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An approach to functional description of mass-operational characteristics in the tasks of quality control of raw materials

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Abstract. The paper presents a mathematical approach to solving an applied problem, namely determining the content of metal-containing impurities in the building industry. An example involving the magnetic control of a quartz sand sample is considered. The results are presented graphically and additionally processed in semi-logarithmic coordinates. Using the proposed method, the total mass of impurities in the sample was calculated, as well as the mass removed during a limited number of operations for extracting metal impurities, and the consistency of the control was assessed.

Keywords: Mathematical approach, Magnetic control, Metal-containing impurities, Semi-logarithmic coordinates, Quality control.

2020 Mathematics Subject Classification: 28Axx, 28Bxx, 28Cxx. MSC2020

1 Introduction

In various raw materials used for construction materials and products (for example, quartz sand, ground talc, dolomite, feldspar, etc.), the presence of impurities of iron and its compounds is inevitable. Therefore, high-gradient magnetic separation [2,8,10,11,13,14,17–19,21,22], including dry-type separation [17–19], is required at industrial sites. These impurities can be introduced into raw materials at various stages: during production (referred to as impurities of natural origin) or during transportation and processing (i.e., impurities of man-made origin). They lead to a decrease in the quality of raw materials; for example, in quartz sand, they reduce the translucency of sheet window glass, while in ceramic products, they cause defective inclusions. Such products are either disposed of, adding to landfills and polluting the environment, or attempts are made to restore them (to save raw materials), which increases energy consumption and reduces the safety and reliability of technological processes and equipment. For example, the use of low-quality potash feldspar (often containing impurities such as hematite, magnetite, white mica, and others) may adversely affect the whiteness, dielectric properties, transparency, and chemical stability of feldspar-based products [7,20].

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In the ceramics, glass, and other construction-related industries, great attention is paid to diagnosing the quality of raw materials of natural origin. One of the key parameters in media quality control is the content of metal impurities. Even in the grading of many materials, this parameter must be specified. The study of particle size distribution and the magnetic properties of such impurities is conducted worldwide [1, 3–6, 9, 15, 16]. This is because it is important to obtain not only statistical information about the particles in a particular sample but also a practical understanding of appropriate methods for monitoring their content and extraction.

Since impurities generally possess magnetic properties, one of the preferred methods for their detection is magnetic control, which requires multiple extraction operations using permanent magnets or electromagnets. Moreover, it is advisable not to simply sum the extracted masses of impurities but to record each extraction at every magnetic control operation [13].

This article includes sections such as *Results*, where experimental characteristics of magnetic control are described and numerical results are obtained; *Discussions*, which considers parameters characterizing the quality of the process; and *Conclusions*, which summarizes the main calculated parameters, particularly the total mass of impurities and the degree of sampling.

2 Results

Figure 1a shows the mass-operational characteristics of magnetic control for impurities in one of these construction materials, with a number of operations exceeding standard requirements. It can be seen that the characteristic is asymptotically decreasing, which means it is practically impossible to achieve complete extraction of magnetic impurities. However, the true content of impurities can be determined mathematically. The solution to this problem involves an approach that assumes a functional description of the dependence of the mass m of impurities on the n-th operation. One effective step to establish the regularity of the obtained empirical dependence of m on n is its graphical illustration in semi-logarithmic coordinates (Fig. 1b). In this case, the quasi-linearization of the approximation is confirmed, indicating that an exponential form of the dependence $m(n)$ is valid [13, 20]:

$$m = A \cdot e^{-kn}, \tag{2.1}$$

where A is the function coefficient, individual for each case of magnetic control (conditionally, at $n = 0$, $m = A$);
k is the function coefficient, individual for each case of magnetic control, depending on the slope of the approximating segment (Fig. 1b).

Figure 2.1: Graphical representation of the magnetic control of the quartz sand sample obtaining the poly-operational dependence of the mass $m(n)$ of ferrous impurities extracted from the sample: (a) in Cartesian coordinates; (b) in coordinates with logarithmic ordinate axis.

