Journal of Innovative Applied Mathematics and Computational Sciences

J. Innov. Appl. Math. Comput. Sci. **3**(2) (2023), 190–202. DOI: 10.58205/jiamcs.v3i2.177

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Adaptive control of a four-dimensional memristor-based Chua's circuit

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Received 22 Mai 2023, Accepted 17 January 2024, Published 21 January 2024

Abstract. This paper investigates the behavior of a four-dimensional memristor-based Chua circuit. Specifically, we emphasize its chaotic and hyperchaotic behavior using the phase portrait and the Lyapunov spectrum. As chaos is deemed undesirable in numerous scientific disciplines, particularly in fields like robotics and electronic sciences, where the analyzed circuit holds potential applications in electronic device construction, we aim to alleviate such behaviors. To achieve this, we put forth an adaptive control strategy involving unknown parameters. The effectiveness of the suggested adaptive chaos control is established using the Lyapunov stability theory. To further illustrate and confirm our findings, we present numerical simulations, providing a visual representation of the successful application of the proposed adaptive control in managing the circuit's dynamics.

Keywords: Memristor, hyperchaotic system, Chua's circuit, stability, Lyapunov function, adaptive control.

2020 Mathematics Subject Classification: 34D20, 34D45, 37G35, 39A10, 67B89. MSC2020

1 Introduction

In 1971, Leon Chua posited the theoretical existence of the memristor as the fourth circuit element alongside the commonly recognized resistor, capacitor, and inductor [8]. Subsequently, in 2008, researchers from Hewlett-Packard (HP) laboratories reported the successful fabrication of a physical memristor implementation [18]. This prototype utilizes TiO_2 thin films with doped and un-doped regions situated between two metal contacts at the nanometer scale. The memristor implementation developed by HP researchers has garnered considerable attention.

A memristor is a passive nonlinear circuit element with two-terminal that connects charge (q) and flux (φ), typically described by its constitutive relation of nonlinear form: $f(\varphi, q) = 0$. When this nonlinear relationship can be transformed to a single-valued function $\varphi(q)$ of the charge q; the memristor is referred to as charge-controlled.

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ISSN (electronic): 2773-4196

The memristor is defined by a memristance function of the form $M(q) = \dfrac{d\varphi(q)}{dq}$.

A memristor is considered flux-controlled when the equation $f(\varphi, q) = 0$ can be expressed as a single-valued function $q = q(\varphi)$. In this scenario, the memristor is defined by its memductance function $M(\varphi) = \dfrac{dq(\varphi)}{d\varphi}$, representing the rate of change of charge with respect to flux [10]. Various memristor-based models have been recently proposed [3–7, 15–17].

Huang et al. [9] have substituted Chua's diode in Chua's circuit with a negative conductance and a flux-controlled memristor characterized by $q(\varphi) = -a\varphi + 0.5b\varphi|\varphi|$ in parallel. This substitution allows for the creation of a new Chua's circuit based on memristors. Utilizing Kirchhoff Laws, the dynamics of the modified memristor-based Chua's circuit can be described by the following set of differential equations

$$\begin{cases} \dfrac{dV_1(t)}{dt} = \dfrac{1}{C_1}\left[\dfrac{V_2(t) - V_1(t)}{R} + GV_1(t) - (-a + b|\phi|)\, V_1(t)\right], \\ \dfrac{dV_2(t)}{dt} = \dfrac{1}{C_2}\left[\dfrac{V_1(t) - V_2(t)}{R} + I_L(t)\right], \\ \dfrac{dI_L(t)}{dt} = \dfrac{1}{L}\left[-V_2(t) - R_L I_L(t)\right], \\ \dfrac{d\phi(t)}{dt} = V_1(t), \end{cases} \tag{1.1}$$

where $V_i, i = 1.2$ voltages, $C_i, i = 1.2$ capacitances, I_L current, R, R_L and G resistances, L is inductance and ϕ is the magnetic flux through the memristor. Set $x = V_1$, $y = V_2$, $z = I_L$, $\omega = \phi$, $C_2 = 1$, $R = 1$, $\alpha = \dfrac{1}{C_1}$, $\beta = \dfrac{1}{L}$, $\gamma = \dfrac{R_L}{L}$ and $\xi = G$ then (1.1) can be transformed into the dimensionless form system (2.1).

In the field of control theory, adaptive control stands out as a widely employed technique for stabilizing systems in situations where the system parameters are not known [19–21]. On the other hand, sliding mode control methods [2, 14, 22, 23], as well as active control methods, are applied when the parameters are accessible and measurable [1, 11, 12, 24–26]

In this study, we counteract the unwanted chaotic and hyperchaotic tendencies, aiming to stabilize the system (2.1) at its equilibrium points. This is achieved through the development of an adaptive control system founded on the principles of Lyapunov stability theory.

