NCMA 2022

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National Conference on Mathematics and Applications

November 29-30, 2022

– Mila – Algeria

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NCMA2022

Online via MEET from November 29 to 30, 2022. 2nd International Conference on Mathematics and Applications, the conference was held online for two days at Abdelhafid Boussouf University Center, Mila, Algeria.

Congress Coordinator : Dr. Bououden Rabah

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Contents

PART A

Algebra and number theory

New diophantine equation involving the sum of positive divisors and Euler's function

Bellaouar Djamel

University 8 Mai 1945-Guelma, Department of Mathematics, Guelma, Algeria
E-mail: bellaouar.djamel@univ-guelma.dz

Abstract
For any positive integer n let $\sigma(n)$, $\tau(n)$ and $\varphi(n)$ stand for the divisor sum function of n, the number of positive divisors of n and the Euler function of n, respectively. In this short note, we show that the only solutions of the equation $\sigma(n) = \tau((\varphi(n))^n)$ are 1 and 3.

1 Introduction

Let $\sigma(n)$ be the sum of the natural number divisors of n, so that $\sigma(n) = \sum_{d|n} d$, where d runs over the positive divisors of n including 1 and itself. If n has the prime factorization $n = q_1^{\alpha_1} q_2^{\alpha_2} ... q_s^{\alpha_s}$ with distinct primes $q_1, q_2, ..., q_s$ and positive integers $a_1, a_2, ..., a_s$, then

$$\sigma(n) = \prod_{i=1}^{s} \frac{q_i^{\alpha_i+1} - 1}{q_i - 1}.$$

Let $\tau(n)$ be the divisor function, which counts the number of positive divisors of n. We also have $\tau(n) = (\alpha_1 + 1)(\alpha_2 + 1) ... (\alpha_s + 1)$.

In our main results, we will use the following inequalities (see; eg. [3]):

- For any positive integer n, we have

$$\tau(n) \leq 2\sqrt{n}. \tag{1.1}$$

- For all positive integers m and n, we have

$$\tau(mn) \leq \tau(m)\tau(n). \tag{1.2}$$

- If $(x, y) > 1$, then

$$\tau(x)\tau(y) > \tau(xy) \tag{1.3}$$

  Moreover, if x divides y, then

$$\tau(y) \geq \tau(x). \tag{1.4}$$

Key Words and Phrases: *Diophantine equations, sum of positive divisors, number of positive divisors, Euler's phi function.*

Let $\varphi(n)$ be the Euler function, which counts the number of positive integers $m \leq n$ with $(m,n)=1$. It is well-known that $\varphi(n)=q_1^{\alpha_1-1}(q_1-1)q_2^{\alpha_2-1}(q_2-1)...q_s^{\alpha_s-1}(q_s-1)$. Recall that $\varphi(n^a)=n^{a-1}\varphi(n)$ for every $a \geq 1$. Several authors have treated diophantine equations involving the sum of divisors function and Euler's function. For example, in [5], it shown that $n=2,3,4$ and 5 are the only solutions of the equation $2\sigma(n!)=m!$, while in [4], it shown that $n=2$ is the only known solution of $\sigma(n)=n+\varphi(n)$. Other similar problems have been discussed in publications such as Guy [3]; e.g., §B-38 $\sigma(n)=\varphi(m)$, and §B-42, $\sigma(\varphi(n))=\varphi(\sigma(n))$, $\varphi(\sigma(n))=n$, $\varphi(\sigma(n))=\varphi(n)$.
The present work is a continuation of the author's articles [1],[2]. Let us define the following sets: $E' := \{n \in \mathbb{N} : \sigma(n)=\tau((\varphi(n))^n)\}$, $L' := \{n \in \mathbb{N} : \sigma(n)<\tau((\varphi(n))^n)\}$ and $G' := \{n \in \mathbb{N} : \sigma(n)>\tau((\varphi(n))^n)\}$. We will characterize the elements of these sets.

2 Main Theorem

Theorem 2.1. *We have* $E' := \{1,3\}$ *and* $G' := \{2,4,6,12\}$.

For the proof we need the following lemma.

Lemma 2.1. *Let $s \geq 2$ and let $n_1,...,n_s$ be positive integers with $n_2 \geq 2$ and $(n_i,n_j)=1$ for $i \neq j$. Then* $\tau(\varphi(n_1)^{n_1})...\tau(\varphi(n_2)^{n_2}) < \tau[(\varphi(n_1...n_s))^{n_1...n_s}]$.

Proofs. The proofs with detail will be in the presentation. □

3 Open problem

Consider the diophantine inequality $3\tau(\varphi(m)^m) > \tau(\varphi(m)^{2m})$, where the first odd terms are $m=15,17,51,85,255,257,771,1285,3855,4369,....$ Are there infinitely many?

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On the Order of the trinomial $x^p - a^{p-1}x - a^p$ over $\mathbb{F}_q$

Soufyane BOUGUEBRINE*

Mailing address: Ecole Nationale Suprieure d'Intelligence Artificielle, Algiers, Algeria
E-mail: soufyane.bouguebrine@ensia.edu.dz

Abstract
In this work, we show that the order of the trinomial $x^p - a^{p-1}x - a^p \in \mathbb{F}_q[x]$ is $\frac{p^p-1}{p-1} \cdot \mathrm{ord}(a)$, where $q = p^m$, p is a prime and $\mathrm{ord}(a)$ is the order of a in $\mathbb{F}_q^*$. As a consequence, we give a criterion for the irreducibility of the polynomial $x^{tp} - a^{p-1}x^t - a^p \in \mathbb{F}_q[x]$ and some families of irreducible polynomials over $\mathbb{F}_p$.

1 Introduction

Irreducible polynomials over finite fields are of great importance in both mathematical theory and practical applications, such as coding theory, cryptography, computer science and computational mathematics (see e.g., [1],[2],[3]).
An important integer attached to an irreducible polynomial over a finite field is its order, sometimes also called the period or the exponent of the polynomial. This work is devoted to the order of the trinomial $x^p - a^{p-1}x - a^p$ over $\mathbb{F}_q$.

2 Main results

In the next theorem, we give the order of the polynomial $x^p - a^{p-1}x - a^p \in \mathbb{F}_q[x]$.

Theorem 2.1. *Let $q = p^m$, where $\gcd(m, p) = 1$. Let $a \in \mathbb{F}_q^*$. Then, the order of the trinomial $x^p - a^{p-1}x - a^p \in \mathbb{F}_q[x]$ is $\frac{p^p-1}{p-1}.\mathrm{ord}(a)$.*

As a consequence of Theorem 2.1, we give a criterion for the irreducibility of the polynomial $x^{tp} - a^{p-1}x^t - a^p \in \mathbb{F}_q[x]$.

Corollary 2.1. *Let $q = p^m$, where $\gcd(m, p) = 1$. Let t be a positive integer. Then, the polynomial $x^{tp} - a^{p-1}x^t - a^p$ is irreducible over $\mathbb{F}_q$ if and only if*

(i) $\gcd(t, \dfrac{p-1}{\mathrm{ord}(a)}) = 1$,

(ii) each prime factor of t divides $\dfrac{p^p-1}{p-1}.\mathrm{ord}(a)$,

(iii) if $t = 0 \bmod 4$, then $p = 1 \bmod 4$.

Key Words and Phrases: *Finite Field, Irreducible Polynomial, Order of a Oolynomial*

Corollary 2.2. *Let t be a positive integer. Suppose that each prime factor of t divides $\frac{p^p-1}{p-1}$. Then the trinomial $f(x^t) = x^{t.p} - x^t - a$ is irreducible over $\mathbb{F}_p$. In particular, we have $x^{\frac{p^p-1}{p-1}.p} - x^{\frac{p^p-1}{p-1}} - a$ is irreducible over $\mathbb{F}_p$. Furthermore, if t is prime, then* $\mathrm{ord}(f(x^t)) = t.\mathrm{ord}(f(x))$.

Corollary 2.3. *For any positive integers k, l and m, we have the following families of irreducible polynomials:*

(i) $x^{11^k.71^l.5} - x^{11^k.71^l} - a \in \mathbb{F}_5[x]$, *for all* $a \in \mathbb{F}_5^*$.

(ii) $x^{29^k.7} - x^{29^k} - a \in \mathbb{F}_7[x]$, *for all* $a \in \mathbb{F}_7^*$, *with order* $\frac{7^7-1}{6}.29^k.\mathrm{ord}(a)$.

(iii) $x^{3^k.7} - x^{3^k} - a \in \mathbb{F}_7[x]$, *for* $a = 2, 4 \in \mathbb{F}_7^*$, *with order* $\frac{7^7-1}{6}.3^{k+1}$.

(iv) $x^{5^k.11} - x^{5^k} - a \in \mathbb{F}_{11}[x]$, *for* $a = 3, 4, 5, 9 \in \mathbb{F}_{11}^*$, *with order* $\frac{11^{11}-1}{10}.5^{k+1}$.

(v) $x^{2^{k+2}.5^l.7^m.13} - x^{2^{k+2}.5^l.7^m} - a \in \mathbb{F}_{13}[x]$, *for* $a = 5, 8, 12 \in \mathbb{F}_{13}^*$.

(vi) $x^{3^k.5^l.7^m.13} - x^{3^k.5^l.7^m} - a \in \mathbb{F}_{13}[x]$, *for* $a = 3, 9 \in \mathbb{F}_{13}^*$.

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New properties of an arithmetic function

Brahim Mittou

Department of Mathematics, University Kasdi Merbah of Ouargla, Algeria
EDPNL & HM Laboratory, ENS of Kouba, Algiers, Algeria
E-mail: mathmittou@gmail.com

Abstract
Let the prime factorization of the positive integer $n > 1$ be $n = \prod_{i=1}^{r} p_i^{e_i}$. For a positive integer α the author and Derbal defined f_α to be the arithmetic function such that $f_\alpha(n) = \prod_{i=1}^{r} p_i^{(e_i,\alpha)}$, where (e_i, α) is the greatest common divisor of e_i and α. New properties of f_α are given in this talk. This paper based on [3].

1 Introduction

Throughout this paper, we let $\mathbb{N}^*$ denote the set $\mathbb{N}\backslash\{0\}$ of positive integers and we let (m, n) denote the greatest common divisor of any two integers m and n. A sequence of positive integers $(a_n)_{n\geq 1}$ is simply denoted by $\mathfrak{a}$.
Let the prime factorization of the positive integer $n > 1$ be $n = \prod_{i=1}^{r} p_i^{e_i}$, where $r, e_1, e_2, \ldots, e_r$ are positive integers and $p_1, p_2, \ldots, p_r$ are different primes. The author and Derbal [2] introduced and studied some elementary properties of the following arithmetic function, for a positive integer α:

$$\begin{cases} f_\alpha(1) = 1, \\ f_\alpha(n) = \displaystyle\prod_{i=1}^{r} p_i^{(e_i,\alpha)}. \end{cases}$$

In our presentation, we will discuss other properties of the functions f_α and will define new integer sequences related to them.

2 Main results

$$f_\alpha(mn) = f_\alpha(m)f_\alpha(n) \text{ whenever } (m, n) = 1, \tag{2.1}$$

which means that f_α is a multiplicative function (see e.g., [1]), for all α. The next theorem gives a condition for m and n (which are not necessarily coprime) to be satisfied the equation (2.1), for all even positive integers α.

Theorem 2.1. *Let α be an even positive integer. Then*

$$f_\alpha(mn) = f_\alpha(m)f_\alpha(n)$$

for all square-free positive integers m and n.

Key Words and Phrases: *Arithmetic function, greatest common divisor, strong divisibility sequence.*

Theorem 2.2. *Let α be a positive integer. Then the two following systems of inequalities:*

$$\begin{cases} m < n \\ f_\alpha(m) < f_\alpha(n) \end{cases} \quad \text{and} \quad \begin{cases} m < n \\ f_\alpha(m) > f_\alpha(n) \end{cases}$$

hold for infinitely many positive integers m and n.

Theorem 2.3. *Let $n > 1$ be an integer and let d be a proper positive divisor of n. Then we have*

1. *If n is a square-free number, then $f_\alpha(d)|f_\alpha(n)$ $(\forall \alpha \in \mathbb{N}^*)$.*
2. *If d is a square-free number, then $f_\alpha(d)|f_\alpha(n)$ $(\forall \alpha \in \mathbb{N}^*)$.*
3. *If n and d are not square-free numbers, then there are infinitely many positive integers α such that $f_\alpha(d)|f_\alpha(n)$.*

Theorem 2.4. *Let the set $\mathfrak{F} := \{\mathfrak{f}_\alpha; \alpha \in \mathbb{N}^*\}$. Then $\mathfrak{f}_1$ is the only strong divisibility sequence in $\mathfrak{F}$, where $\mathfrak{f}_\alpha = \{f_\alpha(n); n \in \mathbb{N}^*\}$.*

Theorem 2.5. *Let $\alpha \geq 2$ be an integer. Then*

1. *f_α is a $\mathfrak{f}_1$-strong divisibility sequence.*
2. *f_α is a $\mathbb{P}$-strong divisibility sequence, where $\mathbb{P}$ is the set of all primes.*

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Class of Non-Associative Algebraic Structures

Abdelkrim Mehenni*

Department of Mathematics, Faculty of Mathematics, University of Sciences and Technology Houari Boumediene, Bab-ezzouar, Alger, Algeria.
E-mail: karim28.usthb@gmail.com

Abstract
In the present work, we study a non-associative algebraic structures like trellises as a generalization of lattices by considering sets with a reflexive and antisymmetric, but not necessarily transitive, relation and by postulating the existence of least upper bounds and greatest lower bounds similarly as for partially ordered sets; and, alternatively, by considering sets with two operations that are commutative, absorptive, and, what will be called, part-preserving. Using this approach we are able to prove theorems analogous to nearly all the basic theorems of lattice theory, thus demonstrating the superfluity of the assumption of associativity.

1 Introduction

The ideas of transitivity and partial order are, without question, fundamental in a wide variety of mathematical theories. The mathematical underground, however, has been simmering for some time with notions of non-transitive relationssome arising from common, every-day observations and some from purely mathematical considerations.
An important step in the theory of partial orderings was the postulation of least upper bounds and greatest lower bounds and the development of the theory of lattices. Transitivity is necessary for the associativity of the operations of least upper bound and greatest lower bound. And associativity has been regarded as essential to the theory of lattices as the proofs of many theorems heavily depend upon it. So it would seem that transitivity is an indispensable requirement for lattice theory. However, starting out with a reflexive and antisymmetric, but not necessarily transitive, order, we can define least upper bounds and greatest lower bounds similarly as for partially ordered sets. With this approach we can prove theorems analogous to nearly all the basic theorems of lattice theory, thus demonstrating the superfluity of the assumption of associativity.
The material herein presented contains a foundation for the theory of non-transitive orderings. By stipulation of the existence of least upper bounds and greatest lower bounds we obtain a structure, called a trellis, having properties similar to those of lattices. It is indeed surprising how much can be done under so few assumptions.
In the present work, we using this approach to prove theorems analogous to nearly all the basic theorems of lattice theory, thus demonstrating the superfluity of the assumption of associativity. Moreover, in the presence of certain additional assumptions, such as distributivity, relative complementation and modularity, or others, associativity follows as a consequence.

Key Words and Phrases: *Poset, Lattice, Psoset, Trellis, Binary operation, Associative element, Transitive element*

2 Main results

A *pseudo-order* relation $\trianglelefteq$ on a set X is a binary relation on X that is reflexive (i.e., $x \trianglelefteq x$, for any $x \in X$) and antisymmetric (i.e., $x \trianglelefteq y$ and $y \trianglelefteq x$ implies $x = y$, for any $x, y \in X$). A set X equipped with a pseudo-order relation $\trianglelefteq$ is called a *pseudo-ordered set* (*psoset*, for short) and denoted by $(X, \trianglelefteq)$.

Remark 2.1. It is easily seen that any order relation on a set X is a pseudo-order relation on X.

The notion of trellis was introduced by Fried [1] and Skala [4, 5] as one of the most important algebraic structures in order theory. Which is considered as an extension of the notion of lattice by dropping the property of transitivity. A trellis is defined as a psoset $(X, \trianglelefteq)$ in which pair of elements has a least upper bound and a greatest lower bound.
In other words, a trellis is an algebra $(X, \wedge, \vee)$, where the binary operations $\wedge$ and $\vee$ satisfy the following properties, for any $x, y, z \in X$.

(i) $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$ (*commutativity*);

(ii) $x \wedge (x \vee y) = x = x \vee (x \wedge y)$ (*absorption identity)*;

(iii) $x \wedge ((x \vee y) \wedge (x \vee z)) = x = x \vee ((x \wedge y) \vee (x \wedge z))$ (*weak-associativity*).

Theorem 2.1. *A set X with two commutative, absorptive, and weak-associative operations $\wedge$ and $\vee$ is a trellis if $a \trianglelefteq b$ is defined as $a \wedge b = a$ and/or $a \vee b = b$. The operations are also idempotent and alternative.*

Theorem 2.2. *Let $(X, \trianglelefteq, \wedge, \vee)$ be a trellis. The following statements are equivalent:*

(i) $\trianglelefteq$ is transitive;

(ii) The meet $(\wedge)$ and the join $(\vee)$ operations are associative;

(iii) One of the operations $(\wedge)$ or $(\vee)$ is associative.

Theorem 2.3. *Let $(X, \trianglelefteq, \wedge, \vee)$ be a trellis. Then any $\wedge$-associative (resp. $\vee$-associative) element is transitive .*

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On the difference of Hermitian least rank solutions of matrix equations $A_1XA_1^* = B_1$ and $A_2XA_2^* = B_2$

Sihem Guerarra*
Faculty of Exact Sciences and Sciences of Nature and Life, Department of Mathematics and informatics, University of Oum El Bouaghi, 04000, Algeria
E-mail: guerarra.siham@univ-oeb.dz

Abstract In this paper we derive the extremal ranks and inertias of the difference $X_1 - X_2$, where X_1 and X_2 are Hermitian least rank solutions of the paire of matrix equations $A_1XA_1^* = B_1$ and $A_2XA_2^* = B_2$ respectively. Then give necessary and sufficient conditions for $X_1 - X_2 > 0\,(\geq 0,\ < 0,\ \leq 0)$ in the Löwner partial ordering. And some Hermitian stuctures on this difference.

1 Introduction

Throughout this paper, $\mathbb{C}^{m\times n}$ and $\mathbb{C}_H^n$ stand for the sets of all $m \times n$ complex matrices and all $n \times n$ complex Hermitian matrices respectively, the symbols, A^*, $r(A)$, $\Re(A)$, stand for the conjugate transpose, the rank, and the range of A respectively. I_m denotes the identity matrix of order m. We write $A > 0$ $(A \geq 0)$ if A is Hermitian positive (nonnegative) definite. The Moore-Penrose generalized inverse of a matrix $A \in \mathbb{C}^{m\times n}$, denoted by $A^\dagger$, is defined to be the unique matrix $X \in \mathbb{C}^{n\times m}$ satisfying the following four matrix equations:

$$\text{(1) AXA=A, (2) XAX=X, (3) } (AX)^* = \text{AX, (4) } (XA)^* = \text{XA.}$$

Results on the Moore-Penrose generalized inverse can be found in [1, 3, 2].
Consider the pair of matrix equations

$$A_1XA_1^* = B_1 \text{ and } A_2XA_2^* = B_2 \tag{1.1}$$

where $A_j \in \mathbb{C}^{m_j\times n}$, $B_j \in \mathbb{C}_H^{m_j}$, $j = 1, 2.$are given matrices and $X \in \mathbb{C}_H^n$ is unknown matrix.

2 Main results

Following the work of Y. Tian in [2], in this work we will give some Hermitian stuctures on the difference of the solutions of the matrix equations (1.1).
For convenience of representation, the following notation for the collection of Hermitian least rank solutions of equation (1.1) is adopted

$$S_i = \{X_i \mid \min r\,(B_i - A_iX_iA_i^*) = \min,\ \text{for } i = 1, 2\},\ \text{for } i = 1,\ 2.$$

Key Words and Phrases: *Matrix equation, Rank formulas, Moore Penrose generalized inverse, Hermitian, Least-rank solution, Inertia.*

Theorem 2.1. *Let $A_i \in \mathbb{C}^{m_i \times n}$, $B_i \in \mathbb{C}_H^{m_i}$, and assume that $A_i X A_i^* = B_i$ are consistent for $i = 1, 2$, and let $S_i = \{X_i \mid A_i X A_i^* = B_i\}$*
Denote,

$$K = \begin{bmatrix} B_1 & 0 & A_1 \\ A_1^* & 0 & 0 \\ 0 & B_2 & -A_2 \\ 0 & A_2^* & 0 \end{bmatrix}, \quad N = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}$$

Then,

$$\max_{X_1 \in S_1,\, X_2 \in S_2} r(X_1 - X_2) = \min\left\{n,\, 2n + r\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} - 2r(M_1) - 2r(M_2)\right\}, \tag{2.1}$$

$$\min_{X_1 \in S_1,\, X_2 \in S_2} r(X_1 - X_2) = r\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} - 2r(K), \tag{2.2}$$

$$\max_{X_1 \in S_1,\, X_2 \in S_2} i_\pm(X_1 - X_2) = n + i_\pm\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} - r(M_1) - r(M_2), \tag{2.3}$$

$$\min_{X_1 \in S_1,\, X_2 \in S_2} i_\pm(X_1 - X_2) = i_\pm\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} - r(K). \tag{2.4}$$

Theorem 2.2. *Let $AXB = C$ and L, K, P be stated as in theorem 2.1. Then,*
a) There exist two Hermitian least rank solutions X_1 and X_2 of (1.1) such that $X_1 > X_2$ $(X_1 < X_2)$ if and only if

$$i_+\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = r(M_1) + r(M_2) \ , \ i_-\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = r(M_1) + r(M_2)$$

b) $X_1 > X_2$ $(X_1 < X_2)$ for any two Hermitian least rank solutions X_1 and X_2 of (1.1) if and only if

$$i_+\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = n + r(K) \ , \ i_-\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = n + r(K)$$

c) There exist two Hermitian least rank solutions X_1 and X_2 of (1.1) such that $X_1 \geq X_2$ $(X_1 \leq X_2)$ if and only if

$$i_-\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = r(K) \ , \ i_+\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = r(K).$$

d) $X_1 \geq X_2$ $(X_1 \leq X_2)$ for any two Hermitian least rank solutions of (1.1) if and only if

$$i_-\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = r(M_1) + r(M_2) - n \ , \ i_+\begin{bmatrix} N & K \\ K^* & 0 \end{bmatrix} = r(M_1) + r(M_2) - n.$$

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A New Family of Generating Functions of Binary Products of Tribonacci Numbers and Ortogonal Polynomials

Sahali Rokiya *

LMAM Laboratory and Department of Mathematics, University of Jijel, Jijel, Algeria
E-mail: rokasali18@gmail.com

Boussayoud Ali

LMAM Laboratory and Department of Mathematics, University of Jijel, Jijel, Algeria
E-mail: aboussayoud@yahoo.fr

Abstract
In this paper, we will recover the generating functions of some products of Tribonacci numbers and orthogonal polynomials. The technique used here is based on the theory called symmetric functions.

1 Introduction

In mathematics, orthogonal polynomials consist of polynomials such that any two different polynomials in the sequence are orthogonal to each other under some inner product. The most widely used orthogonal polynomials are the classical orthogonal polynomials (Tchebyshev polynomials of first and second kinds, Tchebyshev polynomials of third and fourth kind Fibonacci and Lucas polynomials). In [5], the Tribonacci sequence originally was studied in 1963 by M. Feinberg. For any integer $n \geq 0$, the Tribonacci numbers T_n were defined by the recurrence relation

$$\begin{cases} T_0 = 1, T_1 = 1, T_2 = 2 \\ T_{n+1} = T_n + T_{n-1} + T_{n-2}, \ n \geq 2 \end{cases}. \tag{1.1}$$

Consider the characteristic polynomial $x^3 - x^2 - x - 1 = 0$ associated to recursive relation (1.1) with having the roots $\alpha = \frac{1+\sqrt[3]{19+3\sqrt{33}}+\sqrt[3]{19-3\sqrt{33}}}{3}, \beta = \frac{1+w\sqrt[3]{19+3\sqrt{33}}+w^2\sqrt[3]{19-3\sqrt{33}}}{3}, \gamma = \frac{1+w^2\sqrt[3]{19+3\sqrt{33}}+w\sqrt[3]{19-3\sqrt{33}}}{3}$ where $w = \frac{-1+i\sqrt{3}}{2}$.
The Binet formulas for Tribonacci numbers is

$$T_n = -\frac{(\beta+\gamma-\beta\gamma-2)}{(\alpha-\beta)(\alpha-\gamma)}\alpha^n + \frac{(\alpha+\gamma-\alpha\gamma-2)}{(\alpha-\beta)(\beta-\gamma)}\beta^n - \frac{(\alpha+\beta-\alpha\beta-2)}{(\alpha-\gamma)(\beta-\gamma)}\gamma^n.$$

In [4], Chelgham et al. give another definitions of Tribonacci numbers by $T_n = S_n(E)$. In [2], the author derived the different recurrence relations on the Tribonacci numbers and their sums and got some identities of the Tribonacci numbers and their sums by using the companion matrices and generating matrices.

Key Words and Phrases: *Symmetric functions, Generating functions, Tribonacci numbers, Orthogonal polynomials.*

2 Main results

Theorem 2.1. *[1] Let $A = \{a_1, a_2\}$ and $E = \{e_1, e_2, e_3\}$ two alphabets, we have*

$$\sum_{n=0}^{\infty} S_n(A)S_n(E)t^n = \frac{1 - a_1a_2S_2(-E)t^2 - a_1a_2(a_1+a_2)S_3(-E)t^3}{\prod\limits_{e\in E}(1-ea_1t)\prod\limits_{e\in E}(1-ea_2t)}. \tag{2.1}$$

Proposition 2.1. *[1] Let $A = \{a_1, a_2\}$ and $E = \{e_1, e_2, e_3\}$ two alphabets, we have*

$$\sum_{n=0}^{\infty} S_{n-1}(A)S_n(E)t^n = \frac{-S_1(-E)t - (a_1+a_2)S_2(-E)t^2 - ((a_1+a_2)^2 - a_1a_2)S_3(-E)t^3}{\prod\limits_{e\in E}(1-ea_1t)\prod\limits_{e\in E}(1-ea_2t)}. \tag{2.2}$$

Theorem 2.2. *For $n \in \mathbb{N}$, the new generating function of the product of Tribonacci numbers with Lucas polynomials is given by*

$$\sum_{n=0}^{\infty} L_n(x)T_nt^n = \frac{2 - xt - (x^2+2)t^2 - x(x^2+3)t^3}{1 - xt - (x^2+3)t^2 - (x^3+4x)t^3 - (x^2+1)t^4 + xt^5 - t^6}.$$

Theorem 2.3. *For $n \in \mathbb{N}$, the new generating function of the combined Tribonacci numbers and Jacobsthal-Lucas polynomials is given by*

$$\sum_{n=0}^{\infty} j_n(x)T_nt^n = \frac{2x + (1-2x)t + (1-2x-4x^2)t^2 + (1-8x^2)t^3}{1 - t - (6x+1)t^2 - (8x+1)t^3 - 2x(2x+1)t^4 + 4x^2t^5 - 8x^3t^6}.$$

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On the realizability of Cohn's matrix over $SL_3(R[x_1^{\pm},..,x_k^{\pm}])$

Mime Rania*

Mathematics Department, Exact and Applied Sciences Faculty, Oran 1 University, Algeria
E-mail: mime.rania@edu.univ-oran1.dz

Boudaoud Fatima

Mathematics Department, Exact and Applied Sciences Faculty, Oran 1 University, Algeria
E-mail: fboudaoud@yahoo.fr

Abstract
The objective in this work is to develop a new algorithm that can determine the realization of matrices in SL_2 over the multivariate Laurent polynomial ring$R[x_1^{\pm},...,x_k^{\pm}]$; We are interested in Park's algorithm [2] with which one can test whether the matrix A (in our case A is the Cohn matrix) allows a factorization into elementary matrices and if it does give its explicit factorization.

