, 26C10, 65T60. MSC2020

1 Introduction

A variety of linear and nonlinear problems emerge in the domains of science and engineering, frequently represented as second-order ordinary differential equations, capturing the attention of numerous mathematicians and physicists. Generally, obtaining an exact solution for a specific differential equation is impractical. Consequently, it becomes crucial to explore the discretization of these equations, leading to numerical solutions.

In recent times, some researchers have proposed numerical methods for the numerical solutions of second-order ordinary differential equations. For example, the Legendre wavelet collocation method [11], Laguerre Wavelet-Galerkin Method [12], Hermite wavelets method [13], etc.

Wavelet analysis emerged as a prominent discipline in the 1980s, primarily due to its effective use in processing signals and images. This technique entails the systematic translation

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and scaling of a single function, leading to a smooth orthonormal basis that has proven essential for developing compression algorithms specifically designed for signals and images within designated amplitude limits. Notable progress in this domain includes wavelet series expansion in applied mathematics, sub-band coding aimed at voice and image compression, and multiresolution signal processing utilized in computer vision [3].

Spectral methods exhibit excellent spectral localization but lack spatial localization, whereas finite element methods demonstrate strong spatial localization but are deficient in spectral localization. Wavelet bases are employed to merge the strengths of both spectral and finite element bases. A key principle in approximation theory is the representation of a smooth function as a series expansion utilizing orthogonal polynomials. Currently, the exploration of wavelet function bases is being regarded as a promising alternative to traditional piecewise polynomial trial functions in the finite element analysis of differential equations. The Galerkin method is highly esteemed in applied mathematics for its efficiency and practicality.

The Galerkin method utilizing wavelets offers significant advantages over traditional finite difference and finite element methods, resulting in extensive applications across various fields of science and engineering. To some degree, the wavelet approach serves as a formidable alternative to the finite element method. Furthermore, the wavelet technique presents an effective alternative for the numerical solution of differential equations [1,9].

The wavelet-Galerkin method offers advantages over finite difference and finite element methods, resulting in significant applications in science and engineering. The wavelet technique serves as a strong competitor to the finite element method. Moreover, the wavelet method provides an efficient alternative for numerically solving differential equations, particularly boundary value problems.

In this paper, I develop the Galerkin method using Gegenbauer wavelets (GWGM) to numerically solve a specific class of differential equations. This approach involves expanding the solution with Gegenbauer wavelets that have unknown coefficients. The characteristics of Gegenbauer wavelets, combined with the Galerkin method, are employed to determine the unknown coefficients, ultimately resulting in a numerical solution for the given class of differential equations.

The paper is organized as follows: Section 2 discusses Gegenbauer wavelets and function approximation. Section 3 addresses the Gegenbauer wavelet-based Galerkin method for solving certain differential equations. Section 4 presents the numerical implementation, while Section 5 concludes the proposed work.

2 Gegenbauer wavelets and Function approximation

2.1 Gegenbauer wavelets

Gegenbauer wavelets $\psi_{n,m}(x) = \psi(k,n,m,x)$ involve four arguments: $n = 1,2,3,\ldots,2^{k-1}$; m is the degree of Gegenbauer polynomials; x is the normalized time; and k is any positive integer. They are defined on the interval $[0,1)$. Gegenbauer wavelets are defined as [8]

$$\psi_{n,m}(x) = \begin{cases} \frac{2^{k/2}}{\sqrt{R_m^\lambda}} G_m^\lambda(2^k x - \hat{n}), & \frac{\hat{n}-1}{2^k} \le x < \frac{\hat{n}+1}{2^k}, \\ 0, & \text{otherwise}, \end{cases} \tag{2.1}$$

where $\lambda > -\frac{1}{2}$, $\hat{n} = 2n-1$, $m = 0,1,2,\ldots,M-1$, and R_m^λ is the normalization factor given by

$$R_m^\lambda = \begin{cases} 2^{1-2\lambda}\pi\frac{\Gamma(m+2\lambda)}{m!(m+\lambda)(\Gamma(\lambda))^2}, & \lambda \neq 0, m \neq 0, \\ \frac{2\pi}{m^2}, & \lambda = 0, m \neq 0, \\ \pi, & \lambda = 0, m = 0. \end{cases} \tag{2.2}$$