The rest of paper is organized as follows: Section 2 provides an exploration of the dynamics and characteristics of the memristor-based Chua's system. In Section 3, we elaborate on the adaptive control method for regulating the output of the memristor-based Chua's system. Section 4 presents detailed numerical simulations that confirm and illustrate the findings of this paper. Finally, Section 5 offers concluding remarks.

2 Memristor-based four-dimensional Chua's Circuit

A memristor-based Chua's hyperchaotic circuit has been introduced in [9] and mathematically modeled by the following four dimensional differential system

$$\begin{cases} \dfrac{dx(t)}{dt} = \alpha[y(t) - x(t) + \xi x(t) - (-a + b|\omega|)x(t)], \\ \dfrac{dy(t)}{dt} = x(t) - y(t) + z(t), \\ \dfrac{dz(t)}{dt} = -\beta y(t) - \gamma z(t), \\ \dfrac{d\omega\,(t)}{dt} = x(t), \end{cases} \tag{2.1}$$

where x, y, z and ω are the states and α, β, γ, ξ, a and b assumed to be positive constant parameters.

2.1 Dissipativity and existence of an attractor

The system (2.1) can be written in its vector form as

$$\frac{dX}{dt} = f\,(X) = \begin{bmatrix} f_1(x,y,z,\omega) \\ f_2(x,y,z,\omega) \\ f_3(x,y,z,\omega) \\ f_4(x,y,z,\omega) \end{bmatrix}, \tag{2.2}$$

where $X(t) = (x(t), y(t), z(t), \omega(t))$ and

$$\begin{cases} f_1(x,y,z,\omega) = \alpha\,[y(t) - x(t) + \xi x(t) - (-a + b|\omega(t)|)x(t)]\,, \\ f_2(x,y,z,\omega) = x(t) - y(t) + z(t), \\ f_3(x,y,z,\omega) = -\beta y(t) - \gamma z(t), \\ f_4(x,y,z,\omega) = x(t). \end{cases} \tag{2.3}$$

We have

$$divf = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} + \frac{\partial f_4}{\partial \omega} = \alpha(-1 + \xi + a - b|\omega|) - 1 - \gamma. \tag{2.4}$$

For $|\omega| > \dfrac{\xi + a - 1}{b} - \dfrac{\gamma + 1}{\alpha b}$, we have $divf < 0$ then, the system (2.1) is dissipative and exhibits an attractor.

2.2 Equilibrium points

The equilibrium points of the memristor-based Chua's system (2.1) are obtained by solving the following system of equations

$$\begin{cases} \alpha[y(t) - x(t) + \xi x(t) - (-a + b|\omega(t)|)x(t)] = 0, \\ x(t) - y(t) + z(t) = 0, \\ -\beta y(t) - \gamma z(t) = 0, \\ x(t) = 0. \end{cases} \tag{2.5}$$

The resolution of (2.5) gives the set of equilibrium points

$$P_e = \{(x, y, z, \omega); x = 0, y = 0, z = 0 \text{ and } \omega = \omega_e \in \mathbb{R}\}. \quad (2.6)$$

Thus, each point of the ω-axis is an equilibrium point of the system (2.1).
The stability of (2.1) at the equilibrium points was studied by Huang and Wang [9].

3 Chaos and hyper-chaos in the memristor-based Chua circuit

We shall investigate the dynamic of system (2.1) versus the parameter $\alpha \in [0, 9.2]$, using phase portraits, bifurcation diagrams and Lyapunov exponents. The other parameters as set to:

$$\beta = 10, \gamma = 0.11, \xi = 0.1, a = 1.5 \text{ and } b = 1, \quad (3.1)$$

with the initial conditions

$$x(0) = 0, y(0) = 0, z(0) = 0.0001 \text{ and } \omega(0) = -0.98. \quad (3.2)$$

The Lyapunov exponents are numerically calculated using the Wolf-Swift algorithm [**?**]. For $\alpha = 9.15$ one gets

$$L_1 = 0.2976,\ L_2 = 0.0230,\ L_3 = -0.0200,\ L_4 = -6.259. \quad (3.3)$$

Since $L_1 + L_2 + L_3 + L_4 = -5.9582 < 0$, $L_1 > 0, L_2 > 0$, then the system (2.1) is dissipative and hyperchaotic.

The Kaplan-Yorke dimension of the system (2.1) is obtained as

$$\begin{aligned} D_{KY} &= 3 + \frac{L_1 + L_2 + L_3}{|L_4|} \\ &= 3 + \frac{0.2976 + 0.0230 - 0.0200}{6.259} \\ &= 3.0480, \end{aligned} \quad (3.4)$$

which is a fractal dimension.