1 Introduction

the symbol R denote an arbitrary Euclidean ring, which allows a suitable generalization of the Euclidean division algorithm. It's known that every matrix in $SL_n(R[x_1^{\pm},...,x_k^{\pm}])$ for $n \geq 3$ is realizable based on Suslin's stability theorem [3], furthermore, it was proved by P. M. Cohn in [1] that this theorem cannot be applied in the case where $n = 2$. Now if we extend these results over the multivariate Laurent polynomial ring the Cohn matrix is realizable and a factorization is given by Tolhuizen, Hollmann, Kalker in [4];

$$C = \begin{pmatrix} 1 & 0 \\ -\frac{y}{x} & 1 \end{pmatrix} \begin{pmatrix} 1 & x^2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{x}{y} & 1 \end{pmatrix}.$$

In this work we construct a new realization algorithm over $SL_2(R[x_1^{\pm},..,x_k^{\pm}])$ with the appropriate monomial order to determine the realisation of the proposed matrices.

Key Words and Phrases: *Suslin's stability theorem, Laurent polynomial ring, Cohn's matrix monomial orders, Park's algorithm.*

2 Main results

Definition 2.1. 1. $SL_n(R)$ is the set of $n \times n$ matrices of determinant 1 whose entries are elements of R.

2. An elementary matrix E_{ij} over R is defined as follows : for $i \neq j$

$$E_{ij}(a) = I + ae_{ij}, \text{ where } e_{ij} = \begin{cases} 1 & \text{for } (i,j) \\ 0 & Elsewhere \end{cases}$$

3. $A \in SL_n(R)$ is called realizable if it can be written as a product of elementary matrices.

Definition 2.2. A monomial ordering is a relation on $Z^n_{\geq 0}$ that verifies

1. The relation $\geq$ is a total ordering;
2. If $\alpha \geq \beta$, and $\gamma \in Z^n_{\geq 0}$ then $\alpha + \gamma \geq \beta + \gamma$;
3. The relation $\geq$ is a well-ordering.

There are several term orderings. We list two of the most commonly used monomial order : for $\alpha, \beta \in Z^n_{\geq 0}$

Lexicographic ("dictionary") Here $\alpha \geq_{lex} \beta$ if the left-most nonzero entry of $\alpha - \beta$ is positive we write $x^\alpha \geq x^\beta$. The power of the first variable is used to determine the order.

Degree Lexicographic Sort first by the total degree, then by the lexicographic degree. Here $\alpha \geq_{dlex} \beta$ if
$|\alpha| := \sum_{k=1}^{n} \alpha_k \geq |\beta| := \sum_{k=1}^{n} \beta_k$ or $|\alpha| = |\beta|$ and $\alpha \geq_{lex} \beta$.

Theorem 2.1 (Park's theorem [2])**.** *Let* $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \in SL_2(R[x^\pm, .., x_k^\pm])$, *for a fixed monomial order* $' >'$. *If A is a nonconstant realizable matrix, then either A has a zero entry or one of the row vectors of $LT(A)$ is a monomial multiple of the other row.*

Lemma 2.1. *Let* $A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \in SL_2(R[x_1^\pm, .., x_k^\pm])$, *if one of the entries of A is zero or invertible then A is realizable.*
The explicit factorization of A in each case is given as follows

- *If* $a_2^{-1} \in SL_2(R[x_1^\pm, .., x_k^\pm])$

$$A = E_{21}(a_2^{-1}(a_4 - 1))E_{12}(a_2)E_{21}(a_2^{-1}(a_1 - 1)). \tag{2.1}$$

- *If* $a_3^{-1} \in SL_2(R[x_1^\pm, .., x_k^\pm])$

Example 2.1. The Cohn matrix is realizable in $SL_2(R[x^{\pm}, y^{\pm}])$ it has two explicit factorization that we can obtain by using the formula (2.1) or (2.2).
Let $C = \begin{pmatrix} 1+xy & x^2 \\ -y^2 & 1-xy \end{pmatrix}$

$$C = E_{21}(-\frac{y}{x})E_{12}(x^2)E_{12}(\frac{y}{x}).$$

Or

$$C = E_{12}(-\frac{x}{y})E_{21}(-y^2)E_{12}(\frac{x}{y}).$$

In view of these results we extend the realization algorithm for matrices of the special form

$$A = \begin{pmatrix} 1+xy & x^2 & 0 \\ -y^2 & 1-xy & 0 \\ p & q & 1 \end{pmatrix} \in SL_3(R[x_1^{\pm}, ..., x_k^{\pm}]),$$

With $\tilde{A} = \begin{pmatrix} 1+xy & x^2 \\ -y^2 & 1-xy \end{pmatrix} \in SL_2(R[x_1^{\pm}, ..., x_k^{\pm}])$.
By applying elementary operations on A we obtain

$$AE(-p)E(-q) = \begin{pmatrix} 1+xy & x^2 & 0 \\ -y^2 & 1-xy & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \tilde{A} & 0 \\ 0 & 1 \end{pmatrix} \in SL_3(R[x_1^{\pm}, ..., x_k^{\pm}]).$$

The realization of $\begin{pmatrix} \tilde{A} & 0 \\ 0 & 1 \end{pmatrix} \in SL_3(R[x_1^{\pm}, ..., x_k^{\pm}])$ is now reduced to the same problem but for $\tilde{A} \in SL_2(R[x_1^{\pm}, ..., x_k^{\pm}])$. To express $\tilde{A}$ as a product of elementary matrices, we apply the results obtained in the example 2.1.

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Généraliser l'algorithme de Buchberger pour les Anneaux Booléens

Mouseddek Hadjer Ryma*

Adresse postale : Départment de Mathématiques, université Oran 1, Oran, Algérie
E-mail: mouseddekhadjerryma@gmail.com

Boudaoud Fatima

Adresse postale : Départment de Mathématiques, Université Oran 1, Oran, Algérie
E-mail: fboudaoud@yahoo.fr

Abstrait

Nous calculons une Base de Gröbner dynamique sur $\mathbb{F}_2\,[a_1, ..., a_n]\,/\,\langle a_1^2 - a_1, ..., a_n^2 - a_n \rangle$, et résoudre le problème délicat causé par les diviseurs de zéro qui apparaissent comme des coefficients dominants pour répondre à la question ouverte : " comment généraliser l'algorithme de Buchberger pour les Anneaux Booléens " qui est mentionnée par Cai et Kapur dans leur article. Les bases de Gröbner permettent de résoudre algorithmiquement de nombreux problèmes portant sur les idéaux d'anneaux de polynômes.

1 Introduction

Le concept des bases de Gröbner a été introduit par Bruno Buchberger en 1965 dans sa thèse de doctorat et il lui a donné le nom de son directeur de thèse Wolf Gang Gröbner, afin de résoudre le problème d'appartenance d'un idéal pour les anneaux polynomiaux sur un corps. Buchberger a non seulement montré que chaque idéal polynomial a une base de Gröbner mais il a également donné un algorithme à partir de toute base d'idéaux. il a fallu quelques années avant que le concept devienne populaire parmi les mathématiciens et les informaticiens.

Le problème d'appartenance d'un idéal un des principaux problèmes que l'on peut aborder en utilisant les bases de Gröbner, il a reçu une attention considérable de la communauté de l'algèbre constructive résultant en des algorithmes qui généralisent le travail de Buchberger.

Mots clés et Phrases: *Bases de Gröbner dynamique, anneaux Booléens, annulateur, diviseurs de zéro, localisation.*

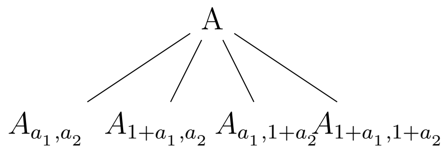
2 Résultat

L'utilisation des bases de Gröbner dynamique permet de résoudre le problème délicat causé par les diviseurs de zéro qui apparaissent comme des coefficients dominants.

Cai et Kapur ont conclut leur article en mentionnant la question ouverte :
comment généraliser l'algorithme de Buchberger pour les Anneaux Booléens. Comme exemple typique d'une situation problématique, ils ont étudié le cas où l'anneau de base est $\mathbb{F}_2\left[a_1, a_2\right] / \left\langle a_1^2 - a_1, a_2^2 - a_2 \right\rangle$.
Dans ce cas la méthode proposée dans leur article ne fonctionne pas en raison du fait qu'un annulateur $a_1a_2 + a_1 + a_2 + \in A$ peut être a_1 ou a_2 et ainsi, il peut exister des multiannulateurs incomparables pour un élément de A.

Les bases de Gröbner dynamique permettent de surmonter équitablement cette difficulté. Dans ce cas ; on a calculé une base de Gröbner dynamique composée de quatre bases de Gröbner sur les localisations de A. On voit qu'à chaque Branche de l'arbre binaire construit, le problème soulevé par Cai et Kapur disparait complètement. Il est simple que les bases de Gröbner dynamique sur les anneaux de Dedekind pourraient tre une solution satisfaisante à ce problème ouvert.

Considérons l'anneau $\mathbf{A} = \mathbb{F}_2\left[a_1, a_2\right] / \left\langle a_1^2 - a_1, a_2^2 - a_2 \right\rangle = \mathbb{F}_2 a_1 a_2 + \mathbb{F}_2 a_1 + \mathbb{F}_2 a_2 + \mathbb{F}_2$ avec les relations $a_1^2 = a_1$ et $a_2^2 = a_2$. En travaillant avec l'anneau A, nous savons à l'avance que l'arbre binaire que nous construirons lors du calcul dynamique d'une base de Gröbner d'un idéal de $A\left[X_1, ..., X_n\right]$ est formé seulement de quatre feuilles comme suit :



Donc calculer une base de Gröbner sur A revient calculer quatre bases de Gröbner sur $\mathbb{F}_2$. qui correspondent $(a_1, a_2) = (1,1) = (1,0) = (0,1) = (0,0)$.

De plus, nous définissons une "base de Gröbner dynamique réduite" comme un ensemble :

$\{(G_1, a_1^{\mathbb{N}} a_2^{\mathbb{N}}), (G_2, a_1^{\mathbb{N}}(1+a_2)^{\mathbb{N}}), (G_3, (1+a_1)^{\mathbb{N}} a_2^{\mathbb{N}}), (G_4, (1+a_1)^{\mathbb{N}}(1+a_2)^{\mathbb{N}})\}$

où chaque G_i est une base de Gröbner dynamique réduite sur $\mathbb{F}_2$.

Et on va travailler sur l'anneau $\mathbb{F}_2\left[a_1, ..., a_n\right] / \left\langle a_1^2 - a_1, ..., a_n^2 - a_n \right\rangle$ pour généraliser l'algorithme de Buchberger pour les anneaux Booléens.

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New Generalization of Gelin-Cesaro's identity for (p, q)-numbers

Mourad Chelgham*
Mailing address: Departement of Mathematics
Faculty of Exact Sciences and informatics
MSBY University
LMAM Laboratory
B.P. 98, 18000, Jijel
Algeria
E-mail: chelghamm@yahoo.fr

Abstract
In this talk, we will give a generalisation of Gelin-Cesaro identity so that of Catalan identity for (p, q)-Fibonacci and (p, q)-Lucas numbers and we show its relationship with triangular numbers of different order. We also present as applications the generalization of this identity for some well-known (p, q)-Fibonacci and (p, q)-Lucas numbers.

1 Introduction

Horadam et al. [3] gave a generalization of Catalan identity for five consecutive terms of the Horadam sequence $(W_n)_{n\in\mathbb{N}}$ defined by the recurrence relation

$$W_{n+1} = pW_n - qW_{n-1},$$

for $n \geq 1$, with $W_0 = \alpha$ and $W_1 = \beta$, where p, q, α and β, with $q \neq 0$ are integer numbers. Mhelam et al. [4] gave other generalization of Catalan identity and get the results obtained by Hadamard. In this study, we are mainly interested by a new generalization of Gelin-Cesaro identity so that of Catalan identity for (p, q)-Fibonacci and (p, q)-Lucas numbers. Recall that the (p, q)-Fibonacci and (p, q)-Lucas numbers noted respectively $(F_{p,q,n})_{n\in\mathbb{N}}$ and $(L_{p,q,n})_{n\in\mathbb{N}}$ are generalization of the well known Fibonacci and Lucas numbers.
The (p, q)-Fibonacci and (p, q)-Lucas numbers, are defined respectively by [5], for $n \geq 2$ and, p, q integer numbers, as follows:

$$F_{p,q,n} = pF_{p,q,n-1} + qF_{p,q,n-2}, \tag{1.1}$$

where $F_{p,q,0} = 0, F_{p,q,1} = 1$,

$$L_{p,q,n} = pL_{p,q,n-1} + qL_{p,q,n-2}, \tag{1.2}$$

where $L_{p,q,0} = 2, L_{p,q,1} = p$.
We call figurate numbers, numbers that can be represented by geometric figures. The study of these numbers is of great interest because of their precursor role in the study of sequences, series and in combinatorial analysis. Among these numbers, we cite in particular Square numbers, Triangular numbers, Pentagonal numbers and Pyramidal numbers, etc.

Key Words and Phrases: *Catalan identity, Gelin-Cesaro identity, (p, q)-Fibonacci number, (p, q)-Lucas number, Triangular number of order j*

Definition 1.1. For all $n, j \in \mathbb{N}$, the triangular number sequence of order j noted $\{T_n^j\}_{n\in\mathbb{N}}$ is defined by

$$T_n^0 = 1 \text{ and } T_n^j = \frac{n(n+1)\ldots(n+j-1)}{j!}, (j \geq 1) \tag{1.3}$$

2 Main results

2.1 Generalisation of Catalan's identity for (p, q)-Fibonacci numbers

Recall that for $n \geq 1$ and p, q integer numbers, the Gelin-Cesaro identity for the (p, q)-Fibonacci numbers, is given in the following proposition:

Proposition 2.1. *For $n \geq 2$ and p, q integer numbers, we have*

$$F_{p,q,n-2}F_{p,q,n-1}F_{p,q,n+1}F_{p,q,n+2} = F_{p,q,n}^4 - B_{1,2}F_{p,q,n}^2 + B_{2,2}, \tag{2.1}$$

where

$$B_{1,2} = \sum_{1\leq j\leq 2} b_j = \sum_{1\leq j\leq 2} (-q)^{n-j} F_{p,q,j}^2,$$
$$B_{2,2} = (-q)^{2n-3}F_{p,q,1}^2F_{p,q,2}^2 = (-q)^{2n-3}F_{p,q,2}^2.$$

The next theorem generalizes the Gelin-Cesaro's identity (2.1), for (p, q)-Fibonacci numbers, i.e., calculate the product: $F_{p,q,n-r}\cdots F_{p,q,n-1}F_{p,q,n+1}\cdots F_{p,q,n+r}$, in terms of $F_{p,q,n}^2, F_{p,q,n}^4, \ldots, F_{p,q,n}^{2r-2}, F_{p,q,n}^{2r}$.

Theorem 2.1. *For all $n, r \in \mathbb{N}$ and p, q integer numbers, we have*

$$\prod_{1\leq j\leq r} F_{p,q,n-j}F_{p,q,n+j} = \sum_{j=0}^{r}(-1)^j B_{j,r}F_{p,q,n}^{2(r-j)},$$

where

$$B_{0,r} = 1,$$
$$B_{1,r} = \sum_{1\leq j\leq r} b_j = \sum_{1\leq j\leq r} (-q)^{n-j}F_{p,q,j}^2,$$
$$B_{j,r} = \sum_{1\leq i_1<i_2<\cdots<i_j\leq r} b_{i_1}b_{i_2}\cdots b_{i_j}, (2 \leq j \leq r-1),$$

and

$$b_{i_s} = (-q)^{n-i_s}F_{p,q,i_s}^2, (1 \leq s \leq j),$$
$$B_{r,r} = (-q)^{rn-\sum_{s=1}^{r} s} \prod_{1\leq s\leq r} F_{p,q,s}^2.$$

2.2 Generalisation of Catalan's identity for (p,q)-Lucas numbers

Recall that for $n \geq 1$ and p,q integer numbers, the Gelin-Cesaro identity for the (p,q)-Lucas numbers, is given in the following proposition:

Proposition 2.2. *For all $n \in \mathbb{N}$ and p,q integer number, we have*

$$L_{p,q,n-2}L_{p,q,n-1}L_{p,q,n+1}L_{p,q,n+2} = L_{p,q,n}^4 + (B_{1,2} - 2a)\, L_{p,q,n}^2 + B_{2,2} - B_{1,2}a + a^2. \quad (2.2)$$

Where $B_{1,2} = \sum\limits_{1 \leq j \leq 2} (-q)^{n-j} L_{p,q,j}^2$, $B_{2,2} = (-q)^{2n-3} L_{p,q,1}^2 L_{p,q,2}^2$ *and* $a = 4(-q)^n$.

The next theorem generalizes the Gelin-Cesaro identity (2.2), for (p,q)-Lucas numbers by calculate the product: $L_{p,q,n-r} \cdots L_{p,q,n-1} L_{p,q,n+1} \cdots L_{p,q,n+r}$, in terms of $L_{p,q,n}^2, L_{p,q,n}^4, \ldots, L_{p,q,n}^{2r-2}$ and $L_{p,q,n}^{2r}$.

Theorem 2.2. *For all $n, r \in \mathbb{N}$ such that $r \neq 0$ and p,q integer numbers, the generalization of Catalan's identity for (p,q)-Lucas numbers is given by*

$$\prod_{1 \leq j \leq r} L_{p,q,n-j} L_{p,q,n+j} = \sum_{j=0}^{r} \sum_{i=0}^{r-j} (-1)^i B_{r-j-i,r} T_{i+1}^{j} a^i L_{p,q,n}^{2j},$$

where

$$B_{0,r} = 1,$$
$$B_{j,r} = \sum_{1 \leq i_1 < i_2 < \cdots < i_j \leq r} b_{i_1} b_{i_2}, \ldots, b_{i_j}, (1 \leq j \leq r);$$

where $b_{i_s} = (-q)^{n-i_s} L_{p,q,i_s}^2, (1 \leq s \leq j)$, *and* $a = 4(-q)^n$. *Such that* T_{i+1}^j *represents the* $(i+1)^{th}$ *triangular number of order* j

3 Applications

As application, we present the generalization of the Gelin Cesaro identity for certain well known (p,q)-numbers such as for example the k-Fibonacci, k-Lucas, k-Mersenne and k-Mersenne-Lucas numbers and the cases for $k=1$.

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PART B

Applications of mathematics

Numerical solution of fractional differential equations with temporal two-point BVPs using reproducing kernel Hilbert space method

yassamine chellouf*

Mailing address: Mathematics, university center of Mila, Mila, Algeria
E-mail: y.chellouf@centre-univ-mila.dz

Banan Maayah

Mailing address: Mathematics, University of Jordan, Amman, Jordan
E-mail: b.maayah@ju.edu.jo

Abstract
In this work, the reproducing kernel Hilbert space method had been extended to model a numerical solution with two-point temporal boundary conditions for the fractional derivative in the Caputo sense, convergent analysis and error bounds are discussed to verify the theoretical results. Numerical examples are given to illustrate the accuracy and efficiency of the presented approach.

1 Introduction

Fractional calculus is an extension of ordinary calculus, in a way that derivatives and integrals are defined for arbitrary real order. In some phenomena, fractional operators allow to model better than ordinary derivatives and ordinary integrals, and can represent more efficiently systems with high-order dynamics and complex nonlinear phenomena. The history of the fractional derivative theory back to the question asked by L'Hopital to Leibniz in 1695, what the result would be to $D^n f$ if n was a fraction. Since that time, many prominent mathematicians, such as Euler, Laplace, Fourier, Abel, Riemann-Liouville, and Laurent, have been interested in fractional calculus where the first definition of the fractional derivative was introduced at the end of the 19-th century by Riemann-Liouville.
In practice, where quantitative results are needed for given real-life problems, numerically approximate solutions can often be demonstrably better, more detailed, efficient, and cost-effective than analytical ones for certain fractional structures. A number of studies were therefore involved in developing approaches for providing estimated solutions, one of these approaches is the Reproducing Kernel Hilbert Space (RKHS) method, which has been successfully applied to various fields of numerical analysis, computational mathematics, probability and statistics, biology, ect.
The two-point BVPs has a strong interest in applied mathematics, this kind of problems arise directly from mathematical models or by transforming partial differential equations into ordinary differential equations. As this type of problems does not have an exact solution, many special techniques have been used to solve it, including the shooting method, the collocation method,

Key Words and Phrases: *reproducing kernel Hilbert space method (RKHSM), fractional differential equations, temporal two-point boundary value problems, numerical method*

the finite difference method, and the quasilinearization method.
In our work, we propose the RKHS method to provide the approximate solutions of these BVP's for the fractional derivative in the Caputo sense.

2 Main results

Example 1

$$\begin{cases} \mathcal{D}^{\alpha}\mathrm{U}(x) &= -4\mathrm{U} + 3\mathrm{V} + 6, \\ \mathcal{D}^{\alpha}\mathrm{V}(x) &= -2.4\mathrm{U} + 1.6\mathrm{V} + 3.6, \quad 0 \leq x \leq 0.5, \quad 0 \leq \alpha \leq 1, \\ \mathrm{U}(0) &= 0, \\ \mathrm{V}(0.5) &= -2.25e^{-1} + 2.25e^{-0.2}, \end{cases}$$

Table 1: Numerical results of problem 1 using Caputo derivative

x	Abs Error of $\mathrm{U}(x)$	Rel Error of $\mathrm{U}(x)$	Abs Error of $\mathrm{V}(x)$	Rel Error of $\mathrm{V}(x)$
0.	0.	Indeterminate	0.	Complex Infinity
0.1	2.8979×10^{-5}	5.3838×10^{-5}	1.7129×10^{-6}	5.3589×10^{-6}
0.2	1.6912×10^{-5}	1.7462×10^{-5}	5.6398×10^{-6}	9.9154×10^{-6}
0.3	7.6470×10^{-6}	5.8341×10^{-6}	1.1152×10^{-5}	1.4659×10^{-5}
0.4	5.4565×10^{-7}	3.4507×10^{-7}	1.5248×10^{-5}	1.6823×10^{-5}
0.5	4.8467×10^{-6}	2.7023×10^{-6}	1.8229×10^{-5}	1.7970×10^{-5}

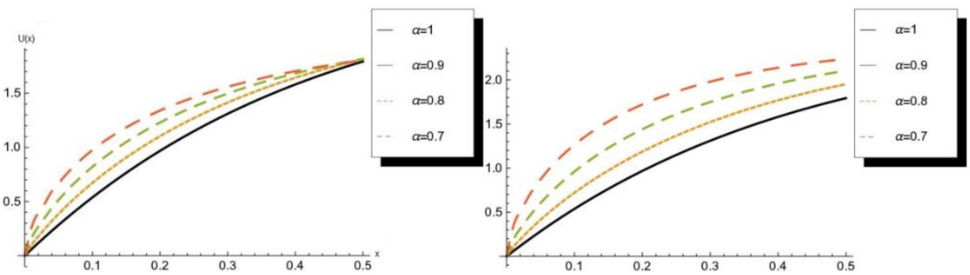

Figure 1: Approximate solution for different values of α for problem 1 using Caputo derivative.

Example 2

$$\begin{cases} \mathcal{D}^{\alpha}\mathrm{U}(x) &= \mathrm{U}^2 - 4(\mathrm{U} - 1) - \cos^2(x) - \sin(x), \\ \mathcal{D}^{\alpha}\mathrm{V}(x) &= \mathrm{UV} - 2\mathrm{V} - x^2\cos(x) + 2x, \\ \mathrm{U}(0) &= 3, \\ \mathrm{V}(1) &= 1, \end{cases} \qquad 0 \leqslant x \leqslant 1,$$

Table 2: Numerical results of problem 2 using Caputo derivative

x	Abs Error of U(x)	Abs Error of V(x)
0.	3.	0.
0.1	$1.44274624 \times 10^{-5}$	$1.362099642 \times 10^{-5}$
0.3	$2.01181257 \times 10^{-5}$	$1.441465488 \times 10^{-5}$
0.5	$2.68598304 \times 10^{-5}$	$1.438980027 \times 10^{-5}$
0.7	$3.44705569 \times 10^{-5}$	$1.178093521 \times 10^{-5}$
0.9	$4.28651606 \times 10^{-5}$	$6.660462894 \times 10^{-6}$

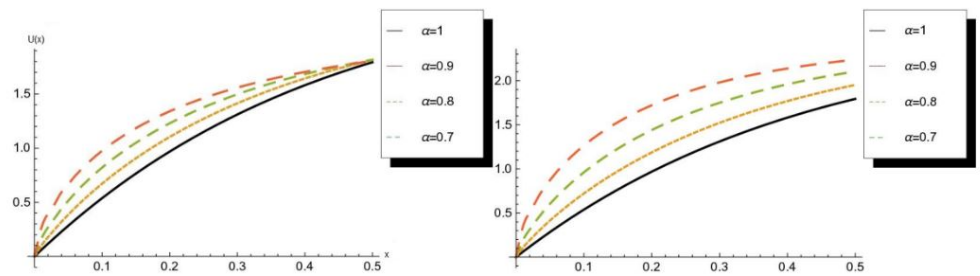

Figure 2: Approximate solution for different values of α for problem 2.

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New kernels implying new general decay for a nonlinear viscoelastic equation

Abdelkarim Kelleche*

Mailing address: Department of mathematics and computer sciences, Djilali Bunaama University, Khemis Miliana, Algeria
E-mail: kellecheabdelkarim@gmail.com

Abstract
This work deals with a nonlinear viscoelastic equation. The aim is to expand the class of the relaxations functions h that ensuring a general deacy. We adopt the following commonly condition $h'(t) \leq -\xi(t)\chi(h(t))$, where χ is an increasing and convex function and ξ is a nonincreasing function on the whole $\mathbb{R}^+$. The paper improves some previous results where χ is assumed to be convex only near the origin.

1 Introduction

The main purpose of this work is to study the stability of the following nonlinear viscoelastic problem

$$\begin{cases} |u_t|^\rho u_{tt} - \triangle u - \triangle u_{tt} + \int_0^t h(t-s)\triangle u(s)ds = |u|^p u, \text{ in } \Omega \times (0,\infty), \\ u = 0, \text{ on } \partial\Omega \times (0,\infty), \\ u(0,x) = u_0(x), \ u_t(0,x) = u_1(x), \ x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain of $\mathbb{R}^n$ $(n \geq 1)$ with a smooth boundary $\partial\Omega$, $\rho, p > 0$ are constants will be specified later. The integral term, accounting for the viscoelastic damping, describes the dependence of the stress on the strain in the past history. The function h is called the kernel or the relaxation function.

2 Main results

First, we state our assumptions:

(A) The constants ρ and p satisfy

$$\begin{cases} 0 < (n-2)\max\{\rho, p\} \leq 2 \text{ if } n \geq 3, \\ \min\{\rho, p\} > 0 \text{ if } n = 1, 2. \end{cases} \tag{2.1}$$

For the kernel h, we assume that:

Key Words and Phrases: *Energy decay, multiplier technique, wave equation, convexity*

(B1) $h(t) \geq 0$ for all $t \geq 0$ and $0 < k = \int_0^\infty h(s)ds < 1$.

(B2) $h'(t) \leq 0$ for almost all $t > 0$.

(B3) $\int_t^\infty h(s)ds \leq \Lambda(t)$ for all $t \geq 0$ where:

$\Lambda : [0, \infty) \to \mathbb{R}^+$ is a given absolutely continuous function satisfying the differential inequality

$$\Lambda'(t) + \xi(t)\chi(\Lambda(t)) \leq 0, \ a.e. t > 0, \tag{2.2}$$

where $\xi : \mathbb{R}^+ \to \mathbb{R}^+$ is decreasing, continuous and not summable in a neighborhood of ∞, while $\chi : [0, \infty) \to [0, \infty)$ is a continuous function with the following properties:
• $\chi(s) = 0$ if and only if $s = 0$.
• χ is increasing and convex.