The Gegenbauer polynomials G_{m+1}^λ are defined as

$$\begin{aligned} &G_0^\lambda(x) = 1, \\ &G_1^\lambda(x) = 2\lambda x, \\ &G_{m+1}^\lambda(x) = \tfrac{1}{(m+1)}[2x(m+\lambda)G_m^\lambda(x) - (m+2\lambda-1)G_{m-1}^\lambda(x)], \quad m = 1,2,\ldots,M-1. \end{aligned}$$

The first few Gegenbauer wavelet bases for $k=1$, $M=3$, and $\lambda = 2$ are as follows:

$$\begin{aligned} &\psi_{1,0}(x) = \tfrac{4}{\sqrt{3\pi}}, \\ &\psi_{1,1}(x) = 4\sqrt{\tfrac{2}{\pi}}(-1+2x), \\ &\psi_{1,2}(x) = 8\sqrt{15\pi}(5-24x+24x^2), \\ &\psi_{1,3}(x) = 4\sqrt{23\pi}(-5+42x-96x^2+64x^3), \quad \text{and so on.} \end{aligned}$$

Function approximation:
Suppose $y(x) \in L^2[0,1)$ is expanded in terms of Gegenbauer wavelets as:

$$y(x) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} c_{n,m}\psi_{n,m}(x). \tag{2.3}$$

Truncating the above infinite series, we get

$$y(x) = \sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x). \tag{2.4}$$

Convergence of Gegenbauer wavelets
Theorem. If a continuous function $y(x) \in L^2(R)$ defined on $[0,1)$ is bounded, that is, $|y(x)| \leq K$, then the Gegenbauer wavelet expansion of $y(x)$ converges uniformly to it [15].
Proof. Let $y(x)$ be a bounded real-valued function on $[0,1)$. The Gegenbauer coefficients of the continuous function $y(x)$ are defined as

$$\begin{aligned} C_{n,m} &= \int_0^1 y(x)\psi_{n,m}(x)dx \\ &= \int_I y(x)\frac{2^{k/2}}{\sqrt{R_m^\lambda}}G_m^\lambda(2^k x - \hat{n})dx, \end{aligned}$$

where $I = \left[\frac{\hat{n}-1}{2^k}, \frac{\hat{n}+1}{2^k}\right)$. Put $2^k x - \hat{n} = z$. Then,

$$C_{n,m} = \int_I y\left(\frac{z+\hat{n}}{2^k}\right)G_m^\lambda(z)dx.$$

Using the generalized mean value theorem for integrals,

$$
\begin{aligned}
C_{n,m} &= \frac{2^{k/2}}{\sqrt{R_m^\lambda}} y\left(\frac{w+\hat{n}}{2^k}\right)\int_{-1}^{1} G_m^\lambda(z)dx, \quad \text{for some } w \in (-1,1) \\
&= \frac{2^{k/2}}{\sqrt{R_m^\lambda}} y\left(\frac{w+\hat{n}}{2^k}\right) h,
\end{aligned}
$$

where $h = \int_{-1}^{1} G_m^\lambda(z)dx$. Thus,

$$
|C_{n,m}| \leq \left|\frac{2^{k/2}}{\sqrt{R_m^\lambda}}\right| \left|y\left(\frac{w+\hat{n}}{2^k}\right)\right| |h|.
$$

Since y is bounded, $\sum_{n,m=0}^{\infty} C_{n,m}$ is absolutely convergent. Hence, the Gegenbauer series expansion of $y(x)$ converges uniformly.