The evolution of the two largest Lyapunov exponents together with the bifurcation diagram of x versus $\alpha \in [0, 9.2]$ are depicted in Figure 3.1, which shows good agreement between them.

From Figure 3.1, one observes that for $\alpha \in (0, 6.15)$ there is a stationary behavior as illustrated in Figure 3.2a for $\alpha = 5$, and a Hopf bifurcation occurs at $\alpha = 6.15$ where the equilibrium point losses its stability in view of a period-one limit cycle (see Figure 3.2b), which in tour bifurcates to a period-two limit cycle at $\alpha = 6.77$ (see Figure 3.2c) and the period doubling scenario becomes very fast at $\alpha = 6.89$ leading to chaotic behavior for a small widows $\alpha \in (6.9, 7.01)$ (see Figure 3.2d), and the stationary behavior return for $\alpha \in (7.01, 9)$ (see Figure 3.2e), followed by a widow of hyper-chaotic behavior via intermittence scenario (see Figure 3.2f).

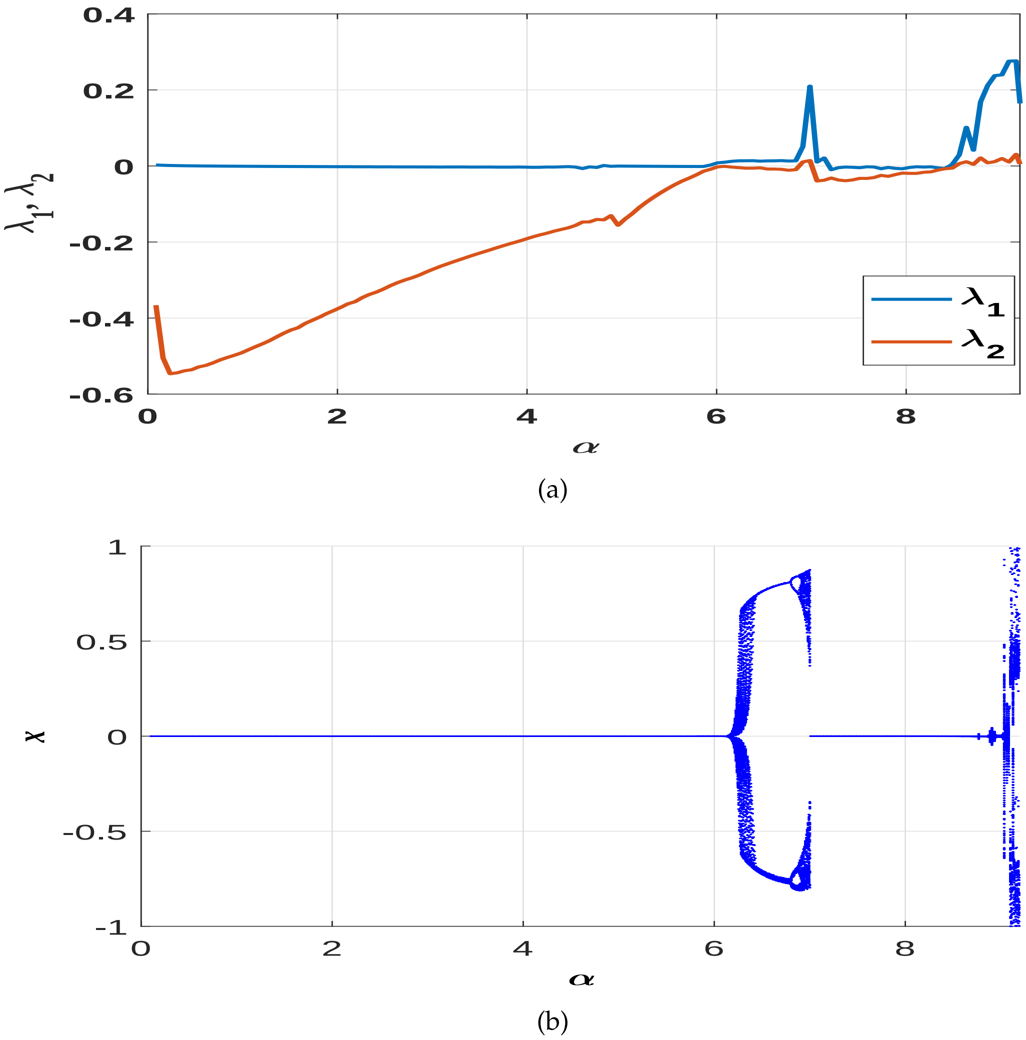


Figure 3.1: The two largest Lyapunov exponents and the bifurcation diagram of x versus $\alpha \in [0, 9.2]$, for $\beta = 10,\ \gamma = 0.11,\ \xi = 0.1,\ a = 1.5$ and $b = 1$.