For an arbitrary, fixed $r > 0$, let

$$\mathcal{B}(x) = \int_x^r \frac{ds}{\chi(s)}.$$

Theorem 2.1. *Assume that (**A**), (**B1**)-(**B3**) hold and that* $\mathcal{R}_h < 1/2$. *Then there exist positive constants* $a < 1$ *and* K *such that*

$$\mathcal{E}(t) \leq K\mathcal{B}^{-1}\left(a\int_{\hat{t}}^t \xi(s)\,ds\right), \ \forall t \geq \hat{t}$$

where K *and* $\hat{t}$ *depend on* $\mathcal{E}(0)$.

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Exponential growth of solution for a system of pseudo-parabolic equations with source terms

Amar OUAOUA

Mailing address: Department of technology, August 20, 1955 University, Skikda, Algeria
E-mail: ouaouaama21@gmail.com

Wissem BOUGHAMSA

Mailing address: Department of Sciences, August 20, 1955 University, Skikda, Algeria
E-mail: boughamsawissem@@gmail.com

Aya KHALDI

Mailing address: Department of Sciences, August 20, 1955 University, Skikda, Algeria
E-mail: ayajhaldi@gmail.com

Abstract
This work is concerned with coupled of semi-linear pseudo-parabolic equations with memory terms in both equations associated with the homogeneous Dirichlet boundary Condition. We prove that the solution has exponential growth for certain conditions regarding the relaxation functions and initial energy. In order to prove the result, we used the energy method based on the construction of a suitable Lyapunov function. The most important behavior for evolution system in the exponentially growth phenomena because of its wide applications in modern science, such like in chemistry, biology, ecology and other areas of engineering and physical sciences.

1 Introduction

We consider the following boundary value problem:

$$\begin{cases} u_t - \Delta u - \Delta u_t + \int_0^t g(t-s)\,\Delta u(s)\,ds + |u|^{m-2} u_t = f_1(u,v), & \text{in } \Omega \times (0,T) \\ v_t - \Delta v - \Delta v_t + \int_0^t h(t-s)\,\Delta v(s)\,ds + |v|^{k-2} v_t = f_2(u,v), & \text{in } \Omega \times (0,T) \\ u(x,\ t) = 0\ , v(x,\ t) = 0, & \text{in } \partial\Omega \times (0,T) \\ u(x\ ,0) = u_0,\ v(x,\ 0) = v_0, & \text{in } \ \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$, $n \geq 1$ with smooth boundary $\partial\Omega$, m and k are real positive constants. The relaxation functions g and h satisfying some conditions we suppose later, and the two functions $f_1(u,v)$ and $f_2(u,v)$ are given by

$$\begin{cases} f_1(u,v) = |u+v|^{2(r+1)}(u+v) + |u|^r u\,|v|^{r+2}. \\ f_2(u,v) = |u+v|^{2(r+1)}(u+v) + |u|^{r+2} v\,|v|^r. \end{cases} \tag{1.2}$$

Recently, Pişkin and Ekinci in [1] treated the following system

$$\begin{cases} u_t - \Delta u + |u|^{q-2} u_t = f_1(u,v), \\ v_t - \Delta v + |v|^{q-2} v_t = f_2(u,v). \end{cases} \tag{1.3}$$

Key Words and Phrases: *Memories term, parabolic equation, Exponential grouth, Source terms*

They proved the exponential growth of the solution with initial negative energy.
In the absence of $|u|^{q-2} u_t$ and $|v|^{q-2} v_t$ terms the system (1.3) becomes

$$\begin{cases} u_t - \Delta u = f_1(u, v), \\ v_t - \Delta v = f_2(u, v). \end{cases}$$

This type of equation is naturally found in physics, chemistry, biology, ecology, and other areas of engineering and physical sciences.
Ouaoua et al in [2] considered the following nonlinear Kirchhoff type reaction-diffusion equation

$$u_t - M\left(\int_\Omega |\nabla u|^2\, dx\right)\Delta u + |u|^{m-2} u_t = |u|^{r-2} u, \qquad (x,t) \in \Omega \times (0,T), \tag{1.4}$$

where $M(s) = a + bs^\gamma$, a, b and γ are positive constants. Under suitable assumptions on the initial data, they obtained global existence and stability of solutions with positive initial energy. In the case of the variable exponents, also, Ouaoua and Maouni in [3] considered the following equation

$$u_t - div\left(|\nabla u|^{p(x)-2} \nabla u\right) + \omega |u|^{m(x)-2} u_t = b |u|^{r(x)-2} u \quad \text{in } \Omega \times (0,T). \tag{1.5}$$

They proved blow-up, exponential growth of solution with negative initial energy in the both case of equation.

2 Main results

Theorem 2.1. *Suppose that assumptions (A_1) and (A_2) hold, $(u_0, v_0) \in (H_0^1)^2$ and (u, v) is a local strong solution of the system (1.1), and $E(0) < 0$.*
Furthermore, we assume that $\max\left(\int_0^\infty g(s)\, ds, \int_0^\infty h(s)\, ds\right) < p/(p+1)$ *and*

$$2(r+2) > \max(m, k). \tag{2.1}$$

Then the solution of the system (1.1) exponentially grows.

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Boundedness of pseudo-differential operators on local variable Herz-type Hardy spaces

Zouheyr Zeghad*

M'sila University Department of Mathematics Laboratory of Functional Analysis and Geometry of Spaces. P.O. Box 166, M'sila 28000, Algeria

E-mail: zgzoheyr@gmail.com

Abstract

In this work, we introduce the local variable Herz-type Hardy spaces where we give their atomic decomposition and we present the boundedness of a class of pseudo-differential operators on such spaces.

1 Introduction

Function spaces with variable exponents have been intensively studied in recent years by a large number of authors. These function spaces are applied in partial differential equations, fluid dynamics and image processing, see for example, [3]. The local Herz-type Hardy spaces $h\dot{K}^{\alpha}_{p(\cdot),q}$ with one variable exponent p were introduced by H. Wang and Z. Liu, [5]. The authors gave their atomic decomposition characterizations also proved the boundedness of a pseudo-differential operators of order zero on these spaces.

2 Main results

Definition 2.1. Let $\alpha \in L^{\infty}(\mathbb{R}^n)$, $p \in \mathcal{P}(\mathbb{R}^n)$, $q \in \mathcal{P}_0(\mathbb{R}^n)$ and $s \in \mathbb{N}_0$. A function a is said to be a central $(\alpha(\cdot), p(\cdot))$-atom, if
(i) $\operatorname{supp} a \subset \overline{B(0,r)} = \{x \in \mathbb{R}^n : |x| \leq r\}, \quad r > 0$,
(ii) $\|a\|_{p(\cdot)} \leq |\overline{B(0,r)}|^{-\alpha(0)/n}, \quad 0 < r < 1$,
(iii) $\|a\|_{p(\cdot)} \leq |\overline{B(0,r)}|^{-\alpha_{\infty}/n}, \quad r \geq 1$,
(iv) $\int_{\mathbb{R}^n} x^{\beta} a(x)dx = 0, \quad |\beta| \leq s$.
A function a on $\mathbb{R}^n$ is said to be a central $(\alpha(\cdot), p(\cdot))$-atom of restricted type, if it satisfies the conditions (iii), (iv) above and $\operatorname{supp} a \subset B(0,r), r \geq 1$

The following atomic decomposition.

Theorem 2.1. *Let α and q are* log-*Hölder continuous, both at the origin and at infinity and $p \in \mathcal{P}^{\log}(\mathbb{R}^n)$ with $1 < p^- \leq p^+ < \infty$. For any $f \in h\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}(\mathbb{R}^n)$, we have*

$$f = \sum_{k=-\infty}^{\infty} \lambda_k a_k, \quad \textit{in the sense of } \mathcal{S}'(\mathbb{R}^n), \tag{2.1}$$

Key Words and Phrases: *Atom, Herz-type Hardy spaces, pseudo-differential operators, variable exponent.*

where for $k \leq 0$, a_k *is a central* $(\alpha(\cdot), p(\cdot))$*-atom, while for* $k > 0$, a_k *is a central* $(\alpha(\cdot), p(\cdot))$*-block, with* $\operatorname{supp} a_k \subset B_k$ *and*

$$\left(\sum_{k=-\infty}^{-1} |\lambda_k|^{q(0)}\right)^{1/q(0)} + \left(\sum_{k=0}^{\infty} |\lambda_k|^{q_\infty}\right)^{1/q_\infty} \leq c\|f\|_{h\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}}.$$

Conversely, if $\alpha(\cdot) \geq n(1 - \frac{1}{p^-})$ *and* $s \geq \left\lfloor \alpha^+ + n(\frac{1}{p^-} - 1) \right\rfloor$, *and if* (2.1) *holds, then* $f \in h\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}(\mathbb{R}^n)$, *and*

$$\|f\|_{h\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}} \approx \inf\left\{\left(\sum_{k=-\infty}^{-1} |\lambda_k|^{q(0)}\right)^{1/q(0)} + \left(\sum_{k=0}^{\infty} |\lambda_k|^{q_\infty}\right)^{1/q_\infty}\right\},$$

where the infimum is taken over all the decompositions of f *as above.*

By using atomic decompositions, we have the following result.

Theorem 2.2. *Let* $q \in \mathcal{P}_0(\mathbb{R}^n)$ *and* $p \in \mathcal{P}^{\log}(\mathbb{R}^n)$, *and let* α *and* q *are* log*-Hölder continuous, both at the origin and at infinity such that* $\alpha \in L^\infty(\mathbb{R}^n)$ *and* $\alpha(\cdot) \geq n\big(1 - \frac{1}{p^-}\big)$. *If*

$$Tf(x) = \int_{\mathbb{R}^n} \hat{f}(x)\, \sigma(x, \xi)\, e^{2\pi i x \xi} d\xi,$$

with $\sigma \in S^0$, *that is* $\sigma \in \mathcal{C}^\infty(\mathbb{R}^n \times \mathbb{R}^n)$ *and* $\left|\partial_x^\gamma \partial_\xi^\beta \sigma(x, \xi)\right| \leq c_{\gamma,\beta}(1 + |\xi|)^{-|\beta|}$, *then*

$$\|T(f)\|_{h\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}} \leq c\,\|f\|_{h\dot{K}^{\alpha(\cdot)}_{p(\cdot),q(\cdot)}}.$$

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The optimal electricity production in the smart grid

Frihi Zahrate El Oula*

Department of Mathematics, Faculty of Science, Badji-Mokhtar University, Annaba, Algeria
E-mail: zahrate.frihi@univ-annaba.org

Choutri Salah Eddine[2], Barreiro-Gomez Julian[3] and Tembine Hamidou[4]

Learning & Game Theory Laboratory, Engineering Division, New York University Abu Dhabi, Saadiyat, UAE
E-mail: (choutri,jbarreiro,tembine)@nyu.edu

Abstract
In this work, we solve a mean-field type hierarchical optimal control problem in electricity market. We consider $n-1$ prosumers and one producer. The ith prosumer, for $1 < i < n$, is a leader of the $(i-1)$th prosumer and is a follower of the $(i+1)$th prosumer. The first player (agent) is the follower at the bottom whereas the nth is the leader at the top. The problem is described by a linear jump-diffusion system of conditional mean-field type, where the conditioning is with respect to common noise, and a quadratic cost functional involving the second moment, the square of the conditional expectation of the controls of the agents. We provide a semi-explicit solution of the corresponding mean-field-type hierarchical control problem with common noise. Finally, we illustrate the obtained result via a numerical example with two different scenarios.

1 Introduction

Leader-follower games were first introduced by Stackelberg [1] in 1934, to model markets where some firms have stronger influence on others. Stackelberg games are non-zero-sum static games with two-level hierarchy as they consist of two players, a major player (the leader) and a minor player (the follower). The minor player chooses a response strategy (assumed rational), for any announced strategy from the major player, such that her own performance criterion is optimized. The major player predicts the best response of the minor player and chooses a strategy to optimize her performance criterion (assuming that she knows the performance criterion of the minor player).
For $n > 2$ the leader-follower problem is called multi-hierarchical, each player is a leader for the previous one and a follower of the next player in the hierarchy. The first and the last players are the leader at the top and the follower at the bottom, respectively.
We consider a hierarchical mean-field type control problem described by the following settings. Let $\mathcal{T} := [0, T]$ be the time horizon with $T > 0$. The energy market in our

Key Words and Phrases: *Hierarchical control, electricity price dynamics, mean-field-type control, smart grids.*

model is described by $n \geq 2$ agents, one producer and $n-1$ prosumers following a hierarchical structure. Each agent $i \in \{1, \ldots, n\}$, at time $t \in \mathcal{T}$, has an output $u_i(t) \geq 0$. The log-price dynamics, as modeled in [2], are given by $p(0) = p_0$ and

$$dp(t) = s[a - D(t) - p(t)]dt + \left(\sigma dB(t) + \int_{\theta\in\Theta} \mu(\theta)\widetilde{N}(dt, d\theta)\right) + \sigma_0 dB_0(t), \qquad (1.1)$$

where

$$D(t) := \sum_{i=1}^{n} u_i(t),$$

is the supply at time t The processes B and B_0 are standard Brownian motions representing, respectively, a local and a global noise in the model.

The conditional price $\bar{p}(t) := E[p(t)|\mathcal{F}_t^{B_0}],\ 0 \leq t \leq \mathcal{T})$.
At time $t \in \mathcal{T}$, agent i gains $\bar{p}(t)u_i(t) - C_i(u_i(t))$ where $C_i : \mathbb{R} \to \mathbb{R}$, represents her instant cost given by

$$C_i(u_i) = c_i u_i(t) + \frac{r_i u_i^2(t)}{2} + \frac{\bar{r}_i \bar{u}_i^2(t)}{2}.$$

The term $\bar{u}_i(t) = E[u_i(t)|\mathcal{F}_t^{B_0}]$ is the conditional expectation of agent i's output given the common noise B_0 (the global uncertainty). The payoff functional (or the long-term revenue) of each agent i is

$$\mathcal{R}_i(p_0, u_1(t), \ldots, u_n(t)) = -\frac{q}{2}e^{-\lambda_i T}(p(T) - \bar{p}(T))^2 + \int_0^T e^{-\lambda_i t}[\bar{p}(t)u_i(t) - C_i(u_i(t))]dt,$$

where $c_i, r_i, \bar{r}_i$ and q are non-negative parameters and λ_i is a discount factor for the agent i.

2 Main results

In this section we present the main result of this work in the form of the following proposition.

Proposition 2.1. *The optimal controls for the n agents are in state-and-conditional mean-field feedback form:*

$$u_i^*(t) = -\frac{s((p(t) - \bar{p}(t))\alpha_i(t) + \xi_i(t)}{e^{-\lambda_i t} r_i} + \frac{e^{-\lambda_i t}(\bar{p}(t) - c_i) - \beta_i(t)s\bar{p}(t) - s\gamma_i(t)}{e^{-\lambda_i t}(r_i + \bar{r}_i)}.$$

The conditional optimal price:

$$\begin{aligned} d\bar{p}(t) =& s\left[a - \sum_{i=1}^{n}\left(\frac{e^{-\lambda_i t}(\bar{p}(t) - c_i) - \beta_i(t)s\bar{p}(t) - s\gamma_i(t)}{e^{-\lambda_i t}(r_i + \bar{r}_i)}\right) - \bar{p}(t)\right]dt + \sigma_0 dB_0(t), \\ \bar{p}(0) =& \bar{p}_0. \end{aligned}$$

The stochastic functions $\alpha_i,\ \beta_i,\ \gamma_i,\ \delta_i$ and ξ_i are $\mathcal{F}_t^{B_0}-$measurable and solve the following stochastic Riccati system: for $1 \leq i \leq n$,

$$
\begin{aligned}
d\alpha_i &= \left(2s\alpha_i - \frac{s^2\alpha_i^2}{e^{-\lambda_i t}r_i} - 2s^2\alpha_i \sum_{j\neq i} \frac{\alpha_j}{e^{-\lambda_j t}r_j}\right) dt + \alpha_{i,0} dB_0,\\
\alpha_i(T) &= -qe^{-\lambda_2 T},\\
d\beta_i &= \left(-\frac{(e^{-\lambda_i t} - \beta_i s)^2}{e^{-\lambda_i t}(r_i + \bar{r}_i)} + 2s\beta_i \sum_{j\neq i} \frac{(e^{-\lambda_j t} - \beta_j s)}{e^{-\lambda_j t}(r_j + \bar{r}_j)} + 2\beta_i s\right) dt + \beta_{i,0} dB_0,\\
\beta_i(T) &= 0,\\
d\gamma_i &= -\Bigg(\beta_i s(a + \sum_{j\neq i} \frac{e^{-\lambda_j t}c_j + s\gamma_j}{e^{-\lambda_j t}(r_j + \bar{r}_j)}) + \sigma_0\beta_{i,0} - s\gamma_i \sum_{j\neq i} \frac{(e^{-\lambda_j t} - s\beta_j)}{e^{-\lambda_j t}(r_j + \bar{r}_j)}\\
&\quad + \sigma_0\beta_{i,0} - s\gamma_i - \frac{(e^{-\lambda_i t} - \beta_i s)(s\gamma_i + e^{-\lambda_i t}c_i)}{(r_i + \bar{r}_i)e^{-\lambda_i t}}\ \Bigg) dt - \beta_i\sigma_0 dB_0,\\
\gamma_i(T) &= 0,\\
d\xi_i &= s\left(\xi_i - s\alpha_i \sum_{j\neq i} \frac{\xi_j}{e^{-\lambda_j t}r_j} - \frac{s\alpha_i\xi_i}{e^{-\lambda_i t}r_i} - s\xi_i \sum_{j\neq i} \frac{\alpha_j}{e^{-\lambda_j t}r_j}\right) dt + \xi_{i,0} dB_0,\\
\xi_i(T) &= 0,\\
d\delta_i &= -\Bigg(\frac{1}{2}\frac{(s\gamma_i + e^{-\lambda_i t}c_2)^2}{e^{-\lambda_i t}(r_i + \bar{r}_i)} + s^2\xi_i \sum_{j\neq i} \frac{\xi_j}{e^{-\lambda_j t}r_j} + \frac{1}{2}\frac{s^2\xi_i^2}{e^{-\lambda_i t}r_i} + \frac{1}{2}\beta_i\sigma_0^2\\
&\quad + \frac{\alpha_i}{2}(\sigma^2 + \int_\Theta \mu^2(\theta)v(d\theta)) + s\gamma_i a + s\gamma_i \sum_{j\neq i} \frac{e^{-\lambda_j t}c_j + s\gamma_j}{e^{-\lambda_j t}(r_j + \bar{r}_j)} + \gamma_{i,0}\sigma_0\ \Bigg) dt\\
&\quad - \sigma_0\gamma_i dB_0,\\
\delta_i(T) &= 0.
\end{aligned}
$$

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Approximating Solutions of First Order Fuzzy Fractional Differential Equations Using Reproducing Kernel Hilbert Space Method

Harrouche Nesrine

Mailing address: Department Of Mathematics, University of Jijel, Jijel, Algeria
E-mail: n.harrouche18@gmail.com

Abstract
This work proposes an advanced numerical-analytical approach for handling a class of fuzzy fractional differential equations involving Caputo-Fabrizio derivative arising in the medical sector. The solution methodology relies on the reproducing-kernel algorithm to generate analytical solutions in the form of a uniformly convergent series in the direct sum of the desired Hilbert spaces.

1 Introduction

- This presentation is concerned with design and comprehensive study of a numerical approach for solving drug pharmacokinetic model under Caputo-Fabrizio fractional derivative.
- The proposed technique is using the reproducing kernel Hilbert space to approximate the solution to these class of fractional fuzzy differential equations in the form of uniformly convergent series with respect to space variables.

2 Main results

The drug concentration within the influence compartment is evaluated in the Caputo-Fabrizio fractional sense under certainty as follows,

$$^{CF}D^{\alpha}y(x) + k_2 y(x) = k_1 A \exp^{-k_1 x}, \;\; 0 \leq \alpha \leq 1, \tag{2.1}$$

along with fuzzy initial condition

$$y(0,r) = [r-1, 1-r],$$

where $y(t)$ is a continuous fuzzy valued function and $^{CF}D^{\alpha}_{0^+}$ is the fuzzy Caputo-Fabrizio derivative of order α.
Assuming $y(x)$ is $^{CF}(1-\alpha)-$differentiable, the fuzzy fractional IVP (2.1) is equivalent to the system

$$^{CF}D^{\alpha}y_{1r}(x) + k_2 y_{1r}(x) = k_1 A e^{-k_1 x},$$

Key Words and Phrases: *Mathematical modeling of medicine, Fuzzy differential equations, Reproducing kernel algorithm, Caputo-Fabrizio fractional derivative, Pharmacokinetics model.*

$$^{CF}D^{\alpha}y_{2r}(x) + k_2 y_{2r}(x) = k_1 A e^{-k_1 x},$$

$$y_{1r}(0) = r - 1, \;\; y_{2r}(0) = 1 - r$$

whose exact solution for arbitrary $\alpha \in (0, 1]$ is

$$y_{1r}(t) = \frac{e^{-xk_1}(k_1\gamma - k_1(\gamma + r - 1)e^{\frac{\gamma k_2 x}{(\alpha-1)k_2-1}} + (r-1)\gamma k_2)}{\alpha k_2 + k_1(-1 + (-1+\alpha)k_2)},$$

$$y_{2r}(x) = \frac{e^{-xk_1}(k_1\gamma - k_1(\gamma + 1 - r)e^{\frac{\gamma k_2 x}{(\alpha-1)k_2-1}} + (1-r)\gamma k_2)}{\alpha k_2 + k_1(-1 + (-1+\alpha)k_2)},$$

where $\gamma = \alpha + (\alpha - 1)k_1$.
Using RKHS technique for $n = 30$, $\alpha = 0.99$ and $A = 1$, the aforementioned system can be solved numerically based on the following two cases:

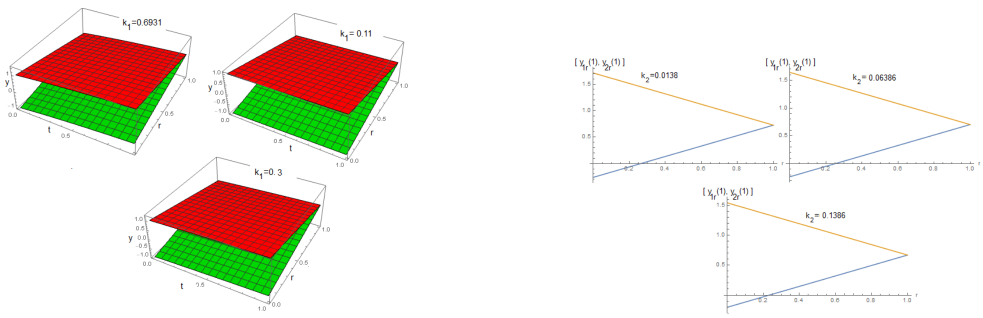

Figure 1: (left) 3D plots of fuzzy approximate solution at $\alpha = 0.99$ under $^{CF}[1-\alpha]-$ differentiability, Case 1, (Right) The level RK-fuzzy approximate solutions at $x = 1$ and $\alpha = 0.99$ under $^{CF}[1-\alpha]-$ differentiability ,Case 2.

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Geodesics in Lorentzian Manifolds

BENSIKADDOUR Djemaia

Department of Mathematics and Computer Science, University Abdelhamid Ibn Badis, Mostaganem, Algeria
E-mail: md.bensikaddour@gmail.com

Abstract
The aim of this work is to provide a detailed presentation of the structure of geodesics on lorentzian manifolds, we are interested in the study of totally geodesic foliations, the geodesically equivalent manifolds and the behavior of the geodesic flow on Heisenberg manifolds equipped with a left invariant metric g.

1 Introduction

Lorentzian geometry is a vivid field of mathematical research that can be seen as part of differential geometry as well as mathematical physics. It represents the framework of general theorem of relativity. To a pseudo-riemannian manifold (M, g), is associated its flow geodesic. A geodesic is the trajectory followed by a point launched with an initial velocity. However, the geodesics are the projections on M of the integral curves of a living vector field on the tangent to M.

Definition 1.1. If (M, g) is pseudo-Riemannian manifold (for metric g on manifold) then the tangent vectors of each point in the manifold can be classified into three different types. A tangent vector $v \neq 0$ is

1/ *timelike if* $g(v, v) < 0$,

2/ *null or lightlike if* $g(v, v) = 0$,

3/ *spacelike if* $g(v, v > 0$.

We focus our study on the structure of geodesic on Heisenberg three dimensional group defined as follow :

Definition 1.2. Lorentzian Heisenberg group $\mathcal{H}_3$ is a three dimensional Cartesian space with respect to the following not commutative product

$$(x, y, z) * (x', y', z') = (x + x',\ y + y',\ z + z' - xy' + x'y), \tag{1.1}$$

This group is equipped with a left invariant flat metric g given by :

$$g = dx^2 + (xdy + dz)^2 - [(1 - x)dy - dz]^2 \tag{1.2}$$

Key Words and Phrases: *Geodesic, Lorentzian manifold, Heisenberg group, foliation*

2 Main results

Theorem 2.1. *Any bundle in an orientable circle of dimension $n \geq 2$ has a totally mixed geodesic foliation of codimension 1.*

Theorem 2.2. *Any totally geodesic* 1*-dimensional foliation space lightlike on a manifold of dimension* 3 *contains incomplete geodesics. In particular, there does not exist on these varieties of geodesic vector field lightlike.*

Theorem 2.3. *For any $m \in \mathcal{H}_3$ which is neither in the t-axis there is a unique geodesic connecting the origin to m.*

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Some inequalities related to numerical radius of semi-Hilbert space operators

Messaoud Guesba

Mathematics Department, Faculty of Exact Sciences, Echahid Hamma Lakhdar University, El oued, Algeria
E-mail: guesbamessaoud2@gmail.com, guesba-messaoud@univ-eloued.dz

Abstract
Let A be a positive bounded operator on a Hilbert space $\mathcal{H}$. The semi-inner product $\langle x,y\rangle_A := \langle Ax,y\rangle$, $x,y\in\mathcal{H}$, induces a semi-norm $\|.\|_A$ on $\mathcal{H}$. Let $\omega_A(T)$ denote the A-numerical raduis of an operator T in semi-Hilbertian space $(\mathcal{H},\|.\|_A)$. Our aim in this work is to give new inequalities of A-numerical raduis of operatos in semi-Hilbertian spaces. These inequalities improve and generalized on some earlier related inequalities. In particular, we show that

$$\frac{1}{4}\left\|T^{\#}T+TT^{\#}\right\|_A \leq \frac{1}{8}\left(\left\|T+T^{\#}\right\|_A^2+\left\|T-T^{\#}\right\|_A^2\right) \leq \omega_A^2(T).$$

Some other related results are also obtained.