3 Method of Solution

Consider the boundary value problem of the form:

$$
\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = \phi(x), \tag{3.1}
$$

with the boundary conditions $y(0) = a$, $y(1) = b$.
Here, P and Q are constants or functions of independent/dependent variables, and $\phi(x)$ is a continuous function. Rewrite Eq. 3.1 as

$$
R(x) = \frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy - \phi(x), \tag{3.2}
$$

where $R(x)$ is the residual of Eq. 3.1. When $R(x) = 0$, the exact solution $y(x)$ satisfies the boundary conditions.
Consider the trial series solution of Eq. 3.1, defined over $[0,1)$, expanded as modified Gegenbauer wavelets satisfying the given boundary conditions and involving unknown parameters as follows:

$$
y(x) = \sum_{n=1}^{2^k-1}\sum_{m=0}^{M-1} c_{n,m}\psi_{n,m}(x), \tag{3.3}
$$

where $c_{n,m}$ are unknown coefficients to be determined. The accuracy of the solution is increased by choosing higher-degree Gegenbauer wavelet polynomials. Differentiating Eq. 3.3 twice with respect to x and substituting the values of y, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ into Eq. 3.2, we obtain:
To find $c_{n,m}$, we choose the weight functions as the assumed basis elements and integrate the product of these with the residual over the boundary values, setting it equal to zero [5]:

$$
\int_0^1 \psi_{n,m}(x)R(x)\,dx = 0, \quad m = 0,1,2,\ldots
$$

From the above equation, a system of linear algebraic equations is obtained. By solving this system using known methods, we determine the unknown parameters. Substituting these parameters into the trial solution, i.e. Eq. 3.3, yields the numerical solution of Eq. 3.1.
In order to assess the accuracy of the Gegenbauer wavelet-based Galerkin method (GWGM) for the test problems, we use the error measure, i.e. the maximum absolute error. The maximum absolute error is calculated by $E_{max} = \max|y(x)_e - y(x)_a|$, where $y(x)_e$ and $y(x)_a$ are the exact and approximate solutions, respectively.

4 Numerical Implementation

Problem 4.1: First, consider the boundary value problem [10]

$$\frac{d^2y}{dx^2} - y = x - 1, \quad 0 \leq x \leq 1 \tag{4.1}$$

$$\text{BCs: } y(0) = 0,\ y(1) = 0. \tag{4.2}$$

The implementation of Eq. 4.1, as per the method explained in Section 3, is as follows. Its residual can be written as:

$$R(x) = \frac{d^2y}{dx^2} - y - (x - 1). \tag{4.3}$$

Now, choosing the weight function $w(x) = x(1 - x)$ for the Gegenbauer wavelet bases to satisfy the given boundary conditions in Eq. 4.2, we have $\psi(x) = w(x) \times \psi(x)$:

$$\psi_{1,0}(x) = \psi_{1,0}(x) \times x(1 - x) = \tfrac{4}{\sqrt{3\pi}} x(1 - x),$$

$$\psi_{1,1}(x) = \psi_{1,1}(x) \times x(1 - x) = 4\sqrt{\tfrac{2}{\pi}}(-1 + 2x)x(1 - x),$$

$$\psi_{1,2}(x) = \psi_{1,2}(x) \times x(1 - x) = \tfrac{8}{\sqrt{15\pi}}(5 - 24x + 24x^2)x(1 - x).$$

Considering that Eq. 4.1's trial solution for $k = 1$ and $m = 2$ is given by

$$y(x) = c_{1,0}\psi_{1,0}(x) + c_{1,1}\psi_{1,1}(x) + c_{1,2}\psi_{1,2}(x). \tag{4.4}$$

Then Eq. 4.4 becomes

$$\begin{aligned} y(x) = c_{1,0}\left\{\frac{4}{\sqrt{3\pi}} x(1 - x)\right\} + c_{1,1}\left\{4\sqrt{\frac{2}{\pi}}(-1 + 2x)x(1 - x)\right\} \\ + c_{1,2}\left\{\frac{8}{15\pi}(5 + 24x + 2x^2)x(1 - x)\right\}. \end{aligned} \tag{4.5}$$