To determine the parameters A and k, it is necessary to solve a system of two equations based on formula (1), using two pairs of $m(n)$ values selected from the approximating trend (according to the program, the approximation reliability value is 0.98). Thus, it becomes possible to determine any current value for n, including, importantly, values outside the actual experiment using extrapolation.

However, it is also important to determine the total mass of impurities in the sample (up to infinity). It is worth noting that in this application, the dependence is discrete (valid for the natural series of numbers $1, 2, 3, \ldots$), where $m_1 = A \cdot e^{-k}$, $m_2 = A \cdot e^{-2k}$, $m_3 = A \cdot e^{-3k}$, and so on. This allows it to be represented as a decreasing geometric progression, with the common ratio found as:

$$q = m_n / m_{n-1} = \exp(-k). \tag{2.2}$$

Therefore, knowing m_1 (as the first term of the progression) and the common ratio q, it is possible to determine the sum of an infinite number of terms the total potential sediment mass $m_{1,\cdots,\infty}$:

$$m_{1\ldots\infty} = \sum_{n=1}^{\infty} m_n = \frac{m_1}{1-q} = \frac{A}{e^k - 1}. \tag{2.3}$$

It is also possible to determine the mass of extracted impurities for a limited number of control operations (formula 4) and the degree of sampling (formula 5):

$$m_{1\ldots n} = \frac{m_1 - m_n q}{1-q} = \frac{\left(1 - e^{-nk}\right)}{e^k - 1}, \tag{2.4}$$

$$\psi_{1\ldots n} = \frac{m_{1\ldots n}}{m_{1\ldots\infty}} = 1 - e^{-nk}. \tag{2.5}$$

The following results were obtained from the magnetic inspection shown in Fig. 1:

1. The general form of the approximating function is expressed as $m = 60.67 \exp(-0.135n)$.
2. The total (potential) sediment mass is $m_{1,\cdots,\infty} = 419.72$ mg.
3. The mass of extracted impurities after three (minimally acceptable) and ten (actually performed) operations is $m_{1,\cdots,3} = 139.78$ mg and $m_{1,\cdots,10} = 310.94$ mg, respectively.

3 Discussions

Thus, with different numbers of control iterations (in other words, levels of thoroughness), the researcher obtains different data on the degree of sampling of impurities relative to their potential total mass in the sample. For example, after three operations, the degree of sampling is only $\psi = 0.33$, while after ten operations it increases to $\psi = 0.74$.

The results of magnetic control can also be evaluated by another parameter, often used in metrology, which characterizes process quality: the relative control error, usually calculated as

$$\varepsilon = \frac{m_{1\ldots\infty} - m_{1\ldots n}}{m_{1\ldots\infty}} \cdot 100\%. \tag{3.1}$$

Thus, with $n = 3$ operations, the error is $\varepsilon = 67\%$, and with $n = 10$ operations, $\varepsilon = 26\%$.

4 Conclusions

A mathematical approach was developed to solve applied tasks in the quality control of raw materials such as quartz sand, ground talc, dolomite, and others. An example of magnetic control of a quartz sand sample was considered. The results were presented graphically, with an exponential dependence obtained in semi-logarithmic coordinates. According to the proposed method, the mass of impurities in the sample was calculated: the total mass equals $m_{1,\cdots,\infty} = 419.72$ mg; the mass for a limited number of operations was $m_{1,\cdots,3} = 139.78$ mg (degree of sampling $\psi = 0.33$) and $m_{1,\cdots,10} = 310.94$ mg (degree of sampling $\psi = 0.74$).

Declarations

Availability of data and materials

Data sharing not applicable to this article.

Funding

Not applicable.

Authors' contributions

All authors contributed equally to this work.

Conflict of interest

The authors have no conflicts of interest to declare.

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