1 Introduction

Throughout this paper, $\mathcal{H}$ denotes a non trivial complex Hilbert space with inner product $\langle .,.\rangle$ and associated norm $\|.\|$. Let $\mathcal{B}(\mathcal{H})$ denote the algebra of all bounded linear operators acting on $\mathcal{H}$. For the sequel, it is useful to point out the following facts. Let $\mathcal{B}(\mathcal{H})^+$ is the cone of positive (semi-definite) operators i.e.,

$$\mathcal{B}(\mathcal{H})^+ = \{A\in\mathcal{B}(\mathcal{H}) : \langle Ax,x\rangle \geq 0, \forall x\in\mathcal{H}\}.$$

Any positive operator $A\in\mathcal{B}(\mathcal{H})^+$ defines a positive semi-definite sesquilinear form

$$\langle .,.\rangle_A : \mathcal{H}\times\mathcal{H}\to\mathbb{C},\quad \langle x,y\rangle_A = \langle Ax,y\rangle.$$

Naturally, this semi-inner product induces a semi-norm $\|.\|_A$ defined by

$$\|x\|_A = \sqrt{\langle x,x\rangle_A} = \left\|A^{\frac{1}{2}}x\right\|, \forall x\in\mathcal{H}.$$

For $T\in\mathcal{B}(\mathcal{H})$, an operator $S\in\mathcal{B}(\mathcal{H})$ is called an A-adjoint of T if for every $x,y\in\mathcal{H}$

$$\langle Tx,y\rangle_A = \langle x,Sy\rangle_A,$$

Key Words and Phrases: *Positive operator, A-numerical radius, Semi-norm, Semi-Hilbert space.*

The existence of an A-adjoint operator is not guaranteed. The set of all operators which admit A-adjoints is denoted by $\mathcal{B}_A(\mathcal{H})$. By Douglas theorem, we get

$$\begin{aligned}\mathcal{B}_A(\mathcal{H}) &= \{T \in \mathcal{B}(\mathcal{H}) : \mathcal{R}(T^*A) \subset \mathcal{R}(A)\} \\ &= \{T \in \mathcal{B}(\mathcal{H}) : \exists\, c > 0; \|ATx\| \leq c\,\|Ax\|, \forall x \in \mathcal{H}\}.\end{aligned}$$

If $T \in \mathcal{B}_A(\mathcal{H})$ then T admits an A-adjoint operators. Moreover, there exists a distinguished A-adjoint operator of T, namely the reduced solution of the equation $AX = T^*A$, i.e., $T^{\#} = A^{\dagger}T^*A$, where $A^{\dagger}$ is the Moore-Penrose inverse of A. The A-adjoint operator $T^{\#}$ verifies

$$AT^{\#} = T^*A,\ \mathcal{R}\left(T^{\#}\right) \subset \overline{\mathcal{R}(A)} \text{ and } \mathcal{N}\left(T^{\#}\right) = \mathcal{N}(T^*A).$$

The concept of the classical numerical radius was generalized to the A-numerical radius as follows

$$\omega_A(T) = \sup\left\{|\langle Tx, x\rangle_A| : x \in \mathcal{H},\ \|x\|_A = 1\right\}.$$

Recently, some refinements of the A-numerical radius inequalities have been proved by many authors (e.g., see [1, 2, 3], and the references therein). In particular, it has been shown that for $T \in \mathcal{B}_A(\mathcal{H})$, we have

$$\frac{1}{2}\sqrt{\|T^{\#}T + TT^{\#}\|_A} \leq \omega_A(T) \leq \frac{\sqrt{2}}{2}\sqrt{\|T^{\#}T + TT^{\#}\|_A}. \tag{1.1}$$

2 Main results

One main of the present work is to give and prove several new A-numerical radius inequlities. In particular, we show some new refinements of the first inequality in (1.1).

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Exact solutions of the Klein-Gordon for trigonometric Pöschl-Teller plus modified ring-shaped harmonic oscillator potential

Sahraoui Asma

Depertement of mathematics and computer sciences, Abdelhafid Boussouf University center, Mila, Algeria
Laboratory of Machematics and thier interactions Abdelhafid Boussouf University center, Mila. Algeria
E-mail: asma.saharoui@centre-univ-mila.dZ

Badredine Boudjedaa

Depertement of mathematics and computer sciences, Abdelhafid Boussouf University center, Mila, Algeria
Lab of Natural Sciences and Materials, University Center Abdelhafid Boussouf, Mila, Algeria
E-mail: b.boudjedaa@centre-univ-mila.dz

Abstract The exact solutions of the Klein Gordon equation for the trigonometric Pöschl-Teller plus modified ring-shaped harmonic oscillator potential are presented. With the change of variables into the cylindrical coordinates and the separation of variables method, the bound states and the associated energy eigenvalues of the problem are obtained explicitly by using the Nikiorov-Uvarov method (NU) and solving the biconfluent Heun equation.

1 Introduction

The Klein-Gordon (KG) equations is an interesting equation in many fields of physics and chemistry especially in relativistic quantum mechanics. The most popular equations that described the particle motions depending on the spin character of the particle are the Klein-Gordon or the Dirac equation. The spin-zero particles like the mesons are described by Klein-Gordon equation. One of the interesting problems in nuclear and high energy physics is to obtain exact solution of the Klein-Gordon equation.
To solve the KG is to determine the relativistic energy eigenvalues and the corresponding wave function of the system which contains all possible information about the physical propreties of a system.
The Klein-Gordon equation with the vector and scalar potentials can be written as follows [3]:

$$\left[\hat{p}^2 + (m + S(r))^2 - (E - V(r))^2\right] \Psi(r) = 0 \tag{1.1}$$

Key Words and Phrases: *The Klein-Gordon (KG) equation, the biconfluent Heun equation, Nikiorov-Uvarov method, the wave functions.*

Where m is the rest mass of the particle, E is the energy eigenvalue and where $\hat{p}$ is the momentum operator.
In our study, we consider the Klein- Gordon equation (1.1) under the Pöschl-Teller plus modified ring-shaped harmonic oscillator potentials [1] and [2]

$$V(r,\theta,\phi) = V(r) + \frac{V(\theta)}{r^2} + \frac{V(\phi)}{r^2 \sin^2\theta}. \tag{1.2}$$

Where

$$\begin{cases} V(r) = a_0 + a_1 r^2. \\ V(\theta) = \frac{a_2 + a_3\cos^2\theta}{\sin^2\theta} + \frac{a_4 + a\sin^2\theta}{\cos^2\theta} + \frac{a_6}{\sin^2\theta\cos^2\theta} + \frac{a_7}{\sin\theta}. \\ V(\phi) = \frac{k^2 a_8(a_8-1)}{\sin^2 k\phi} + \frac{k^2 a_9(a_9-1)}{\cos^2 k\phi}. \end{cases} \tag{1.3}$$

Recently, some authors solved the the Klein-Gordon equation in the case where the scalar potential is equal the vector potential and obtained the bound states with some potentials of interest such us Pöschl-Teller. Different methods such as Nikiorov-Uvarov have been used to solve the differential equation arising from these consideration.

2 Main results

We obtained the exact solution of the radial and angular parts of the Klein-Gordon equation for trigonometric Pöschl-Teller plus modified ring-shaped harmonic oscillator potentials using the NU method and the biconfluent Heun equation. We have also studied some special cases of the potential. Our straightforward study can be simply understood even by the unfamiliar or undergraduate readers. Due to application of this potential in quantum chemistry and nuclear physics.

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Procédure d'accélération pour une classe de problèmes à frontière libre abstrait

Ilhem Mous*

Mailing address: Département des mathématiques, Badji Mokhtar, Annaba, Algérie
E-mail: mousihem@yahoo.fr

Abdelhamid Laouar

Mailing address: Département des mathématiques, Badji Mokhtar, Annaba, Algérie
E-mail: abdelhamid.laouar@univ-annaba.dz

Résumé
Dans ce travaille nous avons voulu tester une telle procédure pour des algorithmes itératifs et faire redémarrer le processus itératif avec une nouvelle estimation. Un gain important en temps a été obtenu pour la résolution numérique grâce à cette procédure. Pour l'illustration, deux exemples ont été traités pour montrer l'efficacité de cette procédure.

1 Introduction

Dans ce travail on propose l'étude d'une procédure d'accélération de convergence (voir [1, 2, 3]) des algorithmes itératifs dans un contexte de monotonie d'ordre partiel (dite aussi la méthode de sur et sous solution).
Pour ce faire, supposons qu'on veut résoudre le système linéaire

$$Ax = b,$$

où A est une matrice carrée de $M_n(\mathbb{R})$ et b, $x \in \mathbb{R}^n$.

Considérons la suite des approximations successives associée est donnée par:

$$u^{k+1} = Tu^k + b, \quad k = 0, 1, ...,$$

où T(avec, $A = I - T$) est une matrice de $M_n(\mathbb{R})$ et b, u^k sont deux vecteurs de $\mathbb{R}^n$.
Ceci est motivé par l'explosion et l'existence actuelle des grands calculateurs (super-ordinateurs) qui nous offrent la possibilité de résoudre des grands systèmes linéaires ou non linéaires avec des milliers de variables. Pour cela, nous sommes donc encouragés à développer des processus d'accélération de convergence pour des algorithmes itératifs afin de réduire les effets des erreurs d'arrondi de la résolution des grands systèmes linéaires. Falcone et Miellou ont proposé initialement une procédure pour

Key Words and Phrases: *procédure d'accélération, méthode de sur et sous solutions, convergence, approximations successives*

résoudre des systèmes linéaires. Ensuite, elle a été étendue par El-Tarazi pour des systèmes non linéaires. Elle consiste à interrompre les itérations de la suite (u^k), $k = 1, 2, ...$, produite par la méthode des approximation successives à la k ième itération en remplaçant u^k par $\widetilde{u}^k$, où l'élément $\widetilde{u}^k$ est obtenu en combinant le vecteur u^{k+1} et le vecteur d'extrapolation $u^k + \eta^k(u^k - u^{k-1})$, avec η^k un paramètre réel à calculer. On redémarre ensuite le processus itératif avec le nouveau $\widetilde{u}^k$ comme une nouvelle estimation. Cette suite $\widetilde{u}^k$ présente une meilleure approximation de la solution que les itérations u^{k+1} et u^k, et de plus elle préserve la propriété de monotonie. Notre objectif principal, ici, est de tester cette procédure pour les grands systèmes. Pour l'illustration, nous allons l'appliquer à une classe de problèmes à frontière libre [1] du type:

Trouver une fonction u tels que

$$(P)\begin{cases} a(u, u - v) \geq (f, v - u)_{L^2(\Omega)}, \\ u \leq M(u),\, v \leq M(u), \end{cases} \tag{1.1}$$

où $a(.,.)$ est une forme bilinéaire, $(.,.)$ étant le produit scalaire dans $L^2(\Omega)$, f une fonction donnée régulière dans $L^2(\Omega)$ et $M(u)$ représente les obstacles (qui sont très importants dans l'étude des gestions des stokes d'énergies).

2 Main results

D'après les résultats obtenu, nous remarquons que notre résultat est meilleur et plus favorable que ceux obtenus dans des travaux existant puisque il y a une meilleure d'accélération de convergence et le gain en temps peut atteindre jusqu'à 50%.

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Heun function a solution of certain potentials of quantum mechanics

Imane Fekrache*

Mailing address: Department of Physices ,Faculty of Exact Sciences ,Université Fréres Mentouri Constantine 1,Aleria
E-mail: fekrache.imane@gmail.com

Farid Benamira

Mailing address:Department of Physices ,Faculty of Exact Sciences ,Université Fréres Mentouri Constantine 1,Aleria
E-mail:

Abstract

The aim of this work is to solve, for bound states, the Schrödinger equation for the potential $v(r) = -\frac{V_1}{\sqrt{1-e^{-\lambda r}}} + \frac{V_2}{1-e^{-\lambda r}}$ where V_1 and λ are real positive parameters and V_2 is a real parameter. The interest of this potential, depending on three parameters, is that it generalizes several physical potentials such as Hulthén potential, Coulomb potential and others, which can be obtained by only adjusting the parameters $V_1 V_2$ and λ

We first show that with an adequate point transformation, the Schrödingerequation associated with this generalized potential reduces to Heun equation with 3 singularities, the solutions of which can be sought as linear combinations of Gauss hypergeometric functions[1,2] ${}_2F_1$.By imposing the integrability constraint for physical solutions, we obtain the possible values of the bound states energies by a graphical resolution with very high precision.

1 Introduction

The goal of this work is to solve exactaly for the bound states the following one_dimentional generalized Hultén potential $v(x) = -\frac{V_1}{\sqrt{1-e^{-\frac{x}{\sigma}}}} + \frac{V_2}{1-e^{-\frac{x}{\sigma}}}$,defined by $x\rangle 0$, where V_1 V_2 and $\sigma$are real positive parameters, by the use of standard approach based on hypergeometric functions.

2 Main results

We have to solve the Schrödinger equation

$$\frac{d^2}{dx^2}\Psi(x) + \frac{2m}{\hbar^2}(E - v(x))\Psi(x) = 0. \quad (1)$$

Using the point transformation defined by

Key Words and Phrases: *Lyapunov Exponents, 0-1 test, FSHPS, Synchronization.*

$$z(x) = \sqrt{1 - e^{\frac{-x}{\sigma}}} \in]0,1[\quad \text{and} \quad \Psi(x) = (z+1)^{\alpha_1}(z-1)^{\alpha_2}U(z). \tag{2}$$

The equation reduces to the following Heun equation

$$\frac{d^2U(z)}{dz^2} + \left(\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\varepsilon}{z+1}\right)\frac{dU(z)}{dz} + \frac{\alpha\beta z - q}{z(z-1)(z+1)}U(z) = 0, \tag{3}$$

with

$$(\gamma, \delta, \varepsilon) = (-1, 1+2\alpha_2, 1+2\alpha_1);\ q = -\alpha_1 + \alpha_2$$
$$\alpha, \beta = \alpha_1 + \alpha_2 \pm \sqrt{-\frac{8m\sigma^2}{\hbar^2}E};\ \alpha_1, \alpha_2 = \sqrt{\frac{2m\sigma^2}{\hbar^2}(-E \pm V_1 + V_2)} \tag{4}$$

Following [3], the physical solution for $U(z)$ my be put as a combination of two hypergeometric functions as

$$U(z) = {}_2F_1\left(\alpha, \beta, \delta, \frac{1+z}{2}\right) - \frac{2\alpha_2}{\alpha_1 + \alpha_2}{}_2F_1\left(\alpha, \beta, \delta - 1, \frac{1+z}{2}\right). \tag{5}$$

The bound states energies are obtained by the constraint that the wave function must vanish at $x \to \infty$and $z \to 0$ this leads to

$$1 - \frac{\alpha\beta + 2\alpha_2 q F_1(\alpha, \beta, \delta - 1, \frac{1}{2})}{2\alpha_2 q_2 F_1(\alpha, \beta, \delta, \frac{1}{2})} = 0. \tag{6}$$

We consider the particular case where $(m, \hbar, V_1, V_2, \sigma = 1, 1, 4, 1, 2)$, we find the bound states energies graphically

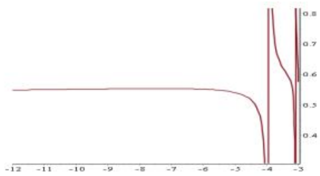

Figure 1: spectrumeq

The technique presented here is efficient and can be used to solve other similar problems.
Key Words and Phrases: Schrödinger equation, Heun equation, Gauss hypergeometric functions, Hulthén potential, Coulomb potential.

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Image Restoration by The Prox-Penalty Method with Bregman Projection

Soulef BOUGUEROUA*
Department of Mathematics, University F. ABBAS Setif 1. 19000 Setif, Algeria
E-mail: soulef.bougueroua@univ-setif.dz

Noureddine DAILI
Department of Mathematics, University F. ABBAS Setif 1. 19000 Setif, Algeria
E-mail: nourreddine.daili@univ-setif.dz

Abstract
Image restoration is an interesting ill-posed problem of crucial importance in the notion of image processing. We are looking for an image close to the original image of a bad quality. In this work we present a new optimization method to solve the problems of image restoration disturbed by additive Gaussian white noise. This resolution method is based on a prox-penalty algorithm. We illustrate the mathematical study of our method with another projection which is the Bregman projection.

1 Introduction

Image processing is a subset of signal processing that focuses on images and video. All procedures done on an image in order to increase readability and facilitate interpretation are referred to as image processing. In the industry, image restoration is an important topic. By applying a proximal algorithm to solve a minimization problem, we propose a unique technique for image restoration. It is believed that the original image has been deteriorated by additive noise. In 1976, Bregman devised a simple and effective method for using the D_g function in the design and analysis of feasibility and optimization algorithms. This has spawned a burgeoning field of research in which Bregman as a technique is used to create and analyze iterative algorithms for non-linear applications, not only to address feasibility and optimization problems, but also to solve variational inequalities and calculate fixed points.

2 Main results

In our problem, we consider the external penalty function $h(x)$ in the following form:

$$h(x) = \max(0, proj_K^{g(x)}(x)),$$

where $proj_K^{g(x)}$ denotes the Bregman projection on $K \subset C$ and C is a closed convex set defined by

$$C := \left\{ div(\varphi(x)) : \varphi \in C_c^1(\Omega; \mathbb{R}^2), \quad \|\varphi\|_\infty \leq 1 \right\}$$

Key Words and Phrases: *Proximal algorithm, Bregman's distance, Bregman projection, Image Processing.*

Definition 2.1. Bregman's distance
Let X be some euclidean space R^N with inner product $\langle x, y\rangle = \sum_i x_i yi$ and $g : X \longrightarrow]-\infty, +\infty[$ a convex function, semi-continuous inferior, clean Legendre type, and $C \subset int(dom(g))$ a non-empty set. Allow g to differentiate from $int(dom(g)) \neq \varnothing$. The Bregman distance D_g is then defined by

$$D_g : X \times int(dom(g)) \longrightarrow [0, +\infty]$$

$$(x, y) \longrightarrow D_g(x, y) = g(x) - g(y) - < \nabla g(y), x - y > .$$

Lemma 2.1. *The Bregman problem $(\mathcal{P}_\alpha)$ admits a single solution, where*

$$(\mathcal{P}_\alpha) \qquad \alpha := \inf_{y \in C} D_g(y, x).$$

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A new model for supporting the design of smart homen

Manel KOLLI

Departement des Informatique et Mathematiques, Ecole Normale Suprieure de Constantine, Algeria

E-mail: E-mail: kollimanel@yahoo.fr

Abstract

The internet has developed significantly especially in recent times. Nowadays the use of the internet is not limited to the management of networks, but has also extended to the management of objects, and that is what is called Internet of things, among the most view of the use of this new technology is the field of home automation what is currently called the smart home. In fact, the smart home market is expected to experience increasing demand, due to the availability of comfort and protection equipment, as well as the drafting of energy cost. As part of this research paper, we aim to remotely control objects in a smart home, and what we consider as a real example is smart lighting.

1 Introduction

The concurrency theory languished for some time, with renewed interest in the early 1970s with the development of the Actor Model in which the main components of a system are actors that send and receive messages.
Indeed, competition is the key assumption underlying the development of bigraphs and many other formalisms such as the Calculus of Syst th Communicants (Calculus of Communicating Systems CCS), Communicating Sequential Processes CSP), Algebra of Communicating Processes (Algebra of Communicating Processes ACP) , -calculation ((pi-calculus), the abstract chemical machine of Berry and Boudol (Chemical Abstract Machine cham), Action Calculi and Ambient Mobiles.
Bigraph theory was developed by Robin Milner. The bigraphs and their reactive systems were developed as a computational graphical model that puts emphasis on both locality and mobile connectivity. The theory was developed with two main objectives: (1) to be able to integrate in the same formalism important aspects of ubiquitous systems; (2) to provide a unification of existing theories by developing a general theory, in which the different calculations existing ones for competition and mobility, in particular the calculation of communicating systems, -calculus, ambient calculation and Petri nets, can be represented, with a uniform behavioral theory.
In this work, we propose a new model that supports the design of smart home. To achieve this goal, our model is based on formal specifications, derived from bigraphical reactive systems, for modelling structural and behavioural aspects of smart home systems. Furthermore, the proposed model reflects on not only structural aspects, but also the multiple reconfigurations concerned in the behaviour of smart home systems.

Key Words and Phrases: *Bigraph, connectivity, locality, smart home.*

2 Main result

In this context, we have developed a home automation system by dealing with the concept intelligence in a small space that of the house. A house which allows controlling domestic devices, locally or remotely. Our project aimed to operate certain domestic appliances automatically (lighting).

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Torsion axisymétrique d'une couche élastique prise en sandwich entre deux demi-espaces élastiques avec deux fissures interfaciales

Fateh MADANI *

Physique Fondamentale Et Appliqué FUNDAPL, Department of Mechanical Engineering, Blida 1, Algeria
E-mail: madanifateh1984@yahoo.com

Abdelkader KIRAD

Physique Fondamentale Et Appliqué FUNDAPL, Department of Mechanical Engineering, Blida 1, Algeria
E-mail: abdelkader_kirad62@yahoo.com

Abstract
Le présent article examine le problème lié à la torsion axisymétrique d'une couche élastique par un disque rigide circulaire au plan de symétrie. La couche est prise en sandwich entre deux demi-espaces élastiques similaires avec deux fissures en forme de penny situées symétriquement aux interfaces entre les deux supports dissemblables collés. La solution générale de ce problème est obtenue en utilisant la méthode des transformées de Hankel. Le problème aux limites doublement mixte correspondant associé au disque rigide et aux fissures circulaires est réduit à un système d'équations intégrales duales, qui se réduisent à des équations intégrales de Fredholm de seconde espèce. En utilisant la règle de quadrature, le système résultant est converti en un système d'équations algébriques infinies. Des résultats numériques et des courbes sont obtenus et discutés selon certains paramètres pertinents..

1 Introduction

Les composites stratifiés ont de nombreuses applications dans une variété de structures d'ingénierie en raison de leur capacité à absorber de l'énergie. Dans les matériaux composites à inclusions planes, de telles interactions peuvent se produire lors de la fracture induite thermiquement et de l'évolution de l'endommagement aux limites de la particule de renforcement.
On peut supposer que la région interfaciale est constituée d'une couche homogène relativement mince avec des propriétés matérielles différentes de celles des matériaux adjacents et que la fissure de séparation est soit intégrée dans la couche interfaciale, soit située le long des interfaces pour les matériaux isotropes et orthotropes liés.
Sous cette hypothèse, Sih et Chen [1]ont considéré la propagation de fissures le long de l'interface de matériaux contigus avec des propriétés élastiques différentes. Dhaliwal et al

Key Words and Phrases: *Torsion axisymétrique Fissure circulaire Équations intégrales duales Équations intégrales de Fredholm Facteur d'intensité de contrainte.*

[2] ont étudié le problème de contrainte de cisaillement anti-plan d'une fissure de Griffith auto-obturante se déplaçant le long de l'interface de deux couches élastiques dissemblables. Dans ce travail, nous avons présenté une situaton liée à la torsion axisymétrique d'une couche élastique par un disque rigide circulaire au niveau du plan de symétrie. La couche est prise en sandwich entre deux élastiques similaires des demi-espaces avec deux fissures situées symétriquement aux interfaces entre les deux milieux dissemblables liés.
Avec l'aide de la méthode de transformation intégrale de Hankel et comme Low[3], Dhawan[4] l'ont fait, Le problème des conditions aux limites mixtes est écrit comme un système des équations intégrales duales.

2 Main results

Les facteurs d'intensité des contraintes au bord de la fissure et au bord du disque peuvent être trouvés respectivement par

$$K^a_{\mathrm{III}} = \frac{-4G_1\omega\sqrt{a}}{(1+\gamma)\sqrt{\pi}}\Phi_N, \qquad K^b_{\mathrm{III}} = \frac{4G_1\omega\sqrt{b}}{\sqrt{\pi}}\Psi_N \tag{2.1}$$

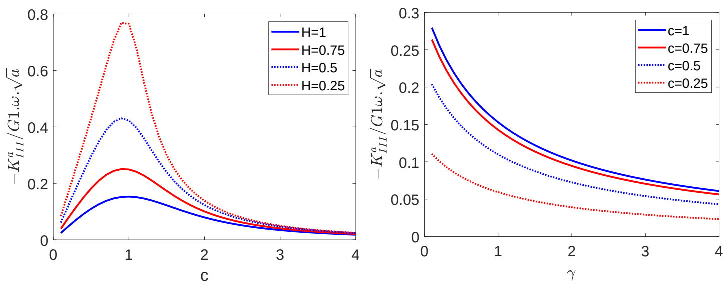

Figure 1: Variation du facteur d'intensité de contrainte normalisé au bord de la fissure K^a_{III} avec a/b et G_1/G_2

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ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTION OF FRACTIONAL P-LAPLACIAN EQUATION

BATAHRI Asma Amira*

Laboratoire d'Analyse Nonlinéaire et Mathématiques Appliquées, Département de Mathématiques, Université Abou Bakr Belkaid, Tlemcen,13000, Algeria
E-mail: batahriamira@gmail.com

ATTAR Ahmed

Laboratoire d'Analyse Nonlinéaire et Mathématiques Appliquées, Département de Mathématiques, Université Abou Bakr Belkaid, Tlemcen,13000, Algeria
E-mail: ahm.attar@yahoo.fr

Abstract
The aim of this work is to prove the existence of two positive sequences of solutions of the following nonlocal problem

$$\begin{cases} (-\Delta)_p^s u + u^{m-1} & = & \lambda u^{q-1} & \text{in } \Omega, \\ u & > & 0 & \text{in } \Omega, \\ u & = & 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

where $(-\Delta)_p^s$ is a fractional p-Laplacian with $s \in (0,1)$, $\Omega \subset \mathbb{R}^N$, $N > ps$, is bounded domain, $q \in [p, p_s^*)$ and λ and m sufficiently large. Then, we investigate the behavior of the solutions as $m \to +\infty$ which will be determied by a limit problem.

1 Introduction

The aim of this work is to generalize the results obtained in [4] by Boccardo et al. in the nonlocal case. When $s = 1$, $p = 2$ $\alpha|\xi|^2 \leq M(x)\xi\xi \leq \beta|\xi|^2$ with $0 < \alpha < \beta$, Boccardo et al. studied the local sub-case of our problem .

Motivated by [4] and the study of nonlocal problems involving fractional p-Laplacian operator ([3],[2], [5]), in this work we prove the existence of two positive sequences of solutions exploiting variational methods to the above problem for m large but fixed. Therefore, we prove a regularity results to be able to perferom our asymptotic analysis, and we will show that the behavior of sequences of solutions is determined by a limit problem as m tends to ∞

Key Words and Phrases: *Fractional p-Laplacian , Asymptotic Analysis, Nonlocal Elliptic problem, increasing power term.*

2 Main results

Now we state our results as follow

Theorem 2.1. *Let $s \in (0,1)$, $N > ps$ and $p < q \leq p_s^* < m$. Then, there exists $\underline{\lambda} > \lambda_* > 0$ such that for each $\lambda > \underline{\lambda}$ there is $m_0 > p_s^*$ such that for every $m \geq m_0$, problem $(P_{\lambda,m})$ has at least two positive nontrivial solutions $z_m, u_m \in X_m^s(\Omega)$ and $z_m \not\equiv u_m$.*

Theorem 2.2. *Let $s \in (0,1)$, $N > ps$ and $p < q \leq p_s^* < m$. Then, there exists $\lambda_* > 0$ such that for each $\lambda > \lambda_*$ there exists $w \in W_0^{s,p}(\Omega) \cap L^\infty(\Omega)$ such that $w \not\equiv 0$, and*

$$\begin{array}{lllll} w_m & \rightharpoonup & w & in & W_0^{s,p}(\Omega), \\ w_m & \to & w & in & L^q(\Omega) \ for\ all\ q \in (1,+\infty). \end{array}.$$

where w is the solution of the variational inequality

$$w \in \mathcal{K} : \langle (-\Delta)_p^s w, v-w \rangle \geq \lambda \int_\Omega w^{p-1}(v-w)$$

with

$$\mathcal{K} := \Big\{ v \in W_0^{s,p}(\Omega) : 0 \leq v(x) \leq 1 \Big\}.$$

In addition, there exists $g_w \in L^\infty(\Omega)$, such that

$$\begin{cases} 0 < g_w \leq \lambda, \\ g_w(x)[1-w(x)] = 0 \ a.e \ \ \Omega. \end{cases}$$

and it results that w is a solution of the following problem

$$(-\Delta)_p^s w + g_w = \lambda w^{q-1} \ in\ \Omega,\ \ w \in W_0^{s,p}(\Omega) \cap L^\infty(\Omega)$$

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Maximum Norm Convergence of Newton-Multigrid Methods for Elliptic Quasi-Variational Inequalities with Nonlinear Source Terms

Belouafi Mohammed Essaid*
Mailing address: University of El Oued, Faculty of Exact Sciences, Department of Mathematics, 39000, El Oued, Algeria
E-mail: belouafi-messaid@univ-eloued.dz

Beggas Mohammed
Mailing address: University of El Oued, Faculty of Exact Sciences, Department of Mathematics, 39000, El Oued, Algeria
E-mail: beggasmr@yahoo.fr

Abstract
In this work, we have applied Newton-multigrid method on adaptive finite element discretisation for solving elliptic quasi-variational inequalities with nonlinear source terms. the two approaches of Newton-Multigrid methods are used: Newton's method as the outer iteration for the global linearization, and a standard multigrid methods for solving the Jacobian system. The uniform convergence of this Non-linear multigrid has been demonstrated successfully.