Differentiating Eq. 4.5 twice with respect to x and substituting the values of y and $\frac{d^2y}{dx^2}$ into Eq. 4.3, the residual of Eq. 4.1 is found. Using the weighted residual approach, if the weight functions in the trial solution are equal to the basis functions, then by the weighted Galerkin method we consider:

$$\int_0^1 \psi_{1,j}(x)R(x)\,dx = 0, \quad j = 0, 1, 2. \tag{4.6}$$

From Eq. 4.6, we get

$$\begin{aligned} \int_0^1 \psi_{1,0}(x)R(x)\,dx = 0, \\ \int_0^1 \psi_{1,1}(x)R(x)\,dx = 0, \\ \int_0^1 \psi_{1,2}(x)R(x)\,dx = 0. \end{aligned} \tag{4.7}$$

From Eq. 4.7, we obtain a system of algebraic equations with unknown coefficients, i.e. $c_{1,0}$, $c_{1,1}$, and $c_{1,2}$. Solving this system, we obtain the values $c_{1,0} = 0.17436$, $c_{1,1} = -0.01803$, and $c_{1,2} = 0.00066$. Substituting these values into Eq. 4.5 gives the numerical solution. Table 4.1 presents the comparison between the numerical solutions and their corresponding absolute

Table 4.1: Comparison between the numerical solution obtained using the proposed method, the solution derived by the method in [10], and the exact solution of Problem 4.1, based on their respective absolute errors.

x	Ref [10]	GWGM	Exact solution	Ref [10] Error	GWGM Error
0.1	0.0264712	0.0265021	0.0265183	4.70e-05	1.62e-05
0.2	0.0443444	0.0443040	0.0442945	5.00e-05	9.50e-06
0.3	0.0546184	0.0545373	0.0545074	1.10e-04	2.99e-05
0.4	0.0583436	0.0582892	0.0582599	8.40e-05	2.93e-05
0.5	0.0565875	0.0565856	0.0565906	3.10e-06	5.00e-06
0.6	0.0504023	0.0504769	0.0504834	8.10e-05	6.50e-06
0.7	0.0407912	0.0408657	0.0408782	8.70e-05	1.25e-06
0.8	0.0286751	0.0286793	0.0286795	4.40e-06	2.00e-07
0.9	0.0148592	0.0147836	0.0147663	9.30e-05	1.73e-05

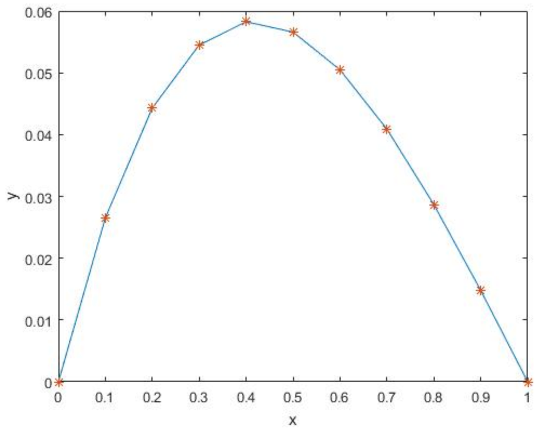

Figure 4.1: Comparison between the numerical solution and exact solution of the problem 4.1.

errors. Additionally, Figure 4.1 illustrates the numerical solution alongside the exact solution of Eq. 4.1, given by

$$y(x) = -\frac{1}{1-e^2}e^x + \frac{e^2}{1-e^2}e^{-x} - x + 1.$$

Problem 4.2: Consider the boundary value problem [6]:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -(e^{x-1} + 1), \quad 0 \le x \le 1, \tag{4.8}$$

$$\text{BCs: } y(0) = 0, \; y(1) = 0. \tag{4.9}$$

As explained in Section 3 and in the previous problem, the obtained values of the coefficients are $c_{1,0} = 0.6077$, $c_{1,1} = 0.0403$, and $c_{1,2} = 0.0042$. Substituting these values into Eq. 4.5 yields the numerical solution. The comparison of the numerical solution and the absolute errors is presented in Table 4.2, and the numerical solution together with the exact solution of Eq. 4.8, which is $y(x) = x(1 - e^{x-1})$, is shown in Figure 4.2.