4 Adaptive control of the memristor-based Chua's system

In this section, we construct an adaptive control law in the objective to stabilize the memristor-based Chua's system with unknown system parameters. The main adaptive control result is established via Lyapunov stability theory.

We consider the memristor-based Chua's system given by

$$\begin{cases} \dfrac{dx(t)}{dt} = \alpha[y(t) - x(t) + \xi x(t) - (-a + b|\omega|)x(t)] + u_1, \\ \dfrac{dy(t)}{dt} = x(t) - y(t) + z(t) + u_2, \\ \dfrac{dz(t)}{dt} = -\beta y(t) - \gamma z(t) + u_3, \\ \dfrac{d\omega(t)}{dt} = x(t) + u_4. \end{cases} \tag{4.1}$$

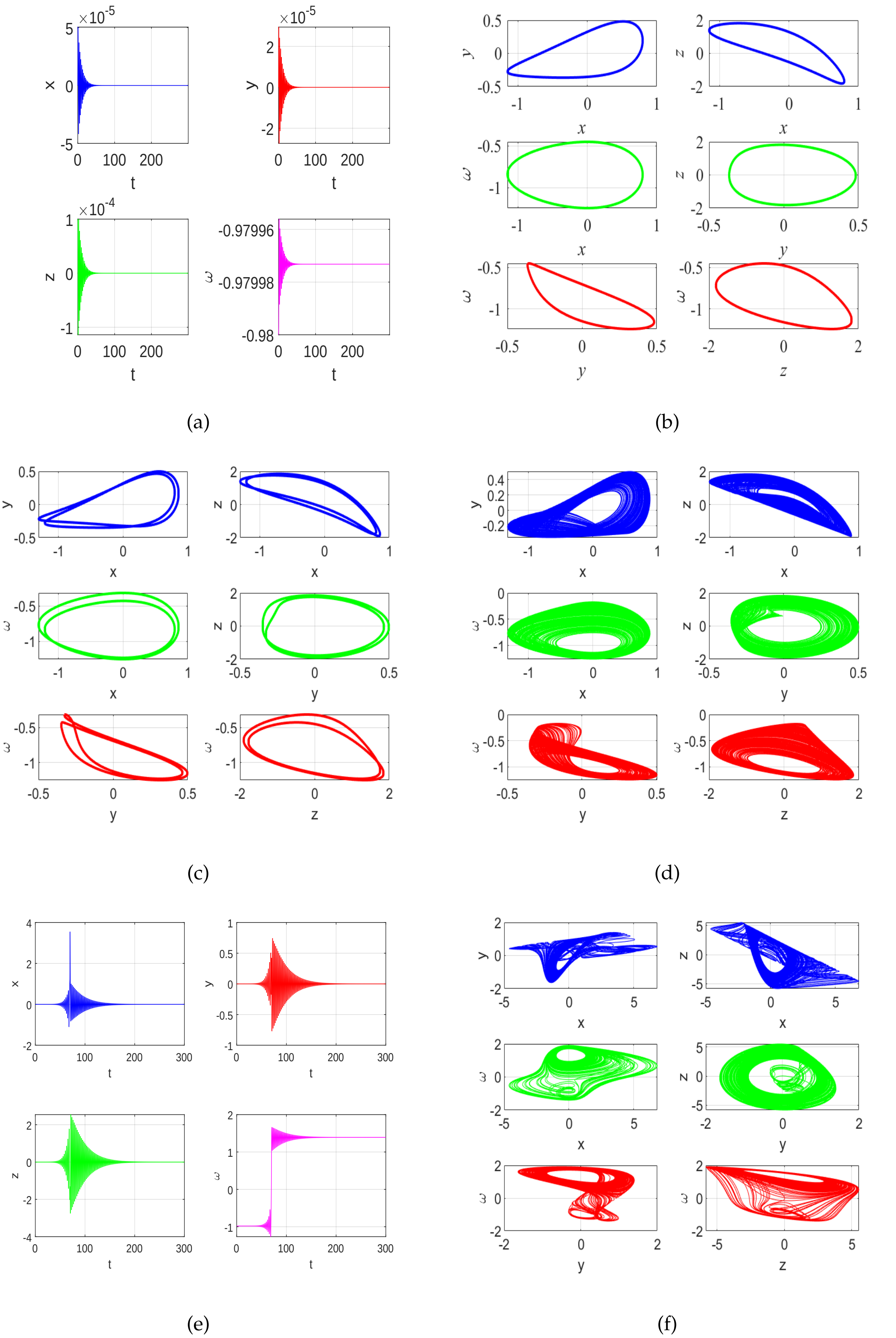


Figure 3.2: Time evolution and phase portrait for: $\beta = 10,\ \gamma = 0.11,\ \xi = 0.1,\ a = 1.5\ b = 1$, and (a) $\alpha = 5$, (b) $\alpha = 6.5$, (c) $\alpha = 7$, (d) $\alpha = 8$, and (e) $\alpha = 9.15$.