1 Introduction

1.1 Continuous problem

Let Ω be an open in $\mathbb{R}^N$, with sufficiently smooth boundary $\partial\Omega$ for $u, v \in V$ $(V = H_0^1(\Omega))$, $a(u, v)$ be a variational form associated with the continuous non-linear operator A. Then, given a nonlinear right-hand side $f(u)$, Consider the following problem: Find $u \in K_g(u)$ solution of

$$\begin{cases} a(u, v-u) \geq \langle f(u), v-u \rangle \qquad v \in K_g(u), \\ \qquad u \leq Mu \qquad Mu \geq 0, \\ \qquad u = g \qquad on \;\; \partial\Omega. \end{cases} \tag{1.1}$$

1.2 Discrete problem

The numerical approximation of the VI (1.1) by finite elements leads to the solution of the discrete QVI in finite dimension. Find $u_k \in K_{g,k}$ such that

$$\begin{cases} \langle A_k u_k, v_k - u_k \rangle \geq \langle f(u_k), v_k - u_k \rangle, \qquad \forall v_k \in K_{g,k}, \\ u_k \leq M_k u_k, \quad v_k \leq M_k u_k, \end{cases} \tag{1.2}$$

2 Description of Newton-multigrid methods for the QVI (1.2) After the HJB-formulation of the discrete problem (1.2), Newton-multigrid iteration of the HJB-equation obtained may be described as the following algorithms 1and 2.

Key Words and Phrases: *Quasi-Variational Inequality, Finite element method, HJB Equation, Multigrid Method, Newton's method*

Algorithm 1 Newton-Muligrid methods

Choose an initial guess u_k^0 and a desired tolerance η.
while $(\mathcal{R}_k < \eta)$ **do**
compute the Jacobian matrix $\mathbb{J}_k$ and the residual vector $\mathcal{R}_k$
Solve the linear system by a Muligrid method $e_k^\nu \leftarrow MGM(\mathbb{J}_k, \mathcal{R}_k, e_k^\nu)$
Set $u_k^\nu \leftarrow u_k^\nu + e_k^\nu$;
Set $\mathcal{R}_k \leftarrow f_k^\nu[u_k^\nu] - A_k^\nu[u_k^\nu]u_k^\nu$;
end while

Algorithm 2 Muligrid methods

$MGM(\mathbb{J}_k, \mathcal{R}_k, e_k, \alpha_1, \alpha_2, \mu)$
$e_k \leftarrow smoother(\mathbb{J}_k, \mathcal{R}_k, e_k, \alpha_1)$; % (presmoothing)
$d_k \leftarrow \mathcal{R}_k - \mathbb{J}_k e_k$; % Computat the residual
$R_k; P_k$; % Define the prolongation and the restriction
$\mathbb{J}_{k-1} \leftarrow R_k \mathbb{J}_k P_k$; % Restrict $\mathbb{J}_k$
$d_{k-1} \leftarrow R_k d_k$; % Restrict d_k
$\mathcal{E}_{k-1} \leftarrow d_{k-1} \cdot 0$; % Define a start value
if $size(\mathbb{J}_{k-1} \leq \mu)$ % Coarsest grid Ω_μ **then**
$\mathcal{E}_{k-1} \leftarrow \mathbb{J}_{k-1}^{-1} d_{k-1}$; % The direct solve on the coarse grid
else
$\mathcal{E}_{k-1} \leftarrow MGM(\mathbb{J}_{k-1}, d_{k-1}, \mathcal{E}_{k-1})$; % Solve the coarse problem
end if
$\mathcal{E}_k \leftarrow P\mathcal{E}_{k-1}$; % Prolongat $\mathcal{E}_{k-1}$
$e_k \leftarrow e_k + \mathcal{E}_k$; % Add correction
$e_k \leftarrow smoother(\mathbb{J}_k, \mathcal{R}_k, e_k, \alpha_2)$; % (Postsmoothing)
return e_k

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Construction d'une nouvelle fonction génératrice pour les produits de certains nombres orthogonaux du plusieurs variables

Krioui Sana*

Laboratoire LMAM, département d'algèbre et de théorie des nombres, Université des sciences et de la technologie Houari Boumediene, Algeria
E-mail: sannakr92@gmail.com

Bellagh Abdelaziz

Laboratoire LMAM, département d'algèbre et de théorie des nombres, Université des sciences et de la technologie Houari Boumediene, Algeria
E-mail: abellagh@hotmail.com

Abstract
Dans ce travail, nous allons récupérer les fonctions génératrices de certains produits des nombres orthogonaux et la fonction symétrique du plusieurs variables. La technique utilisée ici est basée sur la théorie dite des fonctions symétriques.

1 Introduction

Différentes applications des fonctions de généralisation sont assignées à plusieurs branches des mathématiques et de la physique mathématique. Des études antérieures ont présenté plusieurs types de généralisations des nombres de Fibonacci, des nombres de Lucas, des nombres de Jacobsthal et des nombres de Mersenne. L'une des généralisations les plus reconnues de ces nombres est les nombres k-Fibonacci $\{F_{k,n}\}_{n\in N}$, les nombres k-Lucas $\{L_{k,n}\}_{n\in N}$, les nombres k-Jacobsthal $\{J_{k,n}\}_{n\in N}$, et les nombres k-Mersenne $\{M_{k,n}\}_{n\in N}$ donnés par la récurrence suivante (Voir [3, 5])

$$\begin{aligned}
F_{k,n} &= kF_{k,n-1} + F_{k,n-2} \quad n > 2,\ \ F_{k,0} = 1,\ \ F_{k,1} = k\\
L_{k,n} &= kL_{k,n-1} + L_{k,n-2} \quad n > 2,\ \ L_{k,0} = 2,\ \ L_{k,1} = k\\
J_{k,n} &= kJ_{k,n-1} + 2J_{k,n-2} \quad n > 2,\ \ J_{k,0} = 0,\ \ J_{k,1} = 1\\
M_{k,n} &= 3kM_{k,n-1} - 2M_{k,n-2} \quad n > 2,\ \ M_{k,0} = 0,\ \ M_{k,1} = 1
\end{aligned}$$

En 1991, A.Alasco et A. Abderrezzek ont défini l'opérateur $\delta^1_{p_1p_2}$, qui a été appliqué à partir de 2013 par de nombreux chercheurs, car il leur a permis de récupérer de nombreuses identités et fonctions génératrices célèbres, en utilisant la technique des fonctions symétriques. Pour plus d'informations, voir [1, 2, 4]

Key Words and Phrases: *fonctions symétriques, fonctions génératrices, nombres k-Fibonacci, nombres k-Lucas, nombres k-Mersenne, nombres k-Jacobsthal.*

2 Main results

Theorem 2.1. *Etant donné deux alphabets $P = \{p_1, p_2\}$ et $A = \{a_1, a_2, a_3, a_4, a_5\}$, nous avons*

$$\sum_{n=0}^{+\infty} h_n(a_1, a_2, a_3, a_4, a_5) h_n(p_1, p_2) z^n = \frac{\begin{array}{c} e_0^{(5)} - p_1 p_2 e_2^{(5)} z^2 + p_1 p_2 (p_1 + p_2) e_3^{(5)} z^3 \\ -p_1 p_2 \left[(p_1 + p_2)^2 - p_1 p_2 \right] e_4^{(5)} z^4 \\ +p_1 p_2 (p_1 + p_2) \left[(p_1 + p_2)^2 - 2 p_1 p_2 \right] e_5^{(5)} z^5 \end{array}}{\prod_{i=1}^{5} (1 - a_i p_1 z) \prod_{i=1}^{5} (1 - a_i p_2 z)}$$

Theorem 2.2. *Etant donné deux alphabets $P = \{p_1, p_2\}$ and $A = \{a_1, a_2, a_3, a_4, a_5\}$, nous avons*

$$\sum_{n=0}^{\infty} h_n(a_1, a_2, a_3, a_4, a_5) h_{n-1}(p_1, p_2) z^n = \frac{\begin{array}{c} e_1^{(5)} z - (p_1 + p_2) e_2^{(5)} z^2 + \left[(p_1 + p_2)^2 - p_1 p_2 \right] e_3^{(5)} z^3 \\ -(p_1 + p_2) \left[(p_1 + p_2)^2 - 2 p_1 p_2 \right] e_4^{(5)} z^4 \\ +\left[(p_1 + p_2)^4 - p_1 p_2 \left[3 (p_1 + p_2)^2 - p_1 p_2 \right] \right] e_5^{(5)} z^5 \end{array}}{\prod_{i=1}^{5} (1 - a_i p_1 z) \prod_{i=1}^{5} (1 - a_i p_2 z)}$$

Theorem 2.3. *Pour $n \in \mathbb{N}$, la nouvalle fonction génératrice de produit des nombres k-Lucas avec la fonction symétrique de plusieurs variables est donné par*

$$\sum_{n=0}^{\infty} h_n(a_1, a_2, a_3, a_4, a_5) L_{k,n} z^n =$$

$$\frac{2 - k e_1^{(5)} z + (2 + k^2) e_2^{(5)} z^2 - k(k^3 + 3) e_3^{(5)} z^3 + (k^4 + 4k^2 + 2) e_4^{(5)} z^4 - k (k^4 + 5k^2 + 5) e_5^{(5)} z^5}{\prod_{i=1}^{5} (1 - k a_i z - a_i^2 z^2)}.$$

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Coincidence fixed points of two operators in complete metric spaces with An Application in Dynamic Programming

Aouine Ahmed Chaouki*

a.aouine@univ-soukahras.dz: Mathematics and Informatics, Mohamed-Cherif Messaadia University, Souk Ahras, Algeria
E-mail: a.aouine@univ-soukahras.dz, chawki81@gmail.com

Abstract
Our aim in this talk is to prove coincidence fixed points theorem for two self-mappings in complete metric spaces. Our theorem generalizes Theorem 1 of [[3]]. Example is furnished to illustrate the validity of our results. Further, we apply our theorem to realize the existence of common solutions of a system of two functional equations arising in dynamic programming.

1 Introduction

In this chapter, we will recall the main notions and the concepts we will need, as well as the definitions concerning this work.

Definition 1.1. Let $A, S : X \to X$ be two mappings. A point $u \in X$ is said to be
i) a fixed point of A if $Au = u$,
ii) a coincidence point of A and S if $Au = Su$. The point $z = Au = Su$ is called a point of coincidence of A and S.
iv) A and S are weakly compatible iff they commute at their coincidence point.

Definition 1.2. A and S are said to be Quasi weakly Picard-Jungck operators (brievely QWPJO) if:
i) A and S have at least one point of coincidence or coincidence point.
ii) The sequence $\{Sx_n\}$ converges to a point of coincidence for any $x \in X$.

2 Main results

Our purpose in this section to demonstrate coincidence fixed points theorem for two self-mappings in complete metric spaces. Our theorem generalizes Theorem 1 of [4]. Examples are furnished to illustrate the validity of our results. We apply our theorem to realize the existence of common solutions of a system of two functional equations arising in dynamic programming. Now, we state and prove our main result.

Key Words and Phrases: *Coincidence point, weakly compatible mappings, metric space.*

Theorem 2.1. *Let A and S be two mappings of a complete metric space (X,d) into itself verifying*

$$A(X) \subset S(X), \tag{2.4}$$

$$d(Ax,Ay) \leq N(x,y)M(x,y) \tag{2.5}$$

for all $x,y \in X$, where

$$N(x,y) = \frac{\max\{d(Sx,Sy), d(Sx,Ax)+d(Sy,Ay), d(Sx,Ay)+d(Sy,Ax)\}}{d(Sx,Ax)+d(Sy,Ay)+1}, \tag{2.6}$$

$$M(x,y) = \max\{d(Sx,Sy), d(Sx,Ax), d(Sy,Ay), \frac{d(Sx,Ay)+d(Sy,Ax)}{2}\}. \tag{2.7}$$

Suppose that $S(X)$ is a closed subspace of X. So
i) A and S have at least one coincidence point $u \in X$ and the Jungck sequence $\{y_n\} = \{Sx_n\}$ converges to $z = Au$ for each $x \in X$. In this case, A and S are QWPJO.

Proof. Let x_0 be an arbitrary point in X. From (2.4), we can define inductively a sequence $\{y_n\}$ in X such that
.... □

2.1 Application in dynamic programming

We apply our theorem to realize the existence of common solutions of a system of two functional equations arising in dynamic programming.
Let X and Y be Banach spaces, $S \subset X$ be the state space, $D \subset Y$ be the decision space and I_X be the identity mapping on X. $B(S)$ denotes the set of all bounded real valued functions on S and

$$d(f,g) = \sup_{x \in S} |f(x) - g(x)|. \tag{*}$$

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Galerkin method in one dimension and multi-variable dimension for the second kind

BEN DAHMANE Raouia*

Mailing address: Department of mathematics, University of Biskra, Biskra, Algeria
E-mail: raouia.bendahmane@univ-biskra.dz

MOKHTARI Zouhir

Mailing address: Department of mathematics, University of Biskra, Biskra, Algeria
E-mail: z.mokhtari@univ-biskra.dz

KABOUL Hanane

Mailing address: Department of mathematics, University of Biskra, Biskra, Algeria
E-mail: hanane.kaboul@univ-biskra.dz

Abstract
In this work, we present a new method to resolution Fredholm integral equations of the second kind focused on Galerkin method, Kantorovich method and Sloan method for one and multi-dimensional and the difference between them in space, piecewise polynomial interpolation and error estimates .

1 Introduction

Over the last 20 years, since the publication of Sloan's paper on the improvement by the iteration technique, varioums approaches have been proposed for post-processing the Galerkin solution of multi-dimensional second kind Fredholm Integral equation. These method include the iterated Galerkin method proposed by Sloan, the Kantorovich method and the iterated Kantorovich method. Recently, Lin, Zhang and Yan have proposed interpolation as an alternative to the iteration technique. For an integral operator, with a smooth kernel using the orthogonal projection onto a space of discontinuous piecewise polynomials of degree $\leq r$.
For a multi-dimensional integral equation of the second kind with a smooth kernel, using the orthogonal projection onto a space of discontinuous piecewise polynomials of degree r , Atkinson has established an order $r+1$ convergence for the Galerkin solution and an order $2r+2$ convergence for the iterated Galerkin solution, a new method based on projections has been shown to give a $4r+4$ convergence for one-dimensional second kind integral equations.

Key Words and Phrases: *Integral equations, Galerkin method, Fredholm integral equations, Iteration method*

2 Main results

Let

$$u - Tu = f$$

denote a second kind operator equation, where T is a compact linear operator defined on a complex Banach space X and f and u belong to X. We propose to approximate T by the following finite rank operator

$$T_n^M = P_nTP_n + P_nT(I - P_n) + (I - P_n)TP_n.$$

3 Error estimates

3.1 Orthogonal Projection

One dim Let $X = L^2\,[a, b]$ and $\langle ., .\rangle$ denotes the usual inner produt on X. Let T be an integral operator with a kernel $k(., .) \in C^r([a, b] \times [a, b])$.
Let $X_n = S^v_{r,n}$ and $P_n : X \to X_n$ denote the orthogonal projection .

Proposition 3.1. *For $u \in C^r([a, b])$ we have*

$$\| \;\; T(I - P_n)T(I - P_n)u \;\|_\infty \leq ch^{4r}$$

multi dim Let $r \geq 0$ be an integer and X_n be the set of all $\phi \in L^\infty(D)$ such that $\phi \mid \Delta_k$ is a polynomial of degree $\leq r$, for $k = 1, .., n$. The dimension of X_n is nf_r .Let $\tilde{P}_n : L^2(D) \to X_n$ be the orthogonal projection.

Proposition 3.2. *If $k(x, y, ., .) \in C^{r+1}(D)$ for all $(x, y) \in D$ then $\| T(I - \tilde{P}_n) \| \leq c(\delta_n)^{r+1}$ If $k(x, y, ., .) \in C^{r+1}(D)$ for all $(x, y) \in D, k(., ., \xi, \eta) \in C^{r+1}(D)$ for all $(\xi, \eta) \in D$ and $g \in C^{r+1}(D)$, than*

$$\| T(I - P_n)T(I - P_n)u \|_\infty \leq c(\delta_n)^{4r+4}.$$

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On the growth order for meromorphic solutions of ultrametric q-difference equation of shröder type

Houda Boughaba*

Mailing address: Department of mathematics, University Mohamed Seddik Ben Yahia, Jijel, Algeria
E-mail: houdaboughaba5@gmail.com

Tahar Zerzaihi

Mailing address: Department of mathematics, University Mohamed Seddik Ben Yahia, Jijel, Algeria
E-mail: zerzaihi@yahoo.com

Abstract
Let $\mathbb{K}$ be a complete ultrametric algebraically closed field of characteristic zero and let $\mathcal{M}(\mathbb{K})$ be the field of meromorphic functions in all $\mathbb{K}$. In this article, we give some characteristics of the order of growth for transcendantal meromorphic solutions of some ultrametric q-difference equations. These equations arise from the analogue study of the generalized Shröder equation.

1 Introduction

Nevanlinna theory is a far-reaching generalization of Picard's theorem. This theory, roughly speaking, is dependent on two main theorems; The first main theorem is considered as reformulation of the Poisson-Jensen formula for meromorphic functions. The second main theorem is seen as the central part of Nevanlinna theory. These two theorems lead Nevanlinna theory to become a rich and important theory. Recently, Nevanlinna theory has been extended, (see [**?**]), to ultrametric meromorphic functions on an algebraically closed field, complete for an ultrametric absolute value, denoted by $\mathbb{K}$.
There are many papers focused on q-difference equations in the field of the nomber complex $\mathbb{C}$, (see [**?**, **?**]), when the authors studied the solutions of these equations and gave some results and properties of the growth order of their solutions.
Let $\mathbb{K}$ be an ultrametric algebraically closed field of characteristic zero, with the ultrametric absolute value $|.|$. Let α be a point of $\mathbb{K}$, let $R > 0$, $d(\alpha, R^-)$ be the open disk $\{x \in \mathbb{K}|\ |x-\alpha| < R\}$ and $d(\alpha, R^+)$ be the closed disk $\{x \in \mathbb{K}|\ |x-\alpha| \leq R\}$. We denote by $\mathcal{A}(\mathbb{K})$ the $\mathbb{K}$-algebra of entire functions in $\mathbb{K}$ and by $\mathcal{M}(\mathbb{K})$ the field of meromorphic functions in $\mathbb{K}$, i.e. the field of fractions of $\mathcal{A}(\mathbb{K})$.
In the same way, we denote by $\mathcal{A}(d(\alpha, R^-))$ the $\mathbb{K}$-algebra of analytic functions in $d(\alpha, R^-)$ i.e. the set of power series converging inside $d(\alpha, R^-)$ and by $\mathcal{M}(d(\alpha, R^-))$ the field of meromorphic functions in $d(\alpha, R^-)$ i.e. the field of fractions of $\mathcal{A}(d(\alpha, R^-))$.

Key Words and Phrases: *Nevanlinna theory, q-difference equations, Ultrametric meromorphic solution, Order of growth.*

For every $r > 0$, we denote by $|f|(r) = \sup_{n\geq 0}|a_n|r^n$, for every function $f(x) = \sum_{n\geq 0} a_n x^n$ of $\mathbb{K}$, the maximum modulus of f, which is a multiplicative norm on $\mathcal{A}(\mathbb{K})$. If $f \in \mathcal{M}(\mathbb{K})$ and $f = \frac{g}{h}$, where $g, h \in \mathcal{A}(\mathbb{K})$, we write $|f|(r) = \frac{|g|(r)}{|h|(r)}$.
Finally, for every $f \in \mathcal{M}(\mathbb{K})$, suppose that β is a zero or a pole of f, we denote by $\omega_\beta(f) \in \mathbb{Z}$, the order of β i.e if $f(x) = \sum_{n\geq n_\beta} a_n(x-\beta)^n$ and $a_{n_\beta} \neq 0$, then $\omega_\beta(f) = n_\beta$.

2 Main results

In this work, we will study the difference equation of the form

$$\sum_{j=0}^{n} A_j(x) f(q^j x) = A_{n+1}(x), \tag{2.1}$$

where $A_0(x), A_1(x), \ldots, A_n(x)$ are polynomials and $q \in \mathbb{K}$.

Theorem 2.1. *Suppose that the coefficients $A_0(x), A_1(x), \ldots, A_n(x)$ of equation(**??**) are constants, $A_{n+1}(x)$ is a polynomial and $q \in \mathbb{K}$ satisfies $|q| = 1$. If f is a transcendantal entire solution of (**??**), then for each $k \in \{0, \ldots, n\}$, we have*

$$|A_k| \leq \max_{j\in\{0,\ldots,n\}/\{k\}} \{|A_j|\}.$$

Theorem 2.2. *Suppose that f is a entire solution of equation (**??**) such that the coefficients $A_0(x), \ldots, A_n(x)$ are constants, $q \in \mathbb{K}$ satisfies $0 < |q| < 1$ and $A_{n+1}(x) = e^{P(x)}$, where P is polynomial of degree $d \in \mathbb{N}$. Then, we have $\rho(f) = d$.*

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PART C

Partials Differentials equations

Stabilization of the Schrödinger equation with boundary distributed time delay

Fatima Zohra Sidi Ali*

Department of Mathematics, University of Batna 2, Batna Algeria
E-mail: f.sidiali@univ-batna2.dz

Salah-Eddine Rebiai

Department of Mathematics, University of Batna 2, Batna, Algeria
E-mail: rebiai@hotmail.com

Wassila Ghecham

Department of Mathematics, University of Batna 2, Batna, Algeria
E-mail: wassilaghecham@gmail.com

Abstract
We consider a system of the Schrödinger equation with distributed delay terms in the boundary feedbacks. Under suitable assumptions, we prove exponential stability of the solution. This result is obtained by introducing suitable energy functions and by proving some observability estimates

1 Introduction

In this work, we study stability problems for the Schrödinger equation with a distributed delay term in the boundary.
Let Ω be an open bounded domain of $\mathbb{R}^n$ with smooth boundary Γ which consists of two non-empty parts Γ_1 and Γ_2 such that, $\Gamma_1 \cup \Gamma_2 = \Gamma$ with $\overline{\Gamma_1} \cap \overline{\Gamma_2} = \emptyset$.
In addition to these standard hypothesis, we assume the following.

(A) There exists $x_0 \in \mathbb{R}^n$ such that, with $m(x) = x - x_0$,

$$m(x).\nu(x) \leq 0 \text{ on } \Gamma_1, \tag{1.1}$$

where $\nu(.)$ is the unit normal to Γ pointing towards the exterior of Ω.

In Ω, we consider the following system described by the Schrödinger equation with distributed delay term in the boundary feedback:

$$\begin{cases} u_t(x,t) - i\Delta u(x,t) = 0 & \text{in } \Omega \times (0;+\infty), \\ u(x,0) = u_0(x) & \text{in } \Omega, \\ u(x,t) = 0 & \text{on } \Gamma_1 \times (0,+\infty), \\ \frac{\partial u}{\partial \nu}(x,t) = i\alpha_0 u(x,t) + i\int_{\tau_1}^{\tau_2} \alpha(s)u(x,t-s)\,ds & \text{on } \Gamma_2 \times (0,+\infty), \\ u(x,-t) = f_0(x,-t) & \text{on } \Gamma_2 \times (0,\tau_2), \end{cases} \tag{1.2}$$

Key Words and Phrases: *Schrödinger equation, distributed time delay, stability.*

where:
u_0 and f_0 are the initial data which belong to suitable spaces, $\frac{\partial}{\partial\nu}$ is the normal derivative, τ_1 and τ_2 are two real numbers with $0 \le \tau_1 < \tau_2$, α_0 is a positive constant and $\alpha : [\tau_1, \tau_2] \to \mathbb{R}$ is an L^∞ function, $\alpha \ge 0$ almost everywhere.
One of the purposes of this work is to investigate the stability of system (1.2). To this aim, assume as in [2]

$$\alpha_0 > \int_{\tau_1}^{\tau_2} \alpha(s)\, ds, \tag{1.3}$$

which guarantees the existence of a positive constant c_0 such that

$$\alpha_0 - \int_{\tau_1}^{\tau_2} \alpha(s)\, ds - \frac{c_0}{2}(\tau_2 - \tau_1) > 0, \tag{1.4}$$

and define the energy of a solution of system (1.2) by

$$E(t) = \frac{1}{2}\int_\Omega |u(x,t)|^2 dx + \frac{1}{2}\int_{\Gamma_2}\int_{\tau_1}^{\tau_2} s(\alpha(s) + c_0)\int_0^1 |u(x, t - \rho s)|^2\, d\rho\, ds\, d\Gamma. \tag{1.5}$$

Then we have the following stability result for system (1.2).

2 Main results

Theorem 2.1. *Assume* (A) *and* (1.3). *Then, there exist constants* $M \ge 1$ *and* $\delta > 0$ *such that*

$$E(t) \le Me^{-\delta t}E(0).$$

The proof of this result is based on an energy estimate at the $L^2(\Omega)$ level for a fully Schrödinger equation with gradient and potential terms stated in [4], Theorem 2.6.1 and established in [5], Section 10.

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Efficient Compact Scheme for Second-Order Parabolic Equation with Nonlocal Conditions

Nouria Arar*

Mailing address: Mathematics and Decision Sciences Laboratory, Department of Mathematics, University of Frères Mentouri, Constantine, Algeria
E-mail: arar.nouria@umc.edu.dz

Abstract
This work proposes a high-accuracy numerical method based on a compact difference scheme and fourth order Runge-Kutta approach for the parabolic equation with non-local boundary conditions (NLBC). According to this approach, the partial differential equation which represents the heat equation is transformed into several ordinary differential equations. These time dependent ordinary differential equations are then solved using a fourth order Runge-Kutta method. Test problems are examined to demonstrate the accuracy of the current methods. After that, a comparison is done between numerical solutions obtained by the proposed method and the analytical solutions as well as the numerical solutions available in the literature.

1 Introduction

In this work, the following one dimensional problem is considered

$$\frac{\partial w}{\partial \tau} = \frac{\partial^2 w}{\partial \varkappa^2} + f(\varkappa, \tau), \quad 0 < \varkappa < 1, \;\; 0 < \tau \leq \mathcal{T} \tag{1.1}$$

where $f(\varkappa, \tau)$ is a sufficiently differentiable function in time and space, $\mathcal{T} \in \mathbb{R}^+$, and subject to the initial condition

$$w(\varkappa, 0) = \varphi(\varkappa), \quad 0 \leq \varkappa \leq 1 \tag{1.2}$$

and the NLBC

$$\begin{aligned} w(0,\tau) &= \int_0^1 f_1(\varkappa) w(\varkappa,\tau)\,\mathrm{d}\varkappa + g_1(\tau), \quad 0 \leq \tau \leq \mathcal{T} \\ w(1,\tau) &= \int_0^1 f_2(\varkappa) w(\varkappa,\tau)\,\mathrm{d}\varkappa + g_2(\tau), \quad 0 \leq \tau \leq \mathcal{T} \end{aligned} \tag{1.3}$$

where f_1, f_2, g_1, g_2 and φ are known functions.