Problem 4.3: Now consider the singular boundary value problem [4]:

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} - \frac{2}{x^2}y = 4, \quad 0 \le x \le 1, \tag{4.10}$$

Table 4.2: Comparison between the numerical solution obtained using the proposed method, the solution derived by the method in [6], and the exact solution of Problem 4.2, based on their respective absolute errors.

x	FDM	Ref [6]	GWGM	Exact solution	FDM Error	Ref [6] Error	GWGM Error
0.1	0.061948	0.0593827	0.0593163	0.059343	2.61e-03	3.97e-05	2.67e-05
0.2	0.115151	0.1102340	0.1101340	0.110134	5.02e-03	1.00e-04	0
0.3	0.158162	0.1512000	0.1509570	0.151024	7.14e-03	1.76e-04	6.70e-05
0.4	0.189323	0.1806167	0.1804075	0.180475	8.85e-03	1.42e-04	6.75e-05
0.5	0.206737	0.1969833	0.1967255	0.196735	1.00e-02	2.48e-04	9.50e-06
0.6	0.208235	0.1980833	0.1978694	0.197808	1.04e-02	2.75e-04	6.14e-05
0.7	0.191342	0.1816552	0.1815153	0.181427	9.92e-03	2.28e-04	8.83e-05
0.8	0.153228	0.1452000	0.1450578	0.145015	8.21e-03	1.85e-04	4.28e-05
0.9	0.090672	0.0857100	0.0856092	0.085646	5.03e-03	6.40e-05	3.68e-05

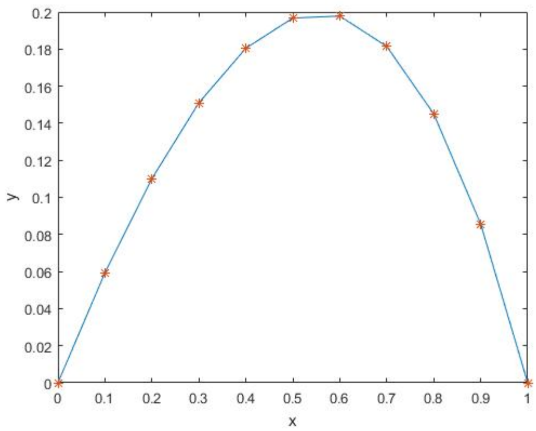

Figure 4.2: Comparison of numerical solution with exact solution of the problem 4.2.

Table 4.3: Comparison between the numerical solution obtained using the proposed method, the solution derived by the method in [2,4], and the exact solution of Problem 4.3, based on their respective absolute errors.

x	Ref [4]	Ref [2]	GWGM	Exact solution	Ref [4] Error	Ref [2] Error	GWGM Error
0.1	-0.08668	-0.091865	-0.0900504	-0.09000	3.32e-03	1.90e-03	5.40e-05
0.2	-0.15682	-0.162047	-0.1600732	-0.16000	3.18e-03	2.00e-03	7.32e-05
0.3	-0.20842	-0.211369	-0.2100770	-0.21000	1.58e-03	1.40e-03	7.70e-05
0.4	-0.24013	-0.240457	-0.2400688	-0.24000	1.30e-04	4.60e-04	6.88e-05
0.5	-0.25119	-0.249739	-0.2500544	-0.25000	1.19e-03	2.60e-04	5.44e-05
0.6	-0.24133	-0.239439	-0.2400384	-0.24000	1.33e-03	5.60e-04	3.84e-05
0.7	-0.21070	-0.209587	-0.2100239	-0.21000	7.00e-04	4.10e-04	2.39e-05
0.8	-0.15977	-0.160010	-0.1600126	-0.16000	2.30e-04	1.00e-05	1.26e-05
0.9	-0.08924	-0.090338	-0.0900049	-0.09000	7.60e-04	3.40e-04	4.90e-06

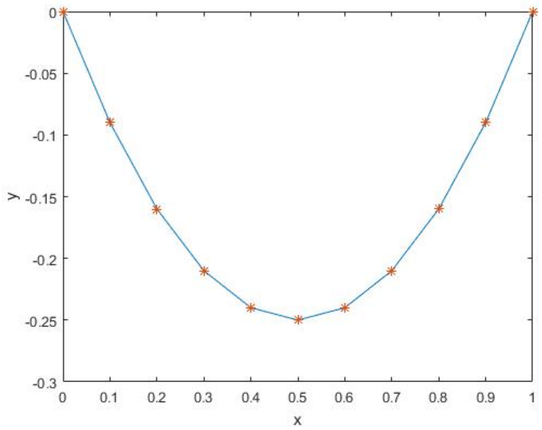

Figure 4.3: Comparison of numerical solution with exact solution of the problem 4.3.