Where, $\alpha,\ \beta,\ \gamma,\ \xi,\ a$ and b are considered as constant unknown parameters, and u_i, $i = 1, ..., 4$ is an adaptive control law that will be formulated using estimates $\hat{\alpha}(t), \hat{\beta}(t)$ and $\hat{\gamma}(t)$ of the unknown parameters α, β and γ respectively.

We propose the adaptive control law defined by

$$\begin{cases} u_1 = -\hat{\alpha}(t)\left[y(t) - x(t) + \xi x(t) - \left(-a + b\left|\omega(t)\right|\right)x(t)\right] - k_1 x(t), \\ u_2 = -x(t) + y(t) - z(t) - k_2 y(t), \\ u_3 = \hat{\beta}(t)y(t) + \hat{\gamma}(t)z(t) - k_3 z(t), \\ u_4 = -x(t) - k_4(\omega(t) - \omega_e), \end{cases} \tag{4.2}$$

where k_1, k_2, k_3 and k_4 are positive gain constants.

Substituting (4.2) into (4.1), we get the closed-loop control system as

$$\begin{cases} \dfrac{dx(t)}{dt} = (\alpha - \hat{\alpha}(t))[y(t) - x(t) + \xi x(t) - (-a + b|\omega|)x(t)] - k_1 x(t), \\ \dfrac{dy(t)}{dt} = -k_2 y(t), \\ \dfrac{dz(t)}{dt} = -(\beta - \hat{\beta}(t))y(t) - (\gamma - \hat{\gamma}(t))z(t) - k_3 z(t), \\ \dfrac{d\omega(t)}{dt} = -k_4(\omega(t) - \omega_e). \end{cases} \tag{4.3}$$

The errors of parameter estimation are

$$\begin{cases} e_\alpha = \alpha - \hat{\alpha}(t), \\ e_\beta = \beta - \hat{\beta}(t), \\ e_\gamma = \gamma - \hat{\gamma}(t). \end{cases} \tag{4.4}$$

Taking the derivative of (4.4) with respect to time t yields:

$$\begin{cases} \dfrac{de_\alpha}{dt} = -\dfrac{d\hat{\alpha}(t)}{dt}, \\ \dfrac{de_\beta}{dt} = -\dfrac{d\hat{\beta}(t)}{dt}, \\ \dfrac{de_\gamma}{dt} = -\dfrac{d\hat{\gamma}(t)}{dt}. \end{cases} \tag{4.5}$$

Using (4.4), the closed-loop system (4.3) can be simplified as

$$\begin{cases} \dfrac{dx(t)}{dt} = e_\alpha\left[y(t) - x(t) + \xi x(t) - (-a + b|\omega|)x(t)\right] - k_1 x(t), \\ \dfrac{dy(t)}{dt} = -k_2 y(t), \\ \dfrac{dz(t)}{dt} = -e_\beta y(t) - e_\gamma z(t) - k_3 z(t), \\ \dfrac{d\omega(t)}{dt} = -k_4(\omega(t) - \omega_e). \end{cases} \tag{4.6}$$

We construct an update law for the parameter estimation, based on Lyapunov direct method.. Namely we adopt the Lyapunov candidate function

$$V = \frac{1}{2}\left(x^2 + y^2 + z^2 + (\omega - \omega_e)^2 + e_\alpha^2 + e_\beta^2 + e_\gamma^2\right). \tag{4.7}$$

It is obvious that the function V is a positive definite on $\mathbb{R}^7$.

We differentiate V on the solutions of (4.6) and (4.5), we obtain

$$\frac{dV}{dt} = -k_1x^2 - k_2y^2 - k_3z^2 - k_4(\omega-\omega_e)^2 + e_\alpha\left[yx - x^2 + \xi x^2 - (-a+b\,|\omega|)\,x^2 - \frac{d\hat{\alpha}}{dt}\right]$$

$$+e_\beta\left(-yz - \frac{d\hat{\beta}}{dt}\right) + e_\gamma\left(-z^2 - \frac{d\hat{\gamma}}{dt}\right). \tag{4.8}$$

In view of (4.8), we take the parameter update law as

$$\begin{cases} \dfrac{d\hat{\alpha}(t)}{dt} = yx - x^2 + \xi x^2 - (-a+b\,|\omega|)\,x^2, \\ \dfrac{d\hat{\beta}(t)}{dt} = -yz, \\ \dfrac{d\hat{\gamma}(t)}{dt} = -z^2. \end{cases} \tag{4.9}$$

Next, we state and prove the main result of this section.