Key Words and Phrases: *Compact finite difference schemes, Nonclassic boundary value problems, Simpson's One-third rule, Runge-Kutta scheme.*

2 Main results

To develop our proposed technique, we shall use the grid points which are detailed by

$$\varkappa_i = ih \in [0,1], \quad i = \overline{0, N+1}, \text{ and } h = \frac{1}{(N+1)}, \qquad \tau_j = j\Delta\tau \qquad j = \overline{0, N_\tau},$$

N and N_τ are integers. Here, N is chosen as an odd number because of the use of Simpson's formula. $\vartheta(\varkappa, \tau)$ denote the numerical solution for the solution $w(\varkappa, \tau)$. Lele in [3] defined compact schemes for approximating 2nd order derivatives at interior nodes as

$$\ell\vartheta''_{i-1} + \vartheta''_i + \ell\vartheta''_{i+1} = \mathfrak{m}_1\vartheta_{i-2} + \mathfrak{m}_2\vartheta_{i-1} - \mathfrak{m}_3\vartheta_i + \mathfrak{m}_2\vartheta_{i+1} + \mathfrak{m}_1\vartheta_{i+2} \tag{2.1}$$

where $\ell = \frac{2}{11}, \quad q = \frac{12}{11}, \quad p = \frac{3}{11}, \quad \mathfrak{m}_1 = \frac{p}{4h^2}, \quad \mathfrak{m}_2 = \frac{q}{h^2}, \quad \text{and} \quad \mathfrak{m}_3 = \frac{p+4q}{2h^2}.$

When $i = 1$, we use

$$\vartheta''_1 + \ell_1\vartheta''_2 = \frac{1}{h^2}\left(\mathfrak{s}_0\vartheta_0 + \mathfrak{s}_1\vartheta_1 + \mathfrak{s}_2\vartheta_2 + \mathfrak{s}_3\vartheta_3 + \mathfrak{s}_4\vartheta_4 + \mathfrak{s}_5\vartheta_5 + \mathfrak{s}_6\vartheta_6\right) \tag{2.2}$$

When $i = N$, we use

$$\vartheta''_N + \ell_1\vartheta''_{N-1} = \frac{1}{h^2}\left(\mathfrak{s}_0\vartheta_{N+1} + \mathfrak{s}_1\vartheta_N + \mathfrak{s}_2\vartheta_{N-1} + \mathfrak{s}_3\vartheta_{N-2} + \mathfrak{s}_4\vartheta_{N-3} + \mathfrak{s}_5\vartheta_{N-4} + \mathfrak{s}_6\vartheta_{N-5}\right) \tag{2.3}$$

$$\ell_1 = \frac{11}{2}, \quad \mathfrak{s}_0 = \frac{131}{360}, \quad \mathfrak{s}_1 = \frac{123}{20}, \quad \mathfrak{s}_2 = -\frac{57}{4}, \quad \mathfrak{s}_3 = \frac{157}{18}, \quad \mathfrak{s}_4 = -\frac{9}{8}, \quad \mathfrak{s}_5 = \frac{3}{20}, \quad \mathfrak{s}_6 = -\frac{1}{90}.$$

The combination of (1.1) with (2.1), (2.2) and (2.3) at time level $\tau = \tau_j$ construct a system of N linear equations with $N+2$ unknowns $\vartheta_0, \vartheta_1, \ldots, \vartheta_{N+1}$. Simpson's formula is used to approximate the integrals in (1.3) and thus eliminate ϑ_0 and ϑ_{N+1}, to obtain

$$\begin{cases} \dfrac{\mathrm{d}\mathcal{V}}{\mathrm{d}\tau} = \mathcal{A}^{-1}(\mathcal{M}\mathcal{V}(\tau) + \mathfrak{L}(\tau)) + \mathcal{F}(\tau) \\ \mathcal{V}(0) = \Phi(\varkappa) \end{cases} \tag{2.4}$$

Example 2.1. We consider equations (1.1)−(1.3) with, $\varphi(\varkappa) = \varkappa^2, \quad f_1(\varkappa) = f_2(\varkappa) = \varkappa,$

$$w(\varkappa,\tau) = \frac{\varkappa^2}{(\tau+1)^2}, \; f(\varkappa,\tau) = -\frac{2(\varkappa^2+\tau+1)}{(\tau+1)^3}, \; g_1(\tau) = -\frac{1}{4(\tau+1)^2}, \; g_2(\tau) = \frac{3}{4(\tau+1)^2}.$$

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Identification of a source term in a degenerate parabolic equation with memory from final observation by optimization method

Maroua Nouar*

ICOSI Laboratory, Department of Mathematics and Informatics,Abbes Laghrour University, P.O. Box 1252, Khenchela, 40004, Algeria
E-mail: nouar.maroua@hotmail.com

Abdeldjalil Chattouh

ICOSI Laboratory, Department of Mathematics and Informatics,Abbes Laghrour University, P.O. Box 1252, Khenchela, 40004, Algeria
E-mail: abdeldjalil chettouh@hotmail.com

Abstract
In this work, we study the inverse problem of identifying a space-wise dependent space source term of a weakly degenerate parabolic integro-differential equation from measurement data at the final time. Based on the optimal framework, the identification problem is formulated as an optimization problem with an adequate cost function. Then, the existence of a minimizer of the cost functional is established. The stability estimate for the unknown parameter is derived by the use of a first-order necessary optimality condition.

1 Introduction

In this paper, we are interested on the identifying of unknown source term in a degenerate parabolic equation with memory term from measurement data at final time. More precisely, we consider the following degenerate parabolic equation on $Q := \Omega \times (0, T]$ with $\Omega := (0, 1)$ and $T > 0$ is final fixed moment if time

$$\begin{cases} \partial_t u - \nabla(d(x)\nabla u) + \displaystyle\int_0^t K(t,s)u(x,s)\mathrm{d}s = f(x), \quad (x,t) \in Q, \\ u(x,0) = u_0(x), \quad x \in \overline{\Omega}, \\ u(0,t) = u(1,t) = 0, \quad 0 \le t \le T, \end{cases} \tag{1.1}$$

where $u_0 \in L^2(\Omega)$, $f(x)$ is an unknown source term supposed to be sufficiently smooth in a sense to be specified later and shall be kept independent of time t and the function $d(x)$ is a diffusive coefficient which degenerates weakly at 0, namely,

$$\begin{cases} d \in C([0,1]) \cap C^1((0,1]), \quad d(0) = 0, \quad d(x) > 0 \text{ for all } x \in (0,1], \\ \exists \alpha \in [0,1), \text{ such that } xd'(x) \le \alpha d(x), \forall x \in [0,1]. \end{cases}$$

Key Words and Phrases: ...., ..., ..., ...

For a wide range of applications, it is frequently necessary to determine one or more parameters that describe specific properties in a system governed by a partial differential equation from additional information that can be observed or measured experimentally.
In this paper, our objective is to establish a stability estimate in determining the term source $f(x)$ in the system (**??**) using the following observation data

$$u(x,T) = u_{obs}(x), \quad \forall x \in [0,1], \tag{1.2}$$

where $u_{obs}(x)$ is a known function supposed to satisfy the homogeneous Dirichlet boundary conditions.
Inverse problems for non-degenerate parabolic systems, including parabolic equations with memory term, are well investigated over the last decades and there have been a great number of works devoted to study this kind of problems in many aspects such as stability, observability and controllability (see [3, 2] and the references therein)
Among the studies devoted to problems which are close to the problem considered in this paper, we mention [1] in which the inverse problem of reconstructing a time-independent coefficient of the first order in an integrodifferential equation from a final time overspecified data similar to. In recent work [5], the authors studied the null controllability and approximate controllability for a class of weakly degenerate parabolic integrodifferential equations. In this line, up to now, the inverse source problems for parabolic equations with memory were never investigated even in the case of non-degenerate parabolic equations.
Motivated by this reason, the present contribution is devoted to the study of the stability and local uniqueness of the identification problem of the term source in the degenerate parabolic equation. In order to achieve our objective, we shall employ the methodology used in [4] for the treatment of the problem of identifying a source term in a classical parabolic problems which is based on the optimal control framework. The basic idea is to consider the solution of the inverse problem as a minimizer of some cost functional which satisfies the first-order necessary optimality condition and make use of this last, we establish a stability estimate which in turn leads to the local uniqueness under some hypothesis.

2 Main results

The main result of this contribution can be briefly stated as follows : let u and $\tilde{u}$ be the solutions of the problem (1.1) associated with the unknown terms f and $\tilde{f}$ respectively, then there exists a positive constant $C > 0$ satisfying

$$\|f - \tilde{f}\|^2_{L^2(\Omega)} \leq C\|u_{obs} - \tilde{u}_{obs}\|^2_{L^2(\Omega)},$$

where u_{obs} and $\tilde{u}_{obs}$ are the values of the solutions u and $\tilde{u}$ respectively at final time $t = T$ expressed by the over-specified condition (1.2).

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Existence and Stability a Thermoelastic Shear Beam Model

Asma Ben Moussa*
Mailing address: Department of Mathematics, University of El-Oued, El-Oued, Algeria
E-mail: benmoussa-asma@univ-eloued.dz

Abdelfeteh Fareh
Mailing address: Department of Mathematics, University of El-Oued, El-Oued, Algeria
E-mail: farehabdelf@gmail.com

Abstract
In this work, we consider a thermoelastic shear beam model. We prove a well posedness result by the use of Faedo–Galerkin method and an exponential decay by the multiplier method.

1 Introduction

in this section, we consider the system

$$\begin{cases} \rho\varphi_{tt} - \kappa(\varphi_x + \psi)_x + \mu\theta_x = 0, & \text{in } (0,L)\times(0,\infty), \\ -b\psi_{xx} + \kappa(\varphi_x + \psi) = 0, & \text{in } (0,L)\times(0,\infty), \\ c\theta_t - \delta\theta_{xx} + \mu\varphi_{xt} = 0, & \text{in } (0,L)\times(0,\infty), \end{cases} \tag{1.1}$$

and we have initial and boundary conditions

$$\varphi(x,0) = \varphi_0(x), \varphi_t(x,0) = \varphi_1(x), \psi(x,0) = \psi_0(x), \theta(x,0) = \theta_0(x),\ x \in [0,L] \tag{1.2}$$

$$\varphi(0,t) = \varphi(L,t) = \psi(0,t) = \psi(L,t) = \theta_x(0,t) = \theta_x(L,t) = 0,\ t \in (0,+\infty). \tag{1.3}$$

The energy of the system is defined by

$$E(t) := \frac{\rho}{2}\int_0^L |\varphi_t|^2 dx + \frac{b}{2}\int_0^L |\psi_x|^2 dx + \frac{c}{2}\int_0^L |\theta|^2 dx + \frac{\kappa}{2}\int_0^L |\varphi_x + \psi|^2 dx. \tag{1.4}$$

We obtain

$$\frac{d}{dt}E(t) = -\delta\int_0^L |\theta_x|^2 dx \leq 0.$$

2 Main results

1)Well-posdness We introduce the phase space:

$$\mathcal{H} := H_0^1(0,L) \times\ L^2(0,L) \times\ H_0^1(0,L) \times H_*^1(0,L) \tag{2.1}$$

Theorem 2.1. *For any initial data $(\varphi_0, \varphi_1, \psi_0, \theta_0) \in \mathcal{H}$ and any $T > 0$, the problem* (1)–(1.3) *has a weak solution (φ, ψ, θ) such that*

$$\begin{gathered} \varphi \in L^\infty(0,T; H_0^1(0,L)),\ \varphi_t \in L^\infty(0,T; L^2(0,L)), \\ \psi \in L^\infty(0,T; H_0^1(0,L)),\ \theta \in L^\infty(0,T; L_*^2(0,L)) \cap L^2(0,T; H_*^1(0,L)). \end{gathered} \tag{2.2}$$

Proof. The proof to tis theorem will be given by the use of Feado–Galerkin method through four steps. □

Key Words and Phrases: *Shear beam model, well–posedness, exponential stability, Faedo–Galerkin method*

2)Exponential decay

Theorem 2.2. *There exists a positive constant* $\gamma > 0$*, such that the energy* $E(t)$ *defined by* (1.4) *satisfies along the solution* (φ, ψ, θ) *the estimate*

$$E(t) \leq E(0)e^{-\gamma t}, \ \forall t \geq 0. \tag{2.3}$$

Proof. The proof of Theorem 2.2 will be given by the use of the multiplier method, in fact we construct a Lyapunov functional $\mathcal{L}(t; U(t;x))$, equivalent to the energy $E(t)$ and $\mathcal{L}_t(t; U(t;x))$ is negative definite. We write $\mathcal{L}$ as a combination of functionals $\{E(t), \mathcal{F}_1(t); \mathcal{F}_2(t)\}$ where each derivative of $E(t), \mathcal{F}_1(t); \mathcal{F}_2(t)$ satisfies an estimate of a term from the energy with negative sign. We obtain, $\mathcal{L}'(t) \leq -\gamma\mathcal{L}(t)$. Thus, an integration we respect to t and the equivalence between $\mathcal{L}$ and $E(t)$ lead to the exponential stability of $E(t)$. $\square$

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Variational Analysis of a Dynamic Electroviscoelastic Problem with Friction

Chahira Guenoune*

Department Of Mathematics, University of Setif 1, Algeria
E-mail: guenounechahire@gmail.com

Aziza Bachmar

Department Of Mathematics, University of Setif 1, Algeria
E-mail: aziza.bachmar@univ-setif.dz

Abstract
A dynamic contact problem is studied. The material behavior is modelled with piezoelectric eects for electro-visco-elastic constitutive law. The body may come into contact with a rigide obstacle. Contact is described with the Signorini condition, a version of Coulombs law of dry friction, and a regularized elec- trical conductivity condition.We derive a variational formulation of the problem, then, under a smallness assumption on the coecient of friction, we prove an existence and uniqueness result of a weak solution for the model. The proof is based on arguments of evolutionary variational inequalities and xed points of operators.

1 Introduction

The piezoelectric eect is the apparition of electric charges on surfaces of particular crystals after deformation. It reverse eect consists of the generation of stress and strain in crystals under the action of the electric eld on the boundary. A deformable material which presents such a behavior is called a piezoelectric material. Piezoelectric materials are used extensively as switches and actualy in many engineering systems. Dierent models have been developed to describe the interaction between the electric and mechanical elds. General models for elastic materials with piezoelectric eects can be found in [1-2] and more recently in [3], viscoelastic piezoelectric materials in [3]. In this work, we consider a general model for the dynamic process of frictional contact between a deformable body and a rigid obstacle. The material obeys an electro-viscoelastic constitutive law with piezoelectric eects. which is set as a system coupling a variational second order evolution inequality. We establish the existence of a unique weak solution of the model. The idea is to reduce the second order evolution inequality of the system to rst order evolution inequality. After this,we use classical results on rst order evolution inequalities and equations and the xed point arguments.

Key Words and Phrases: *piezoelectric, frictional contact, visco-elastic, xed point, dynamic process, variational inequality.*

2 Main results

Our main result which states the unique solvability of Problem are the following.
Theorem 1. The Problem has a unique solution$\{u, \sigma, \phi, D\}$ which satises

$$\begin{aligned}
&u \in C^1(0.T; H) \cap W^{1.2}(0.T; V) \cap W^{2.2}(0.T; V'),\\
&\phi \in W^{1.2}(0.T; W),\\
&\sigma \in \mathcal{L}^2(0.T; H), Div\sigma \in \mathbb{L}^2(0.T; V'),\\
&D \in W^{1.2}(0.T; \mathcal{W}_1).
\end{aligned} \tag{2.1}$$

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Solvability of a dynamic contact problem with adhesion

Imane Ouakil*

Mailing address:Laboratory of Pure and Applied Mathematics, University of Mohamed Boudiaf, M'sila
E-mail: imane.ouakil@univ-msila.dz

Benyattou Benabderrahmane

Mailing address:Laboratory of Pure and Applied Mathematics, University of Mohamed Boudiaf, M'sila
E-mail: benyattou.benabderrahmane@univ-msila.dz

Yamna Boukhatem

Mailing address: Laboratory of Pure and Applied Mathematics, University of Laghouat
E-mail: y.boukhatem@lagh-univ.dz

Abstract
A frictionless dynamic contact problem with adhesion between a viscoelastic body and an obstacle is considered. The contact is modelled with a version of normal compliance. We provide a variational formulation to the model then we prove the existence of a unique weak solution. The proof is based on on nonlinear evolution equations with monotone operators and fixed point arguments.

1 Introduction

Consider the following contact problem
Problem P Find a displacement field $u : \Omega \times [0,T] \to \mathbb{R}^d$ and a stress field $\sigma : \Omega \times [0,T] \to S^d$ and a bonding field $\beta : \Omega \times [0,T] \to [0,1]$ such that

$$\begin{array}{lll}
\sigma = A\varepsilon(\dot{u}) + G\varepsilon(u) + \int_0^t B\,(t-s)\,\varepsilon\,(u\,(s))\,ds & \text{in} & \Omega \times (0,T), \\
\rho\ddot{u} = Div\sigma + f_0 & \text{in} & \Omega \times (0,T), \\
u = 0 & \text{on} & \Gamma_1 \times (0,T), \\
\sigma\nu = f_2 & \text{on} & \Gamma_2 \times (0,T), \\
-\sigma_\nu = p_\nu\,(u_\nu) - \gamma_\nu \beta^2 R_\nu(u_\nu) & \text{on} & \Gamma_3 \times (0,T), \\
-\sigma_\tau = 0 & \text{on} & \Gamma_3 \times (0,T), \\
\dot{\beta} = -\left(\beta\,(\gamma_\nu (R_\nu(u_\nu))^2 + \gamma_\tau \|R_\tau(u_\tau)\|^2) - \varepsilon_a\right)_+ & \text{in} & \Gamma_3 \times (0,T), \\
\beta(0) = \beta_0 & \text{in} & \Gamma_3, \\
u(0) = u_0, \dot{u}(0) = v_0 & \text{in} & \Omega.
\end{array} \tag{1.1}$$

In the study of problem (1.1), we consider the different assumptions on problem data given in ([1, 2]).

Key Words and Phrases: *Dynamic contact, adhesion, weak solution, normal compliance*

2 Main results

Lemma 2.1. *There exists a unique solution to problem PV_η satisfying the following regularity*

$$u_\eta \in W^{1,2}(0,T;V) \cap C^1(0,T;H), \ddot{u}_\eta \in L^2\left(0,T;V'\right).$$

Moreover, if u_i represents the solution of problem PV_η for $\eta = \eta_i \in L^2\left(0,T;V'\right), i = 1,2,$ then there exists $C > 0$ such that

$$\int_0^t \|\dot{u}_1(s) - \dot{u}_1(s)\|_V^2 ds \leq C \int_0^t \|\eta_1(s) - \eta_2(s)\|_{V'}^2 ds \quad \forall t \in [0,T].$$

Lemma 2.2. *There exists a unique solution $\beta_\eta \in W^{1,\infty}(0,T;L^2(\Gamma_3)) \cap \mathcal{Z}$ to problem PV_η^β .*

Now, we introduce the operator $\Lambda : L^2\left(0,T;V'\right) \to L^2\left(0,T;V'\right)$ defined by

$$(\Lambda\eta(t), v)_{V'\times V} = \left(G\varepsilon\left(u_\eta(t)\right), \varepsilon(v)\right)_{\mathcal{H}} + \left(\int_0^t B(t-s)\varepsilon\left(u_\eta(s)\right)ds, \varepsilon(v)\right)_{\mathcal{H}} + j_{ad}(\beta_\eta, u_\eta, v).$$

Lemma 2.3. *The operator Λ has a unique fixed point $\eta^* \in L^2\left(0,T;V'\right)$.*

We have now all the ingredients to state and prove our principal theorem

Theorem 2.1. *Problem PV has a unique solution (u, σ, β) which satisfies*

$$u \in W^{1,2}(0,T;V) \cap C^1(0,T;H), \ddot{u} \in L^2\left(0,T;V'\right),$$

$$\sigma \in L^2\left(0,T;\mathcal{H}\right), Div\sigma \in L^2\left(0,T;V'\right),$$

$$\beta \in W^{1,\infty}(0,T;L^2(\Gamma_3)) \cap \mathcal{Z}.$$

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Polynomial Stability and Optimality for a transmission problem of waves under a nonlocal boundary control

Sabah Baibeche*

Mailing address: Department of Mathematics, 20 August 1955 University, skikda, Algeria
E-mail: sabahbaibeche@gmail.com

Lamine Bouzettouta

Mailing address: Department of Mathematics, 20 August 1955 University, skikda, Algeria
E-mail: lami750000@yahoo.fr

Abstract
In this work we considered the stabilization for the following wave equation with dynamic boundary control of fractional derivative type. We give an existence result and decay properties of solutions for the initial boundary value problem and prove the global existence of its solutions in Sobolev spaces by means of the semigroup theory. To prove decay estimates, we use a technique based on a resolvent estimate and Borichev-Tomilov Theorem. We have shown that the transmission wave system is not exponentially stable and we prove that it is polynomially stable with an optimal rate of decay when $\eta > 0$.

1 Introduction

In this section we study a transmission wave system with boundary control of nonlocal type given by

$$\begin{aligned}
&\rho_1 u_{tt}(x,t) - \tau_1 u_{xx}(x,t) = 0 \text{ in } (0,l_0)\times(0,+\infty),\\
&\rho_2 v_{tt}(x,t) - \tau_2 v_{xx}(x,t) = 0 \text{ in } (l_0,L)\times(0,+\infty),\\
&.
\end{aligned} \tag{1.1}$$

where ρ_1, ρ_2, τ_1 and τ_2 are positive constants that represent the densities and tensions of the strings u and v, respectively, and the initial conditions are

$$u(x,0) = u_0(x), u_t(x,0) = u_1(x), \quad v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x). \tag{1.2}$$

The transmission condition is

$$u(l_0,t) = v(l_0,t), \quad \rho_2\tau_1 u_x(l_0,t) = \rho_1\tau_2 v_x(l_0,t) \quad \forall t \in (0,+\infty), \tag{1.3}$$

Key Words and Phrases: *transmission wave system, polynomially stable, boundary control of fractional derivative type, semigroup theory*

followed by the boundary conditions

$$u(0,t) = 0, \quad \tau_2 v_x(L,t) + \gamma\rho_2 \partial_t^{\alpha,\eta} v(L,t) = 0 \quad \forall t \in (0,+\infty) \tag{1.4}$$

and conditions of compatibility

$$u_0(l_0) = v_0(l_0), u_1(l_0) = v_1(l_0), \quad , \rho_2\tau_1 u_{0x}(l_0) = \rho_1\tau_2 v_{0x}(l_0), \tag{1.5}$$

where $\gamma > 0$, the initial data (u_0, u_1, v_0, v_1) belong to a suitable function space. The notation $\partial_t^{\alpha,\eta}$ stands for the generalized Caputo's fractional derivative of order α, $0 < \alpha < 1$, with respect to the time variable (see Choi and MacCamy [**2**] and E. Blanc, G. Chiavassa, and B. Lombard [**4**]).

2 Main results

we show an optimal energy decay rate depending on the parameter α. The proof heavily relies on a precise estimate of the resolvent of the generator associated to the semi-group and Borichev-Tomilov Theorem
Our main result is as follows:
The semigroup $S_{\mathcal{A}}(t)_{t\geq 0}$ is polynomially stable and

$$E(t) = \|S_{\mathcal{A}}(t)U_0\|_{\mathcal{H}}^2 \leq \frac{1}{t^{2/(1-\alpha)}}\|U_0\|_{D(\mathcal{A})}^2. \tag{2.1}$$

Moreover, the rate of energy decay $t^{-2/(1-\alpha)}$ is optimal for any initial data in $D(\mathcal{A})$. Besides, we prove that the decay rate is optimal. Indeed, the decay rate is consistent with the asymptotic expansion of eigenvalues which shows a behavior of the real part like $k^{-(1-\alpha)}$.

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FIRST ORDER EVOLUTION INCLUSIONS GOVERNED BY TIME AND STATE DEPENDENT MAXIMAL MONOTONE OPERATOR IN HILBERT SPACE

F. Selamnia*

Mailing address: University of Mohammed Seddik Benyahia-Jijel, Algeria
E-mail: fatihamenigher@gmail.com

D. Azzam-Laouir

Mailing address: University of Mohammed Seddik Benyahia-Jijel, Algeria
E-mail: laouir.dalila@gmail.com

Abstract
In this paper, we prove the existence of solutions of evolution problems governed by time and state dependent maximal monotone operator in a separable Hilbert space, of the form:

$$(\mathcal{P}) \quad \begin{cases} -\dot{u}(t) \in A(t,u(t))u(t) & \text{a.e.} t \in [0,T] \\ u(0) = u_0 \in D(A(0,u_0)) \end{cases}$$

1 Introduction

The aim of this paper is to prove existence results of solutions for the first order differential inclusion in separable Hilbert space H of the form

$$(\mathcal{P}) \begin{cases} -\dot{u}(t)) \in A(t,u(t))u(t), & a.e.\ t \in I = [0,T] \\ u(0) = u_0 \in D(A(0,u_0)); \end{cases}$$

governed by a time and state dependent maximal monotone operator $A(t,x)$. For this purpose we consider the existence problem of Lipschitz continuous solutions to $(\mathcal{P})$ by assuming that $(t,x) \mapsto A(t,x)$ is of Lipschitz continuous variation, in the sense that there exists a nonnegative constant $\lambda < 1$ and a function $\alpha : I \to [0,+\infty[$, which is continuous on I and nondecreasing, such that

$$dis(A(t,x),A(t,y)) \leqslant |\alpha(t) - \alpha(s)| + \lambda\|x - y\|, \ \forall t,s \in I, \text{ and } \forall x,y \in H$$

where $dis(\cdot,\cdot)$ is the pseudo-distance between maximal monotone operators

Key Words and Phrases: *Differential inclusions, maximal monotone operators , state-dependent, pseudo-distance, perturbation*

2 Main results

For the statement of our theorems of this work, we have to assume the following hypotheses.

(H_A^1) There exists a nonnegative constant $\lambda < 1$ and a function $\alpha : I \to [0, +\infty[$, which is continuous on I and nondecreasing, such that

$$\operatorname{dis}(A(t,x), A(s,y)) \leqslant |\alpha(t) - \alpha(s)| + \lambda \|x - y\|, \ \forall t, s \in I, \ \forall x, y \in H. \tag{2.1}$$

(H_A^2) There exists a positive constant c such that

$$\|A^0(t,x)y\| \leqslant c(1 + \|x\| + \|y\|). \tag{2.2}$$

(H_A^3) For any bounded subset $B \subset H$, the set $D(A(I \times B))$ is relatively ball compact.

Now, we present our main result.

Theorem 2.1. *Let for every $(t,x) \in I \times H$, $A(t,x) : D(A(t,x)) \to 2^H$ be a maximal monotone operator satisfying (H_A^1), (H_A^2) and (H_A^3). Then for any $u_0 \in D(A(0,u_0))$, the differential inclusion*

$$(\mathcal{P}) \qquad \begin{cases} -\dot{u}(t) \in A(t, u(t))u(t) & a.e. \ t \in [0,T] \\ u(0) = u_0 \end{cases}$$

has a Lipschitz solution $u : I \to H$. Moreover, we have for almost every $t \in I$

$$\|\dot{u}(t)\| \leqslant L(1 + \dot{\alpha}(t))$$

for some nonnegative constant L depending on $\|u_0\|, \lambda, c, T$, and $\alpha(\cdot)$.

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Differential equations of elliptic type with variable operators and general Robin boundary condition, in UMD spaces

Author Rabah HAOUA*

Mailing address: Department of Mathematical, University of Mostaganem, Algeria
E-mail: rabah.haoua@univ-mosta.dz

Co-author Laid DJILALI

Mailing address: Department of Mathematical, University of Mostaganem, Algeria
E-mail: laid.djilali@univ-mosta.dz

Abstract
In this paper we study an abstract second order differential equation of elliptic type with variable operator coefficients and general Robin boundary conditions, in the framework of UMD spaces. These problems presents for example the linearized stationary case of a model describing information diffusion in online social networks. Existence and regularity results are obtained when the Labbas-Terreni assumption is fulfilled using semi-groups theory and interpolation spaces.