$$\text{BCs: } y(0) = 0, \ y(1) = 0. \tag{4.11}$$

As explained in Section 3 and in the previous problem, the obtained values of the coefficients are $c_{1,0} = -0.76768$, $c_{1,1} = 0.00007$, and $c_{1,2} = -0.00002$. Substituting these values into Eq. 4.5 yields the numerical solution. The comparison of the numerical solution and the absolute errors is presented in Table 4.3, and the numerical solution together with the exact solution of Eq. 4.10, which is $y(x) = x^2 - x^3$, is shown in Figure 4.3.

Problem 4.4: Now consider the nonlinear boundary value problem [7]:

$$\frac{d^2y}{dx^2} - y^2 = 2\pi^2\cos(2\pi x) - \sin^4(2\pi x), \quad 0 \leq x \leq 1, \tag{4.12}$$

$$\text{BCs: } y(0) = 0, \ y(1) = 0. \tag{4.13}$$

The exact solution of Eq. 4.12 is $y(x) = \sin^2(\pi x)$. Table 4.4 and Figure 4.4 present the exact solution alongside the numerical solution, which was derived as described in Section 3.

Table 4.4: Comparison of numerical solution and absolute error with the exact solution of the problem 4.4.

x	Ref [2]	GWGM	Exact solution	Ref [14] Error	GWGM Error
0.1	0.097457	0.096787	0.0954920	1.97E-03	1.30E-03
0.2	0.351009	0.350839	0.3454920	5.52E-03	5.35E-03
0.3	0.657342	0.656318	0.6545082	2.83E-03	1.81E-03
0.4	0.906851	0.905968	0.9045082	2.34E-03	1.46E-03
0.5	0.997985	0.998985	1	2.01E-03	1.02E-03
0.6	0.910379	0.910215	0.9045082	5.87E-03	5.71E-03
0.7	0.658956	0.656335	0.6545082	4.45E-03	1.83E-03
0.8	0.348898	0.346849	0.3454920	3.41E-03	1.36E-03
0.9	0.097684	0.097656	0.0954920	2.19E-03	2.16E-03

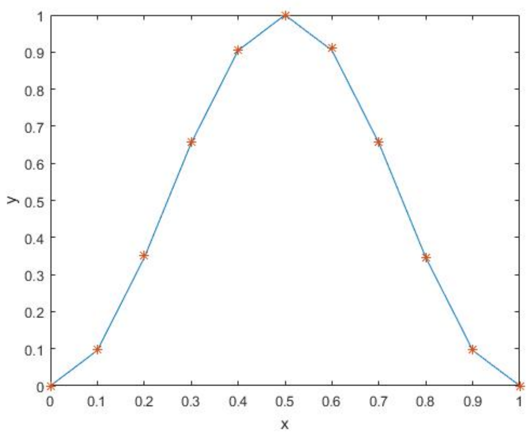

Figure 4.4: Comparison of numerical solution with exact solution of the problem 4.4.

5 Conclusion

This study proposes the Galerkin method for the numerical solution of a certain class of differential equations using Gegenbauer wavelets (GWGM). From the tables and figures presented, the following observations can be made:

- The numerical solutions derived from this method outperform those obtained through the finite difference method (FDM) and other techniques (i.e., Refs. [2,4,6,10,14]).
- Additionally, the absolute error associated with this method is significantly lower compared to FDM and other methods (i.e., Refs. [2,4,6,10,14]).

Therefore, the Galerkin method using Gegenbauer wavelets is highly effective for solving this specific class of differential equations.

Declarations

Availability of data and materials

Not applicable

Funding

Not applicable

Authors' contributions

The author solely contributed to the preparation and writing of this paper.

Conflict of interest

The author has no conflicts of interest to declare.

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