Theorem 4.1. *The memristor-based Chua's system (4.1) with unknown parameters is globally exponentially stabilized by the adaptive control law (4.2), and the parameter update (4.9), where k_1, k_2, k_3 and k_4 are positive gain constants.*

Proof. The quadratic Lyapunov function V defined by (4.7) is positive definite on $\mathbb{R}^7$.

Substituting (4.9) into (4.8), we obtain the time derivative of V as

$$\frac{dV}{dt} = -k_1x^2 - k_2y^2 - k_3z^2 - k_4(\omega-\omega_e)^2, \tag{4.10}$$

which is negative semi definite on $\mathbb{R}^7$.

We define $k = \min(k_1, k_2, k_3, k_4)$.

Then it follows from (4.10) that

$$\frac{dV}{dt} = -k_1x^2 - k_2y^2 - k_3z^2 - k_4(\omega-\omega_e)^2 \le -kx^2 - ky^2 - kz^2 - k(\omega-\omega_e)^2 = -k\|X\|^2, \tag{4.11}$$

where $X = (x, y, z, (\omega-\omega_e))$.

That is,

$$\|X\|^2 \le -\frac{dV}{dt}. \tag{4.12}$$

Integrating the inequality (4.12) from 0 to t, we get

$$\int_0^t \|X(\tau)\|^2 d\tau \le V(0) - V(t). \tag{4.13}$$

From (4.13), it follows that $X(t) \in L_2$, while from (4.6), it can be deduced that $\dfrac{dX}{dt} \in L_\infty$.

Thus, by Barbalat's lemma [13], we conclude that $X(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $X(0) \in \mathbb{R}^4$.

This completes the proof. □

5 Numerical simulations

To validate the theoretical findings mentioned above, we implement the proposed adaptive control law (4.2) to stabilize two unstable equilibria: specifically, the origin $(x, y, z, \omega) = (0, 0, 0, 0)$ and $(x, y, z, \omega) = (0, 0, 0, 1.32)$.

The system of differential equations (4.1) and (4.9), along with the control law (4.2), are solved numerically using a Matlab code based on the classical fourth-order Runge-Kutta method with a step size of $h = 0.01$.

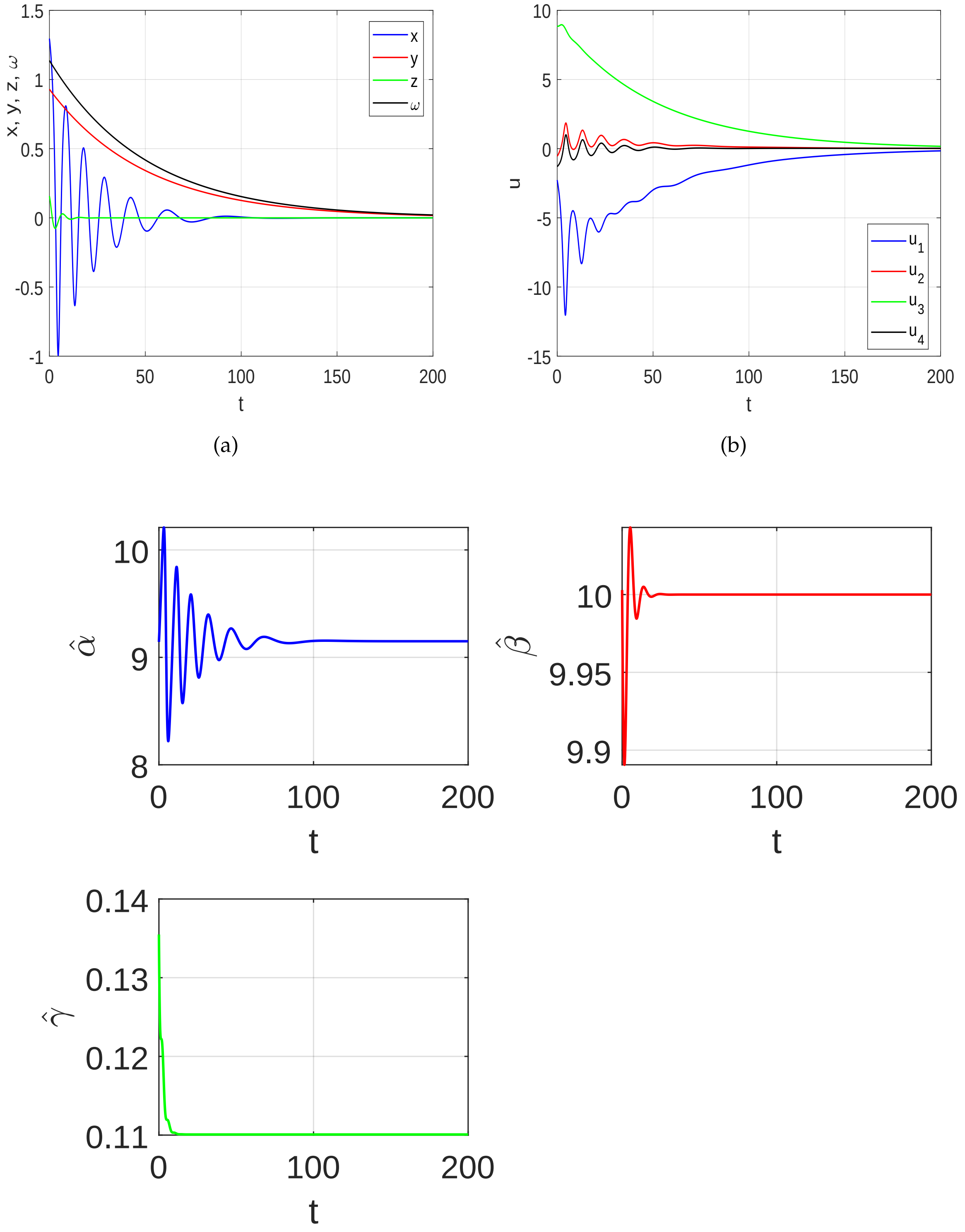
We take the parameter values as in (3.1), and set $\alpha = 9.15$, together with the initial conditions $x(0) = 1.2956$, $y(0) = 0.9294$, $z(0) = 0.1573$, $\omega(0) = 1.1365$.
Additionally, regarding the initial conditions for the parameter estimates, we select $\hat{\alpha}(0) = 9.1439$, $\hat{\beta}(0) = 10.0029$ and $\hat{\gamma}(0) = 0.1355$, and the gain constants as $k_1 = 0.1$, $k_2 = 0.02$, $k_3 = 0.5$, $k_4 = 0.02$.