1 Introduction and hypotheses

This paper is devoted to study the following general problem

$$\left\{\begin{array}{ll} u''(x) + A(x)\,u(x) - \omega u(x) = f(x), & x \in (0,1) \\ u'(0) - Hu(0) = d_0 & \\ u(1) = u_1, & \end{array}\right. \tag{1.1}$$

with $f \in L^p(0,1,E)$, $1 < p < +\infty$, where E is a complex Banach space, d_0, u_1 are given elements in E and $(A(x))_{x\in[0,1]}$ is a family of closed linear operators whose domains $D(A(x))$ are dense in E. H is a closed linear operator in E, ω is a positive real number. The results proved here in the L^p case complete our recent paper concerning the hölderian case, see [2].
For all $x \in [0,1]$, set:

$$A_\omega(x) = A(x) - \omega I.$$

We will seek for a classical solution u to (1.1), i.e. a function u such that

$$\left\{\begin{array}{l} \text{a.e } x \in (0,1), \quad u(x) \in D(A(x)) \text{ and} \\ x \mapsto A(x)\,u(x) \in L^p(0,1;E) \\ u \in W^{2,p}(0,1;E) \\ u(0) \in D(H), \end{array}\right. \tag{1.2}$$

Key Words and Phrases: *Differential equation, Robin boundary conditions, analytic semigroup, Dore-Venni theorem.*

The method is essentially based on Dunford calculus, interpolation spaces, the semigroup theory and some techniques as in [3], [2].
We will assume that

$$E \text{ is a } UMD \text{ space.} \tag{1.3}$$

We suppose that:

$\exists\omega_0 > 0$, $\exists C > 0 : \forall x \in [0,1]$, $\forall z \geq 0$, $(A_{\omega_0}(x) - zI)^{-1} \in L(E)$ and

$$\left\| (A_{\omega_0}(x) - zI)^{-1} \right\|_{L(E)} \leq \frac{C}{1+z}; \tag{1.4}$$

we suppose also that:
$\exists C,\, \alpha,\, \mu > 0 : \forall x,\, \tau \in [0,1]$, $\forall \omega \geq \omega_0$:

$$\begin{cases} \left\| A_\omega(x)(A_\omega(x) - zI)^{-1}\left(A_\omega(x)^{-1} - A_\omega(\tau)^{-1}\right) \right\|_{L(E)} \leq \dfrac{C\,|x-\tau|^\alpha}{|z+\omega|^\mu} \\ \text{with } \alpha + \mu - 2 > 0; \end{cases} \tag{1.5}$$

this hypothesis is well known as Labbas-Terreni assumption.

2 Main results

We obtain the following theorem.

Theorem 2.1. *Assume (1.3)∼(1.5). Let* $f \in L^p(0,1;E)$, $1 < p < +\infty$ *and*

$$(Q_\omega(0) - H)^{-1} d_0 \in (D(A(0)), E)_{\frac{1}{2p},p}, \quad u_1 \in (D(A(1)), E)_{\frac{1}{2p},p}.$$

Then there exists $\omega^* > 0$ *such that for all* $\omega \geq \omega^*$, *the problem (1.1) has a unique solution* $w(\cdot) = Q_\omega(\cdot)^2 u(\cdot)$ *verifying*

1. $Q_\omega(\cdot)^2 u(\cdot) \in L^p(0,1;E)$.
2. $u'' \in W^{2,p}(0,1;E)$.

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Topological degree for fractional system

Chaima Saadi*

LAMAHIS Laboratory, University of 20 Aot 1955-Skikda, Algeria
E-mail: saadichaima248@gmail.com

Hakim Lakhal

LAMAHIS Laboratory, University of 20 Aot 1955-Skikda, Algeria
E-mail: h.lakhal@univ-skikda.dz

Abstract
In this research, semilinear fractional system involving a distributional Riesz fractional gradient is tackled. The existence of a distributional solution is demonstrated in fractional Sobolev space and Leray-Schauder degree method is used to deal with existence of this system with some condition on the semilinear term.

1 Introduction

Classical partial differential equations can be generalized to fractional partial differential equations (FPDEs). In recent years, fractional differential equations have received a lot of attention from researchers and this was due to its applications in various field, such as: image processing, mechanics, biophysics, finance.
In the last decade, the fractional derivative has been defined in a variety of ways, one of wich is the Riesz fractional derivative. Tis drew the attention of a number of authors. The problems involving the Riesz fractional gradient, in particular, have piqued curiosity. Where Shieh and Spector in ([5]) were the first to look into partial differential eqution using this derivative, and they looked into the existence and uniqueness results of linear fractional problem relating the distributional Riesz fractional gradient, proving it with Lax-Milgram theorem. Studies on these issues have continued since then. We will mention the some work, C. Saadi et all [3] interested to prove the existence and uniqueness of distribtional solution for semilinear fraction problem involving the Riesz derivative and we use topological degree method to deal it, after that, Belhadi et all [2], using the variational method to prove the existence result for partial differentional equation involving this derivative. Then Abada et all in [1] study the existence solution for non-linear fractional problem and they suggest Leray-Schauder degree theorem, in the last, Slimani et all [4] using fixed point theorem to prove the existence result for convection-reaction fractional problem.
Recently, the fractional partial different equation associated to the distributional Riesz fractional gradient has been extensively studied, because it has new spry properties,

Key Words and Phrases: *partial differential equations, semilinear elliptic system, Leray-Schauder degree, distributional solution*

among its amazing features, enables us to conduct more effective and precise numerical analyses, it is also, the fractional gradient is similar to the ordinary gradient in appearance.
In this document, we are interesting to study the existence of distributional solution for the semilinear fractional system, in fractional Sobolev space and Leray-Schauder degree method is used to deal with existence of this system with some condition on the semilinear term.

2 Main results

In this section, we present somme results about the condition of the Schauder fixed point theorem

Lemma 2.1. *Thanks to assumption. we will show that* $\exists R > 0$, $\forall (\overline{u}, \overline{v}) \in L^2(\Omega) \times L^2(\Omega)$ *such that*

$$\begin{cases} H(\lambda, \overline{u}, \overline{v}) = (\overline{u}, \overline{v}) \\ \lambda \in [0,1], (\overline{u}, \overline{v}) \in L^2(\Omega) \times L^2(\Omega) \end{cases} \Rightarrow \|(\overline{u}, \overline{v})\|_{L^2(\Omega)\times L^2(\Omega)} < R + 1.$$

Lemma 2.2. *Thanks to assumption,* $H\colon [0,1] \times L^2(\Omega) \times L^2(\Omega) \to L^2(\Omega) \times L^2(\Omega)$ *is continuous.*

Lemma 2.3. *Thanks to assumption,* $\{H(\lambda, \overline{u}, \overline{v}), t \in [0,1], (\overline{u}, \overline{v}) \in \overline{B}_{R+1}\}$ *is relatively compact in* $L^2(\Omega) \times L^2(\Omega)$.

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STABILITY RESULT FOR LAME SYSTEM WITH FRACTIONAL TIME-VARIYING AND BOUNDERY FEEDBACK

S. BOUSSERHANE REDA*

University of Constantine, Algeria
E-mail: redasoufiane2000@gmail.com

A. MEMOU

University of Constantine, Algeria
E-mail: k.bouhali@qu.edu.sa

A. BERKANE

University of Constantine, Algeria
E-mail: berkane@usa.com

A. BERKANE

University of Qassim University, Ar-Rass, Saudi Arabia
E-mail: khaledzennir4@gmail.com

Abstract
The Lam'e system with fractional time-variation and boundery feedback is the focus of the paper. A general explanation of the problem's well-posedness, its stability, and an evaluation of the findings are provided.

1 Introduction

The advancement of the natural sciences in the contemporary era is intrinsically tied to the creation and application of mathematical models resulting from process analysis of the material's disruptions spreading. These models typically rely on the mathematical physics equations. Depending on whether or not the proliferation The equations are either hyperbolic or linear, depending on the velocity of these disturbances. type parabolic. Wave propagation is used when these perturbations have a consistent internal structure in spacetime. Taking into account the stable modes of oscillations at a specific rate

Key Words and Phrases: *Damped wave equation, Strong nonlinear system, Global solution.*

2 Main results

The existence and uniqueness results.

Theorem 2.1. *Assume that*

1. $Y = D(\mathrm{A}(0))$ *is a dense subset of* X
2. $D(\mathrm{A}(t)) = D(\mathrm{A}(0))$ *for* $\forall t > 0$
3. *For all* $t \in [0, T]$, $\mathrm{A}(t)$ *generates a strongly continuous semigroup on* X *and family* $\mathrm{A} = \{\mathrm{A}(t) : t \in [0, T]\}$ *is stable with stability constants* C *and* m *independent of* t *(i.e the semigroup* $(S_t\,(s))_{s\geq 0}$ *generated by* $\mathrm{A}(t)$ *satisfies* $\|S_t\,(s)\,u\|_{\mathrm{X}} \leq Ce^{ms}\|u\|_{\mathrm{X}}$ *for all* $u \in \mathrm{X}$ *and* $s \geq 0$*)*
4. $\partial_t \mathrm{A}$ *belongs to* $L^{\infty}_{*}\left([0, T], B(Y, \mathrm{X})\right)$, *which is the space of equivalent classes of essentially bounded, strongly measurable function from* $[0, T]$ *into the set* $B(Y, \mathrm{X})$ *of bounded operators from* Y *into* X.

   Then, the problem has a unique solution $U \in C\left([0, T], Y\right) \cap C^1\left([0, T], \mathrm{X}\right)$ *for any intial data in* Y.

Theorem 2.2. *For any* $a_2 > 0$, *There exist* C_1, C_2 *such that for any solution of problem we have*

$$E(t) \leq C_1 e^{-C_2 t} \quad t > 0$$

The constants C_1, C_2 *are independent of the intial data, but they depend on* a_2 *and on the geometry of* Ω.

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On a nonlinear partial differential system with fractional order

Esma Abada*

University of 20 Août 1955 Skikda, Laboratory of Applied Mathematics and History and Didactics of Mathematics (LAMAHIS), Department of Mathematics, Algeria.
E-mail: esmaa.abada@gmail.com

Hakim Lakhal

University of 20 Août 1955 Skikda, Laboratory of Applied Mathematics and History and Didactics of Mathematics (LAMAHIS), Department of Mathematics, Algeria.
E-mail: h.lakhal@univ-skikda.dz

Abstract
This paper presents some results concerning the existence and uniqueness of weak solutions for a nonlinear fractional Laplacian system with Dirichlet boundary conditions under certain nonlinear function conditions. The topological degree in infinite dimension is used to prove the existence of weak solutions, and the Banach fixed point theorem is used to prove uniqueness of weak solution.

1 Introduction

One of the most important fields in fractional calculus that has piqued the interest of many researchers is fractional partial differential systems. Particularly, semilinear fractional elliptic systems involving fractional Laplacian have piqued the interest of researchers, owing to the interesting property of Fractional Laplacian, which is the non-local property, and because this operator appears in a variety of phenomena such as flame propagation and mathematical finance, as demonstrated by the work of HongGuang Sun, Yong Zhang, Dumitru Baleanu, Wen Chen, and YangQuan Chen (2018).
X. Wang (2020) investigated the Liouville type theorem of solutions for the nonlinear fractional Laplacian system shown below.

$$\begin{cases} (-\Delta)^s u = f(u,v) & \text{in } \mathbb{R}^n, \\ (-\Delta)^s v = g(u,v) & \text{in } \mathbb{R}^n. \end{cases}$$

We use in this work the Leray-Schauder degree method to investigate the existence of weak solutions for a semilinear fractional elliptic system involving fractional Laplacian with Dirichlet boundary conditions. In one particular case, we use the Banach fixed point theorem to investigate the uniqueness of a solution.

Key Words and Phrases: *Nonlocal operator, Leray Schauder degree, fractional order, fixed point*

2 Main results

2.1 Position of problem

In this work, we study the following problem

$$\begin{cases} (-\Delta)^s u(x) = g_1(x, u(x), v(x)) & \text{in } \Omega, \\ (-\Delta)^s v(x) = g_2(x, u(x), v(x)) & \text{in } \Omega, \\ u = v = 0, & \text{on } \mathbb{R}^n \backslash \Omega, \end{cases} \tag{2.1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded open set with a Lipschitz boundary,
with $s \in (0,1)$ such that $n > 2s$ and $g_1, g_2 : \Omega \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are satisfying the Caratheodory conditions and the following assumptions :

(H_1) There exist $a, b \in L^2(\Omega)$ and $K_1, K_2, r_1, r_2 \in \mathbb{R}_+^*$ such that

$$|g_1(x, s, p)| \leq a(x) + K_1|s| + K_2|p|, \forall s, p \in \mathbb{R} \text{ and a.e. } x \in \Omega.$$
$$|g_2(x, s, p)| \leq b(x) + r_1|s| + r_2|p|, \forall s, p \in \mathbb{R} \text{ and a.e. } x \in \Omega.$$

(H_2)

$$\begin{cases} \lim\limits_{s \to \pm\infty} \frac{g_1(x,s)}{s} = 0, \ \forall x \in \Omega. \\ \lim\limits_{s \to \pm\infty} \frac{g_2(x,s)}{s} = 0, \ \forall x \in \Omega. \end{cases}$$

The following theorems are the main results

Theorem 2.1. *Under the assumptions (H_1) and (H_2), the problem (2.1) has at least one weak solution $(u, v) \in D^{s,2}(\Omega) \times D^{s,2}(\Omega)$.*

Particular case
In this particular case we used the following assumption

(H_3) There exists $C_i > 0$, for almost every $x \in \Omega$ and for all $p_i, q_i \in L^2(\Omega), i \in \{1, 2\}$.

$$\|g_i(x, p_i) - g_i(x, q_i)\|_{L^2(\Omega)} \leq C_i \|p_i - q_i\|_{L^2(\Omega)}, i \in \{1, 2\}.$$

g_1, g_2 satisfy the Caratheodory conditions, i.e. $g_1(., s), g_2(., z)$ are measurables for all $s, z \in \mathbb{R}^2$ and

Theorem 2.2. *Under the assumption* (H_3) *and for* $C_iC_{emb}^2 < 1, i \in \{1, 2\}$*, the problem* (2.1) *has a unique weak solution* $(u, v) \in D^{s,2}(\Omega) \times D^{s,2}(\Omega)$.

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Existence results for an unsteady non-Newtonian incompressible flow problem with mixed boundary conditions

Hanene Debbiche*

Department of mathematics, Mohamed El Bachir El Ibrahimi University, Bordj Bou Arréridj, Algeria
E-mail: hanane.debbiche@gmail.com

Mahdi Boukrouche and Laetitia Paoli

Lyon University, F-42023 Saint-Etienne, Institut Camille Jordan UMR 5208, 42023 Saint-Etienne cedex 2, France
E-mail: mahdi.boukrouche@univ-st-etienne.fr and laetitia.paoli@univ-st-etienne.fr

Abstract
We consider non-stationary flow problems for general incompressible fluids in a bounded domain $\Omega \subset \mathbb{R}^3$. The conservation of mass and momentum lead to a unsteady Stokes system. We assume non-standard mixed boundary conditions with a given time dependent velocity on a part of the boundary and Tresca's friction law on the other part. From the latter condition, we obtain that the fluid velocity and pressure satisfy a non-linear parabolic variational inequality and belong to Hilbert spaces. We prove the existence of a solution by using Schauder's fixed point theorem, the notion of semigroup and monotony methods. Then, we conclude by applying De Rham's theorem to construct the pressure term.

1 Introduction

Fluid flow problems are involved in several physical phenomena and play an important role in many industrial applications. Motivated by applications to industrial processes like lubrication or extrusion/injection. We study non-stationary Stokes system with boundary conditions of friction type.

2 Main results

We decompose the boundary of Ω as $\partial\Omega = \Gamma_D \cup \Gamma_0$. We define

$$V_{0.div} = \{\varphi \in (H^1(\Omega))^3; \quad \varphi = 0 \quad \text{on} \quad \Gamma_D, \quad \varphi \cdot n = 0 \quad \text{on} \quad \Gamma_0 \quad \text{and} \quad div(\varphi) = 0 \quad \text{in} \quad \Omega\},$$

and

$$H = \{\psi \in (L^2(\Omega))^3; \quad \psi \cdot n = 0 \quad \text{on} \quad \partial\Omega \quad \text{and} \quad div(\psi) = 0 \quad \text{in} \quad \Omega\}.$$

Key Words and Phrases: *Tresca's friction law, Non-linear variational parabolic inequality, Monotonicity methods, Schauder's fixed point theorem.*

The variational formulation of the problem is given by

Problem (P): Find $\overline{v} \in C([0,T];H) \cap L^2(0,T;V_{0.div})$ with $\overline{v}' \in L^2(0,T;(V_{0.div})')$ satisfay

$$\begin{aligned}&[\overline{v}', \overline{\varphi} - \overline{v}] + [\mathcal{A}\overline{v}, \overline{\varphi} - \overline{v}] + J(\overline{\varphi}) - J(\overline{v}) \geq [\overline{f}, \overline{\varphi} - \overline{v}], \quad \forall \overline{\varphi} \in L^2(0,T;V_{0.div})\\ &\overline{v}(0) = 0 \quad \text{in} \quad \Omega,\end{aligned}$$

where $\overline{v} = v - v_0\xi$, $[.,.]$ denotes the duality product between $L^2(0,T;V_{0.div})$ and $L^2(0,T;(V_{0.div})')$,

$$J(\overline{\varphi}) = \int_0^T \int_{\Gamma_0} k|\overline{\varphi} - \tilde{s}|\, dx'\, dt,$$

and

$$[\mathcal{A}\overline{v}, \overline{\varphi}] = \int_0^T \int_\Omega 2\mu(\theta, \overline{v} + v_0\xi, |D(\overline{v} + v_0\xi)|) d_{ij}(\overline{v} + v_0\xi) d_{ij}(\overline{\varphi}) dx\, dt.$$

For all $\mathbf{u} \in L^2(0,T;(L^2(\Omega))^3)$, we consider the following problem

Problem ($\mathrm{P}_\mathbf{u}$): Find $\overline{v} \in C([0,T];H) \cap L^2(0,T;V_{0.div})$ with $\overline{v}' \in L^2(0,T;(V_{0.div})')$ satisfay

$$\begin{aligned}&[\overline{v}', \overline{\varphi} - \overline{v}] + [\mathcal{A}_\mathbf{u}\overline{v}, \overline{\varphi} - \overline{v}] + J(\overline{\varphi}) - J(\overline{v}) \geq [\overline{f}, \overline{\varphi} - \overline{v}], \quad \forall \overline{\varphi} \in L^2(0,T;V_{0.div}),\\ &\overline{v}(0) = 0 \quad \text{in} \quad \Omega,\end{aligned}$$

where

$$[\mathcal{A}_\mathbf{u}\overline{v}, \overline{\varphi}] = \int_0^T \int_\Omega 2\mu(\theta, \mathbf{u} + v_0\xi, |D(\overline{v} + v_0\xi)|) d_{ij}(\overline{v} + v_0\xi) d_{ij}(\overline{\varphi}) dx\, dt.$$

We prove the existence and uniqueness of a solution of problem ($\mathrm{P}_\mathbf{u}$) by using the notion of semigroup and monotony methods [3].
Finally, we prove the existence of a solution to Problem (P) by using Schauder's fixed point theorem and we use De Rham's theorem to establish the existence of the pressure.

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Existence and uniquness results for a micropolar fluid flow in a thin domain with boundary conditions of friction type

Fatima Ziane*

Mailing address: Department of mathematics, Ziane Achour University, Djelfa, Algeria
E-mail: f.ziane@univ-djelfa.dz

Mahdi Boukrouche and Laetitia Paoli

Mailing address: Institut Camille Jordan, Lyon University, UJM F-42023 Saint-Etienne, CNRS UMR 5208, Institut Camille Jordan, 23 rue Paul Michelon, 42023 Saint-Etienne Cedex 2, France
E-mail: mahdi.boukrouche@univ-st-etienne.fr, laetitia.paoli@univ-st-etienne.fr

Abstract
We consider a micropolar fluid flow in a two-dimensional domain. We assume that the velocity field satisfies a non-linear slip boundary condition of friction type on a part of the boundary while the micro-rotation field satisfiesnon-homogeneous Dirichlet boundary conditions. We prove the existence and uniqueness of a solution.

1 Introduction

Several industrial problems involve nowadays complex fluids like polymers, colloidal fluids, ferro-liquids or liquid crystals. Such fluids contain suspensions of rigid particles that undergo rotations and the classical Navier-Stokes theory is inadequate since it does not take into account the effects of the micro-structures. The micropolar fluid model has been introduced by A.C. Eringen in [1] in order to describe the macroscopic behaviour of such fluids under the assumptions that the particles are randomly oriented or spherical and the deformation of the particles is neglected. The unknowns are the fluid velocity $u = (u_1, u_2, u_3)$, the pressure p and the micro-rotation field $w = (w_1, w_2, w_3)$ which can be interpreted as the angular velocity field of the micro-particles. Then the equilibrium of momentum, of mass and moment of momentum lead to a system of coupled partial differential equations for the triplet (u, p, w). Motivated by lubrication problems we consider a flow in an infinite journal bearing. The cross section of the domain is thus given by the gap between two non-concentric discs which is much smaller that the discs radii. By assuming that the flow and the external excitation fields do not depend on the coordinate along the longitudinal axis of the bearing we obtain a 2D problem.

$$\frac{\partial u^\varepsilon}{\partial t} - (\nu + \nu_r)\Delta u^\varepsilon + (u^\varepsilon \cdot \nabla)u^\varepsilon + \nabla p^\varepsilon = 2\nu_r rot(w^\varepsilon) + f^\varepsilon \text{ in }]0, T[\times\Omega^\varepsilon,$$

$$div(u^\varepsilon) = 0 \text{ in }]0, T[\times\Omega^\varepsilon,$$

Key Words and Phrases: *Micropolar fluid, Tresca friction law, Galerkin method.*

$$\frac{\partial w^\varepsilon}{\partial t} - \alpha \Delta w^\varepsilon + (u^\varepsilon \cdot \nabla) w^\varepsilon + 4\nu_r w^\varepsilon = 2\nu_r rot(u^\varepsilon) + g^\varepsilon \text{ in }]0,T[\times \Omega^\varepsilon,$$

with the initial conditions

$$u^\varepsilon(0,.) = u_0^\varepsilon, \quad w^\varepsilon(0,.) = w_0^\varepsilon \text{ in } \Omega^\varepsilon.$$

We decompose $\partial\Omega^\varepsilon$ as $\Gamma_0 = z \in \partial\Omega^\varepsilon : z_2 = 0, \quad \Gamma_1^\varepsilon = z \in \partial\Omega^\varepsilon : z_2 = \varepsilon h^\varepsilon(z_1)$, and Γ_L^ε is the lateral part of the boundary. Due to the original geometry of the flow domain, we have $u^\varepsilon, w^\varepsilon, p^\varepsilon$ are L-periodic with respect to z_1. A first study for non-homogeneous Dirichlet conditions on Γ_0 and non-standard free boundary conditions on Γ_1^ε. Nevertheless, experimental studies have shown that non-linear slip boundary conditions of friction type are more realistic for such complex fluids. Hence we will consider in this presentation non-homogeneous Dirichlet boundary conditions on Γ_1^ε and Tresca friction boundary conditions for the fluid velocity on Γ_0.

2 Main results

Theorem 2.1. *Let* $(U_0, W_0, s_0) \in H^1(0,T)^3$, $f^\varepsilon \in (L^2((0,T)\Omega^\varepsilon))2$, $g^\varepsilon \in L^2((0,T)\Omega^\varepsilon)$, $k^\varepsilon \in L^\infty(0,T; L_+^\infty(\Gamma^0))$ *and* $(v_0^\varepsilon, Z_0^\varepsilon) \in H^\varepsilon \times H^{0,\varepsilon}$. *Then our problem admits an unique week solution.*

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On stability result for a Kirchhoff type reaction-diffusion equation

Aya Khaldi*

Department of mathematics, Laboratory of Applied Mathematics and History and Didactics of Mathematics(LAMAHIS), University of 20 august 1955, Skikda, Algeria
E-mail: a.khaldi@univ-skikda.dz

Mesaoud Maouni and Amar Ouaoua

Department of mathematics, Laboratory of Applied Mathematics and History and Didactics of Mathematics(LAMAHIS), University of 20 august 1955, Skikda, Algeria
E-mail: m.maouni@univ-skikda.dz and a.ouaoua@univ-skikda.dz

Abstract
We consider a class of Kirchhoff type reaction-diffusion equations with source terms. Under suitable assumptions on the initial data we prove the stability solution with positive initial energy. The stability being based on the Komornik's integral inequality.

1 Introduction

We consider the following value problem

$$\begin{cases} u_t - M\left(\int_\Omega |\nabla u|^2 dx\right)\Delta u + |u|^{m-2} u_t = |u|^{r-2} u, & (x,t) \in \Omega \times (0,T), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n$, $n \geq 1$ with smooth boundary $\partial\Omega$ and $M(s) = a + bs^\gamma$ with positive parameters a, b, γ, the constant components $r, m > 2$.
In the recent years, much effort has been devoted to nonlocal problems because of their wide applications in both physics and biology. For example, the hyperbolic equation with a nonlocal coefficient is

$$\varepsilon u_{tt}^\varepsilon + u_t^\varepsilon - M\left(\int_\Omega |\nabla u^\varepsilon|^p dx\right)\Delta_p u^\varepsilon = f(x,t,u^\varepsilon), \tag{1.2}$$

where $M(s) = a + bs$, $a > 0$, $b > 0$ and $p > 1$. In a bounded domain $\Omega \subset \mathbb{R}^n$ it is a potential model for damped small transversal vibrations of an elastics string with uniform density ε. For $p = 2$, such nonlocal equations were first proposed by Kirchhoff in 1883. In the case $\varepsilon = 0$, the equation (1.2) becomes a Kirchhoff type parabolic equation

$$u_t - M\left(\int_\Omega |\nabla u|^p dx\right)\Delta_p u = f(x,t,u). \tag{1.3}$$

Key Words and Phrases: *Kirchhoff equation, reaction-diffusion equation,positive initial energy, Komornik's integral inequality.*

Equation (1.3) can also be used to describe the motion of a nonstationary fluid or gas in a nonhomogeneous and anisotropic medium.
Ouaoua and Maouni, in [1] considered the following nonlinear parabolic equation with $p(x)$ −Laplacian

$$u_t - div\left(|\nabla u|^{p(x)-2}\nabla u\right) + \omega |u|^{m(x)-2} u_t = b\,|u|^{r(x)-2} u, \tag{1.4}$$

they proved a finite blow up result for the solutions in the case $\omega = 0$, and exponential growth in the case $\omega > 0$, with negative initial energy. Many authors have studied the existence and nonexistence of solutions for the problem with variable exponents or constants.

2 Main results

Lemma 2.1. *Assume that u be a solution of* (1.1) *. Then, we have*

$$E'(t) = -\|u_t\|_2^2 - \int_\Omega |u(t)|^{m-2}|u_t(t)|^2\,dx \le 0, \ \ t\in[0,T] \tag{2.1}$$

and

$$E(t) \le E(0). \tag{2.2}$$

Theorem 2.1. *Let the assumptions of Lemma* 2.1 *hold. Then, there exists constants* $C > 0$, *such that*

$$E(t) \le E(0)\left(\frac{C+qt}{C+qC}\right)^{-\frac{1}{q}}, \ \textit{for all } t \ge C. \tag{2.3}$$

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Numerical Treatment Based on Finite Differences and Galerkin Method for the One-Dimensional Wave Equation

LAOUAR Zineb*
Laboratoire des Mathématiques Appliquées et Didactique, Ecole Normale Supérieure, Constantine, Algeria.
Centre Universitaire Abdelhafid Boussouf, Mila, Algeria.
E-mail: z.laouar@centre-univ-mila.dz

ARAR Nouria
Département des Mathématiques, Université des Frères Mentouri, Constantine, Algeria.
E-mail: arar.nouria@umc.edu.dz

Abstract
This work aims to provide an efficient numerical method to solve one-dimensional wave equation. The proposed technique is based on a combination of finite differences method for the time derivative and a Galerkin method using a basis composed of Legendre polynomials for spatial discritization. This method turns the studied problem into a simple system of equations that can be easily solved. Numerical simulation is provided via some examples to validate the effenciency and reliability of the proposed algorithm.