Figure 5.1, illustrates the stabilization of the origin using the proposed adaptive control law (4.2), the time-histories of the controlled states x, y, z and ω of the memristor-based Chua's hyperchaotic system are depicted in Figure (5.1a), and the evolution of the control effort in Figure (5.1b), whereas the time-histories of the estimated $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are depicted in Figure (5.1c).

Figure 5.2, highlights the stabilization of the unstable equilibrium point $(x, y, z, \omega) = (0, 0, 0, 1.32)$, using the proposed adaptive control law (4.2). The evolution of the controlled states x, y, z and ω is depicted in Figure (5.2a), and the time-histories of the control effort in Figure (5.2b), whereas the evolution of the estimated $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ are depicted in Figure (5.2c).

6 Conclusion

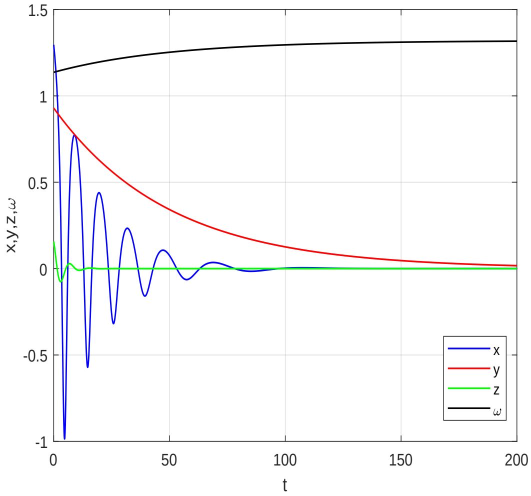
In this paper, the dynamic properties of a 4-dimensional memristor-based Chua's circuit are analyzed, particularly the existence of the chaotic and hyperchaotic behavior is demonstrated using the Lyapunov spectrum and phase portrait. To address these undesirable behaviors, we developed an adaptive control law that stabilizes unstable equilibrium points, taking into account unknown system parameters. The efficacy of the proposed approach is validated through numerical simulations.



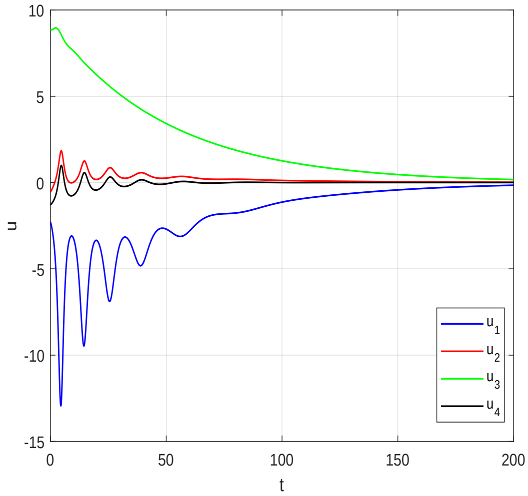
(a) (b)

(c)

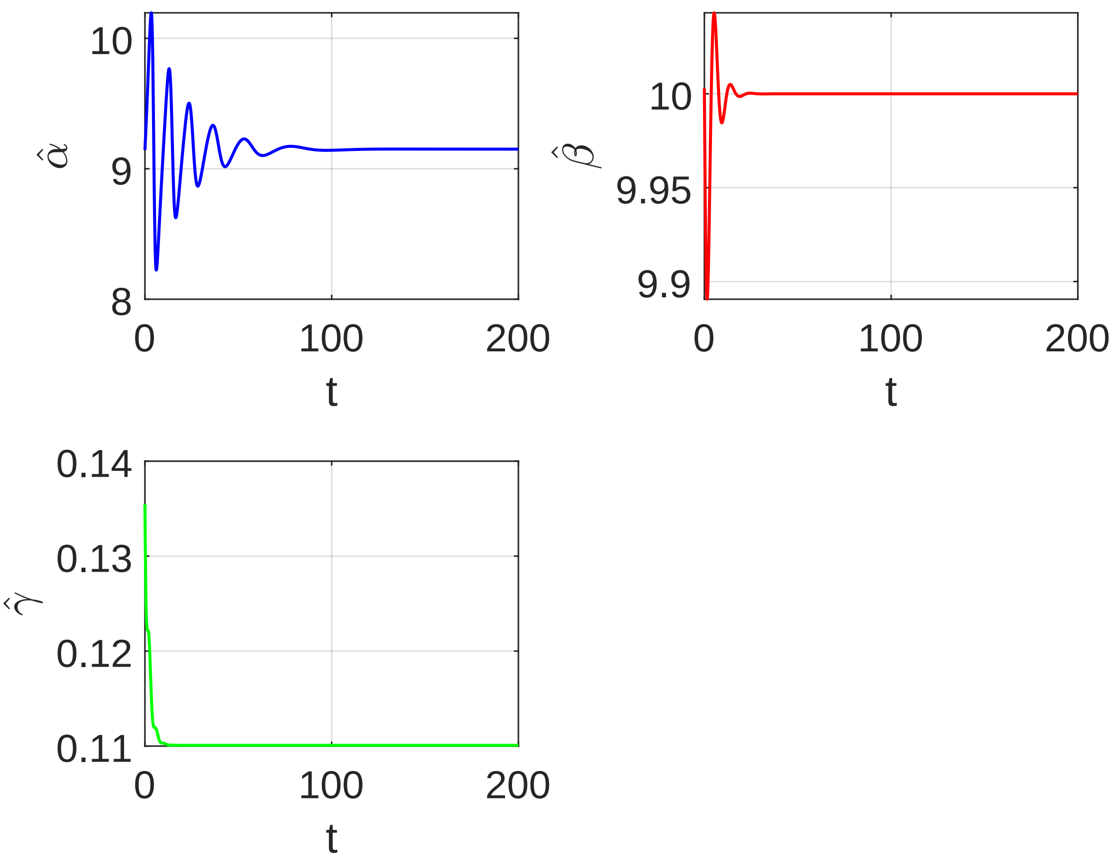
Figure 5.1: Stabilization of the origin $(x, y, z, \omega) = (0, 0, 0, 0)$, using the adaptive control law (4.2), with the initial conditions for the parameter estimates, $\hat{\alpha}(0) = 9.1439$, $\hat{\beta}(0) = 10.0029$, $\hat{\gamma}(0) = 0.1355$, and the constant gains $k_1 = 0.1$, $k_2 = 0.02$, $k_3 = 0.5$, $k_4 = 0.02$. (a) Time-history of the controlled states x, y, z and ω. (b) Time-history of the control effort u_1, u_2, u_3 and u_4. (c) Time-history of the estimated parameters $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$.



(a)



(b)



(c)

Figure 5.2: Stabilization of the unstable equilibrium $(x, y, z, \omega) = (0, 0, 0, 1.32)$, using the adaptive control law (4.2), with the initial conditions for the parameter estimates, $\hat{\alpha}(0) = 9.1439$, $\hat{\beta}(0) = 10.0029$, $\hat{\gamma}(0) = 0.1355$, and the constant gains $k_1 = 0.1$, $k_2 = 0.02$, $k_3 = 0.5$, $k_4 = 0.02$. (a) Time-history of the controlled states x, y, z and ω. (b) Time-history of the control effort u_1, u_2, u_3 and u_4. (c) Time-history of the estimated parameters $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$.

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