1 Introduction

Many real world phenomena present a wave propagation behaviour such as waves on the surface of sea, sound waves, electromagnetic waves, etc. This behaviour is characterised by the well-known wave equation related to Jean Le Rond d'Alembert (1747).
We consider in this study a problem of wave equation in one dimension

$$\begin{aligned}
&\frac{\partial^2 u}{\partial t^2}(x,t) - \alpha \frac{\partial^2 u}{\partial x^2}(x,t) = 0, \quad (x,t) \in [a,b] \times [0,T], \alpha > 0 \\
&u(a,t) = u(b,t) = 0, \quad 0 < t < T \\
&u(x,0) = f(x), \quad a < x < b \\
&\frac{\partial u}{\partial t}(x,0) = g(x), \quad a < x < b.
\end{aligned} \tag{1.1}$$

In those days, numerical methods gain a lot of interest from researchers because of its effectiveness and reliability. Since the majority of equations that describe real phenomena don't have an analytical solution, the seek of suitable algorithms that approximate this solution becomes essential. Spectral methods are known for there fast convergence rate. They are characterized by using orthogonal polynomials or combination of orthogonal polynomials as basis functions to express the approximate solution which allows us to take advantage from their important properties to acheive accuracy with small data. A combaination of Galerkin method with a 2-centred finite differences scheme form the current study.

Key Words and Phrases: *One-dimensional wave equation, Spectral methods, Legendre polynomials, Finite differences scheme.*

2 Main results

Temporal discretization: We start first by subdividing the time interval $[0,T]$ into q subintervals using a step $\Delta t = t_j - t_{j-1}$ for $j = 0,\ldots,q$ with $t_0 = 0$, and using a centred finite differences scheme of order 2 to approximate $\frac{\partial^2 u}{\partial t^2}$ we obtain for $j = 1,\ldots,q$

$$\begin{aligned}&\frac{1}{\Delta t^2}U_{j+1} = \frac{2}{\Delta t^2}U_j - \frac{1}{\Delta t^2}U_{j-1} + \alpha^2\frac{\mathrm{d}^2U_{j+1}}{\mathrm{d}x^2}, \quad x \in [a,b];\\ &U_0(x) = f(x);\ U_1(x) = f(x) + \Delta t.g(x) + \frac{\Delta t^2}{2}\alpha^2 f''(x).\end{aligned} \tag{2.1}$$

Spatial discretization: We start by writing the weak formulation of (2.1), then we express the approximate solution as a finite serie of a special spectral basis $\{\varphi_i(x)\}$ composed of Legendre polynomials that verifies the boundary conditions

$$U^N_{j+1}(x) = \sum_{i=0}^{N-2} u_i^{j+1}\varphi_i(x).$$

By subtituting the approximation in the weak formulation, we obtain the following matrix system that we solve by recurrence on j where $u^{j+1} = (u_0^{j+1},\ldots,u_{N-2}^{j+1})^{\mathrm{T}}$ is the unkown vector

$$\left(\frac{1}{\Delta t^2}\mathbf{A} + \alpha^2\mathbf{B}\right)u^{j+1} = \frac{1}{\Delta t^2}\left(2C^j - D^{j-1}\right), \tag{2.2}$$

$$\begin{aligned}\text{such that}\quad &\mathbf{A}_{ki} = \int_a^b \varphi_k(x)\varphi_i(x)\mathrm{d}x,\ \mathbf{B}_{ki} = \int_a^b \varphi_k'(x)\varphi_i'(x)\mathrm{d}x.\\ &C^j = (C_0^j, C_1^j,\ldots,C_{N-2}^j),\ C_k^j = \int_a^b \varphi_k(x)U_j(x)\mathrm{d}x,\\ &D^{j-1} = (D_0^{j-1}, D_1^{j-1},\ldots,D_{N-2}^{j-1}),\ D_k^{j-1} = \int_a^b \varphi_k(x)U_{j-1}(x)\mathrm{d}x,\end{aligned} \tag{2.3}$$

Numerical simulation: $[a,b] = [-1,1],\ T = 2,\ \alpha = 1,\ f(x) = 0,\ g(x) = \sin(\frac{3\pi}{2}(x+1))$ and the analytical solution is given by $u(x,t) = \frac{2}{3\pi}\sin(\frac{3\pi}{2}(x+1))\sin(\frac{3\pi}{2}t)$.

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STUDY OF THE INVERSE PROBLEM FOR A LINEAR PARABOLIC EQUATION UNDER INTEGRAL OVER DETERMINATION CONDITION AND NEWMAN-TYPE BOUNDARY CONDITION

Benguesmia Amal*

Mailing address: Departement of Mathematics and Informatics, Oum El Bouaghi University, Oum El Bouaghi, Algeria
E-mail: benguesmia.amal@univ-oeb.dz

Oussaeif Taki-Eddine

Mailing address:Departement of Mathematics and Informatics ,Oum El Bouaghi University , Oum El Bouaghi , Algeria
E-mail: taki_maths@live.fr; takieddine.oussaeif@gmail.com

Abstract
the inverse problem of a linear parabolic equation with integral overdetermination as a supplementary condition is investigated in this study.
For the solvability of direct problem we apply the energy inequality method and for the inverse problem we employ the fixed point technique..

1 Introduction

Inverse boundary value problem arise in various areas of human activity such as mineral exploration, biology, medicine, etc. Inverse problems for parabolic equation satisfying nonlocal integral overdetermination condition were first investigated in [9, 14, 18, 22, 28] and [21, 24, 29, 34] for equations with coefficient independent of time and boundary condition of the first and third kind. In this paper we investigate the one -of-Kind solvability of the inverse problem of determining a pair of function $\{u, f\}$ satisfying the equation

$$u_t - \partial_x(a(x,t)u_x) + bu = f(t)h(x,t) \quad (x,t) \in \Omega \times (0,T) \tag{1.1}$$

with the initial condition

$$u(x,0) = \varphi(x) \quad x \in \Omega \tag{1.2}$$

The boundary condition

$$u_x(0,t) = u_x(d,t) = 0 \quad (x,t) \in \partial\Omega \times (0,T) \tag{1.3}$$

Key Words and Phrases: *Inverse nonlinear problem, nonlocal integral condition, fixed point theorem, regularisation*

and the nonlocal overdetermination condition

$$\int_{\Omega} v(x)u(x,t)dx = E(t) \quad t \in (0,T) \tag{1.4}$$

Where Ω is a bounded domain of $\mathbb{R}^n$ with smoth boundary $\partial\Omega$. The functions h, φ, E are known functions.
Here supplementary or additional information about the solution to the inverse problem comes in the form of the integral condition (1.4).
Many authors have studied the theory of the existence and uniqueness of the investigation problem (see, for example [2]-[10],[1, 4, 11-13, 17] and other papers). Based on these previous works, and in order to further develop these theories and works. The present paper is devoted to study the existence and the uniqueness for the inverse problem with integrale condition of second type by reducing the problem to fixed point principle.

2 Main results

the study of the existence,uniqueness and continuous dependence of the solution upon the data of inverse problem

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Thermo-viscous elastic free boundary problem of fractional PDEs of nonlinear acoustics

Rabah Djemiat*

Laboratory of Pure and Applied Maths, Mohamed Boudiaf University of M'sila, Algeria.
E-mail: rabahdjemiat19@gmail.com

Abstract
In this paper, we examine the existence and uniqueness of solutions under the traveling wave forms for a free boundary Cauchy problem of space-fractional Jordan-Moore-Gibson-Thompson equations of nonlinear acoustics, which describe sound propagation in thermo-viscous elastic terms. It does so by applying the properties of Banach's fixed point theorem, while Caputo's fractional derivative is used as the differential operator.

1 Introduction and statement of results

Several mathematical models are used to describe nonlinear acoustics phenomena. For example, In this work, we shall give a fractional model of nonlinear acoustics that is named the space-fractional Jordan-Moore-Gibson-Thompson (JMGT) equation. This equation results from modeling high-frequency ultra sound waves, and is written as follows:

$$\begin{cases} \tau\psi_{ttt} + \mu\psi_{tt} - \kappa^2\partial_x^\alpha\psi - \eta\partial_x^\alpha\psi_t = \varphi\left(x,t,\psi,\psi_t,\psi_{tt},\psi_{xx},(\psi_t)_{xx}\right), & (x,t)\in\Omega, \\ \psi(x,0) = \psi_0(x),\ \psi_t(x,0) = \psi_1(x),\ \psi_{tt}(x,0) = \psi_2(x), & \psi_0,\psi_1,\psi_2\in\mathbb{C}, \\ \psi_x(\kappa t,t) = 0,\ \psi_{xx}(\kappa t,t) = 0, & \kappa>0, \end{cases} \tag{1.1}$$

where $\tau,\mu,\kappa,\eta\in\mathbb{R}_+^*$ and $\varphi:\Omega\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\to\mathbb{C}$ is a nonlinear function.
The major goal of this work is to determine the existence and uniqueness of the fractional-order's partial differential equation (1.1), under the traveling wave form

$$\psi(x,t) = \exp\left(-\frac{\kappa^2}{\eta}t\right)u(x-\kappa t),\ \text{with } \kappa,\eta\in\mathbb{R}_+^*. \tag{1.2}$$

The basic profile u is not known in advance and is to be identified.
We represent the role of Free Boundary Problems in the real world as a significant source of new ideas in modern analysis. With the help of a model problem, we illustrate the use of analytical techniques to obtain the existence and uniqueness of weak solutions via the use of the traveling wave method. This method permits us to reduce the fractional-order's PDE (1.1) to a fractional differential equation; the idea is well illustrated with examples in our paper. This approach (1.2) is promising and can also bring new results for other applications in fractional-order's PDEs.
For the forthcoming analysis, we impose the following assumptions:

Key Words and Phrases: *Traveling wave; nonlinear acoustics; fractional order; existence; uniqueness.*

$(A1)$ φ is a continuous function that is invariant by the change of scale (1.2). It gives us

$$\varphi\left(x,t,\psi,\psi_t,\psi_{tt},\psi_{xx},(\psi_t)_{xx}\right)=\exp\left(-\frac{\kappa^2}{\eta}t\right)\left(\eta\kappa f\left(\xi,u\left(\xi\right),u'\left(\xi\right),u''\left(\xi\right)\right)-\kappa^3\tau u'''\left(\xi\right)\right), \quad (1.3)$$

where $\xi=x-\kappa t$ and $f:[0,\ell]\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\to\mathbb{C}$ is a continuous function.
$(A2)$ There exist three positive constants $\beta,\gamma,\lambda>0$ so that the function f given by (1.3) satisfies

$$\left|f\left(\xi,u,v,w\right)-f\left(\xi,\bar{u},\bar{v},\bar{w}\right)\right|\leq\beta\left|u-\bar{u}\right|+\gamma\left|v-\bar{v}\right|+\lambda\left|w-\bar{w}\right|,\text{ for }\beta,\gamma,\lambda>0,$$

for any $u,v,w,\bar{u},\bar{v},\bar{w}\in\mathbb{C}$.
Now, we give the principal theorems of this work.

Theorem 1.1. *Assume that the assumptions $(A1)$, $(A2)$ hold. We give*

$$\tau\kappa^5+\beta\eta^4-\mu\eta\kappa^3\neq0 \text{ and } \omega\in(0,1)$$

If we put

$$\ell<\left(\frac{\eta^4\Gamma\left(\alpha+2\right)\left(1-\omega\right)}{\left|\tau\kappa^5+\beta\eta^4-\mu\eta\kappa^3\right|}\right)^{\frac{1}{\alpha+1}}, \quad (1.4)$$

then the Cauchy problem (1.1) admits a unique solution in the traveling wave form (1.2) on Ω.

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Modeling and Mathematical Analysis of a Degenerate Reaction-Diffusion Model Applied in Biology and Ecology

Samiha Djemai*
Department of Mathematics, Ferhat Abbas University, Setif, Algeria
E-mail: samiha.djemai@univ-setif.dz

Salim Mesbahi
Department of Mathematics, Ferhat Abbas University, Setif, Algeria
E-mail: salim.mesbahi@univ-setif.dz

Abstract
This paper deals with a degenerate reaction-diffusion system with coupled nonlinear localized sources subject to weighted nonlocal Dirichlet boundary conditions. We obtain the conditions for global and blow-up solutions. The model studied appears in the modeling of many diffusion phenomena in various sciences, particularly in biology and ecology.

1 Introduction

In this paper we study the following degenerate parabolic reaction-diffusion system with coupled nonlinear localized sources subject to weighted nonlocal Dirichlet boundary conditions

$$\begin{cases} u_t - \beta \Delta u^m = \phi(t,x)\, u^{p_1} v^{q_1} & , \ (x,t) \in \Omega \times (0,T) \\ v_t - \delta \Delta v^n = \psi(t,x)\, v^{p_2} u^{q_2} & , \ (x,t) \in \Omega \times (0,T) \\ u = \int_\Omega f(x,y)\, u(y,t)\, dy & , \ (x,t) \in \partial\Omega \times (0,T) \\ v = \int_\Omega g(x,y)\, v(y,t)\, dy & , \ (x,t) \in \partial\Omega \times (0,T) \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x), \quad x \in \Omega \end{cases} \tag{1.1}$$

where $\Omega \in \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $x_0 \in \Omega$ is a fixed point. $m, n > 1$, β, δ, q_1, q_2, r_1, $r_2 > 0$, p_1, $p_2 \geqslant 0$ which ensure that equations in (1.1) are completely coupled with nonlinear localized reaction terms, while the weight functions $f(x,y)$, $g(x,y)$ in the boundary conditions are continuous nonnegative on $\partial\Omega \times \Omega$ and $\int_\Omega f(x,y)\, dy$, $\int_\Omega g(x,y)\, dy > 0$ on $\partial\Omega$. The functions ϕ and ψ are positive. The initial values $u_0(x)$, $v_0(x) \in C^{2+\alpha}(\Omega) \cap C(\overline{\Omega})$ with $0 < \alpha < 1$ are nontrivial nonnegative and satisfy the compatibility conditions.

2 The main result

Theorem 2.1. *If $m > p_1$, $n > p_2$ and $q_1 q_2 < (m - p_1)(n - p_2)$, then the nonnegative solution of* (1.1) *is global.*

Key Words and Phrases: *degenerate reaction-diffusion system, nonlocal boundary conditions, blow-up, blow-up rate.*

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Existence of solution of Non-Periodic Snap BVP in the G-Caputo Sense

Dr.Berhail Amel

Department of Mathematics, University 08 May 1945, Guelma, Algeria
E-mail: berahailam@gmail.com

Abstract
In the present paper, we consider a nonlinear fractional snap model with respect to a G-Caputo derivative and subject to non-periodic boundary conditions. Some qualitative analysis of the solution, such as existence and uniqueness, are investigated in view of fixed-point theorems.

1 Introduction

The second derivative of the accelaration (fourth derivative of position) is a physical quantity called a snap or jounce. In fact, the terms jerk and snap are exceptionally rare for most individuals, counting physicists and engineers. Scientists jerk and snap are the third and fourth derivatives of our position with regard to time, respectively.
The corresponding fractional model is a chieved by using the fractional derivative(of order less than ore qual 1) in stead of the standard derivative. Many types of fractional derivatives can be used here, such as the RiemannLiouville, Caputo and Hadamard. We prefer to use the generalized fractional derivative with respect to differentiable increasing function G.

2 Main results

In this paper, we dened a new fractional mathematical model consisting of a fractional snap equation with non-periodic boundary conditions in the framework of the generalized G-operators. Thus, some investigations on the qualitative behaviors of its solutions, including existence and uniqueness. To obtain the uniqueness of the solution, we used Banach contraction theorem, and for the general existence of at least one solution, we used the Shauder fixed-point theorem.

Key Words and Phrases: *snap problem, G-Caputo fractional differential equation, boundary value problem,Ulam HyersRassias stability*

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Steklov–Poincaré Operator for A System of Coupled Abstract Cauchy Problems

Bachir Alliti

Mailing address: Department Mathematics , Houari Boumediene University of Science and Technology, Bab el oued, Algeria
E-mail: bachir.alliti.math@gmail.com

Toufik Laadj

Mailing address: Department Mathematics , Houari Boumediene University of Science and Technology, Algeria
E-mail: laadjt@vt.edu

Abstract
We are interested in a coupled system of two abstract Cauchy problems; one of them is posed in small time interval. The aim is to obtain a condition that replaces the effect of this small interval. This condition is constructed from an approximation of Steklov–Poincaré operator. In this Presentation , we analyze the construction and the approximation of this operator. Finally, the results obtained will be applied to a problem of diffraction of an electromagnetic wave by a perfectly conducting planar obstacle coated with thin layer of dielectric material.

1 Introduction

The study of evolution problems is of major interest in many research areas such as propa- gation of waves and flow of fluids. In this Presentation , we are particularly interested in abstract Cauchy problems where the model shows variability during time evolution. We shall refer to these models as coupled abstract Cauchy problems, which consist in solving a system of two equations set in two non- overlapped time intervals. Solving numerically these equations is challenging since it requires discretizing on the scale of the small interval. The mesh then contains a very large number of elements, which makes the calculations long and sometimes imprecise . For this reason, we try to replace our problem by another problem that does not bring in any more the small interval.

Key Words and Phrases: *Abstract Cauchy problem Steklov-Poincaré operator Electromagnetic scattering*

2 Main results

we start with presenting briefly our problem which consists of a coupled system of two abstract Cauchy problems; one of them is posed in small time interval.
Let E be a Banach space, and let $A, B, F, G, A_0, B_0, F_0, G_0$ be densely defined linear operators in E. Let $\mathtt{T}$ be in the interval $(0, \infty]$, and let δ be a small nonnegative real number. We consider the following system of coupled linear evolution equations.

$$\left.\begin{array}{ll}\frac{d}{dt}v_1(t) = A_0 v_1(t) + G_0 v_2(t) & \text{in } \mathcal{C}((-\delta, 0); E), \\ \frac{d}{dt}v_2(t) = F_0 v_1(t) + B_0 v_2(t) & \text{in } \mathcal{C}((-\delta, 0); E), \\ \alpha v_1(-\delta) + \beta v_2(-\delta) = g, & \\ u_1(0) = v_1(0), \quad u_2(0) = v_2(0), & \\ \frac{d}{dt}u_1(t) = A u_1(t) + G u_2(t) & \text{in } \mathcal{C}((0, \mathtt{T}); E), \\ \frac{d}{dt}u_2(t) = F u_1(t) + B u_2(t) & \text{in } \mathcal{C}((0, \mathtt{T}); E), \\ + \text{ condition at } t = \mathtt{T}, & \end{array}\right\} \tag{2.1}$$

with α, β and g are given.

Then we reformulate our problem using Steklov–Poincaré operator, after that, we determine the exact formula of this operator. The third section is devoted to constructing approximations of the SP operator using two approaches: the first one consists of writing Taylor expansions in the small time interval and the second approach concerns the asymptotic analysis of the problem with respect to the length of this small interval.
In the last section we apply the results obtained in the third section to a problem of diffraction of an electromagnetic wave by a perfectly conducting planar obstacle coated with thin layer of dielectric material.

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Well-posedness and general energy decay of solutions for a piezoelectric beams system with magnetic effects and a nonlinear damping term

Hassan Messaoudi *

Laboratory of Informatics and Mathematics, Department of Mathematics and Informatics, University of Souk-Ahras, P.O. Box 1553, Souk Ahras 41000, Algeria

E-mail: hassanmessaoudi50conf@gmail.com

Abstract

In this work, we consider the one-dimensional system of piezoelectric beams with magnetic effects in the presence of a nonlinear damping term acting on the mechanical equation. Under suitable assumptions on the nonlinear damping term, we prove the existence and uniqueness of the solution using the semigroup theory more precisely by Hille-Yosida theorem. And by introducing a suitable Lyapunov functional and using some properties of convex functions, we show the general stability of the solution of the system whose exponential and polynomial decays are only special cases. Furthermore, our results are independent of any relationship between system parameters.

1 Introduction

Piezoelectricity is the property that certain bodies have of being electrically polarized under the action of mechanical stress. In 1880, piezoelectricity was discovered by the brothers Pierre and Jacques Curie. These have shown that certain crystals (such as: quartz, lithium niobate, sugar cane, barium titanate and Rochelle salt) generate an electrical polarization under mechanical stress, that is to say produce an electrical charge under mechanical stress (the direct piezoelectric effect). The reversible phenomenon (the reverse piezoelectric effect), was theoretically stated by Gabriel Lippmann in 1881 and verified experimentally by the Curie brothers in the same year. Piezoelectric materials have been widely used in actuators or sensors because of their ability to convert electrical energy into mechanical energy and vice versa. These so-called intelligent materials can be used in various applications such as injection mechanisms, piezoelectric motors, sonars. In [5], Morris and Özer proposed a variational approach to construct a coupled model of piezoelectric beams with magnetic effect given by

$$\begin{cases} \rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} = 0, \text{ in } (0,L)\times(0,\infty), \\ \mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, \text{ in } (0,L)\times(0,\infty), \end{cases} \tag{1.1}$$

where α, ρ, μ, γ, β and L are positive constants represent, respectively, elastic stiffness, the mass density, magnetic permeability, piezoelectric coefficient, water resistance coefficient of the beam and the length of the beam. Moreover, we consider the relationship

$$\alpha = \alpha_1 + \gamma^2\beta \quad \text{with} \quad \alpha_1 > 0, \tag{1.2}$$

Key Words and Phrases: *Piezoelectric beams, Nonlinear damping, Semigroup approach, General decay*

and the system (1.1) is equipped by the following boundary and initial conditions

$$\begin{cases} v(0,t) = p(0,t) = \alpha v_x(L,t) - \gamma\beta p_x(L,t) = 0, \\ \beta p_x(L,t) - \gamma\beta v_x(L,t) = -V(t)/h, \\ v(x,0) = v_0(x), \ v_t(x,0) = v_1(x), \ p(x,0) = p_0(x), \ p_t(x,0) = p_1(x), \end{cases} \tag{1.3}$$

where h, $V(t)$ represent, respectively, thickness of the beam, voltage applied at the electrode. Motivated by the above work, in this paper we consider the following problem

$$\begin{cases} \rho v_{tt} - \alpha v_{xx} + \gamma\beta p_{xx} + \chi(t) f(v_t) = 0, \text{ in } (0,L) \times (0,\infty), \\ \mu p_{tt} - \beta p_{xx} + \gamma\beta v_{xx} = 0, \text{ in } (0,L) \times (0,\infty), \\ v(0,t) = v_x(L,t) = p(0,t) = p_x(L,t) = 0, \ t \in (0,\infty), \\ v(x,0) = v_0(x), \ v_t(x,0) = v_1(x), \ p(x,0) = p_0(x), \ x \in (0, L), \\ p_t(x,0) = p_1(x), \ x \in (0, L), \end{cases} \tag{1.4}$$

where the functions v and p represent respectively, the longitudinal displacement of the center line, the total load of the electric displacement along the transverse direction at each point x. Other problems related to systems with nonlinear term [1, 2, 3, 4].

2 Main results

The main result of this work is to study the asymptotic behavior of a piezoelectric beam system with a nonlinear damping term. First, by using the semi-group technique, we show the existence and uniqueness of the solution. Also, by using some properties of convex functions and Lyaponov functionals, we obtain general stability estimates.

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Genral decay of solution for a nonlinear wave equation with fractional damping

Radhouane Aounallah*

Mailing address: Laboratory Analysis and Control of PDEs, Djillali Liabes University, P.O.BOX 89 Sidi Bel Abbes 22000, Algeria.
E-mail: radhouaneaounallah@gmail.com

Abstract
The paper deals with the study of global existence of solutions and the general decay in a bounded domain for nonlinear wave equation with fractional derivative boundary condition by using the Lyaponov functional.

1 Introduction

Let Ω is a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$ of class C^2 and we assume that $\partial\Omega = \Gamma_0 \cup \Gamma_1$, where Γ_0 and Γ_1 are closed subsets of $\partial\Omega$ with $\Gamma_0 \cap \Gamma_1 = \phi$ The system is given by :

$$(P)\begin{cases} y_{tt} - \Delta y(t) + au_t = |y|^{p-2}y, & \text{in } \Omega \times (0,\infty), \\ \frac{\partial u}{\partial \nu} = b\partial_t^{\alpha,\beta} y(t) & \text{on } \partial\Gamma_0 \times (0,\infty), \\ y = 0, & \text{on } \partial\Gamma_1 \times (0,\infty), \\ y(x,0) = y_0(x), \quad y_t(x,0) = y_1(x), & \text{in } \Omega. \end{cases}$$

where $a, b > 2$,$p > 2$ and $\partial\nu$stands for the unit outward normal to $\partial\Omega$. The notation $\partial_t^{\alpha,\beta}$ stands for the generalized Caputo's fractional derivative (see [1]) defined by the following formula:

$$\partial_t^{\alpha,\beta} u(t) := \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} e^{-\beta(t-s)} u_s(s) ds, \quad 0 < \alpha < 1, \beta \geq 0.$$

Then, by using an augmented system, problem (P) takes the form :

$$(P')\begin{cases} y_{tt} - \Delta y(t) + au_t = |y|^{p-2}y, & x \in \Omega,\ \mathrm{t} > 0, \\ \partial_t \phi(\xi,t) + (\xi^2+\beta)\phi(\xi,t) - y_t(x,t)\eta(\xi) = 0, & \xi \in \mathbb{R},\ \mathrm{t} > 0, \beta \geq 0, \\ \frac{\partial u}{\partial \nu} = -b\int_{-\infty}^{+\infty} \phi(\xi,t)\eta(\xi)d\xi, & x \in \Gamma_0,\ \mathrm{t} > 0 \\ y = 0, & x \in \Gamma_1,\ \mathrm{t}, \\ y(x,0) = y_0(x), \quad y_t(x,0) = y_1(x), & x \in \Omega, \\ \phi(\xi,0) = 0. & x \in \Omega, \xi \in \mathbb{R}. \end{cases}$$

Key Words and Phrases: *wave equation, Lyaponov functional,fractional derivative boundary, Semigroup theory.*

2 Main results

The main results of this work is the following:

Theorem 2.1. *Suppose that* $2 < p < \dfrac{2(n-1)}{n-2}$, *if* $n \geq 3$.
Then for any

$$U_0 \in \mathcal{H}_{\Gamma_0}$$

the problem (P') has a unique local solution

$$U \in C\left([0,T), \mathcal{H}_{\Gamma_0}\right).$$

Theorem 2.2. *Suppose that* $2 < p < \dfrac{2(n-1)}{n-2}$, *if* $n \geq 3$.
Then ther exist positive constants k *and* K*such that the global soulution of problem (P') satisfies*

$$E(t) \leq Ke^{-kt}$$

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ASYMPTOTIC STABILITY OF POROUS ELASTIC SYSTEM WITH FRACTIONAL DAMPING

Abderrahmane KASMI

LMA, Hassiba Benbouali University of Chlef, Algeria
E-mail: a.kasmi@univ-chlef.dz

Abstract
We study the asymptotic stability of a porous-elastic system with **only one** fractional derivative damping acting in the system. We prove the strong stability of the system using the semigroup theory of linear operators and a result obtained by Arendt-Batty.

1 Introduction

In this work, we investigate the existence and decay properties of solutions for the initial boundary value problem of the **porous-elastic system** of the type

$$(P) \quad \begin{cases} \rho u_{tt}(x,t) - \mu u_{xx}(x,t) - b\phi_x(x,t) = 0 & \text{in } (0,L)\times\mathbb{R}^+ \\ J\phi_{tt}(x,t) - \delta\phi_{xx}(x,t) + bu_x(x,t) + a\phi(x,t) + \tau\partial_t^{\alpha,\eta}\phi(x,t) = 0 & \text{in } (0,L)\times\mathbb{R}^+ \end{cases}$$

where the functions u and ϕ represent, respectively, the displacement of the solid elastic material and the volume fraction. In addition, the ρ, μ, b, J, δ, a and τ are (strictly) positive constants characterizing the physical properties of porous-elastic system (P), where μ and a satisfy $\mu a > b^2$. The notation $\partial_t^{\alpha,\eta}$ stands for the generalized Caputo's fractional derivative of order α with respect to the time variable. It is defined as follows

$$\partial_t^{\alpha,\eta} w(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha} e^{-\eta(t-s)} \frac{dw}{ds}(s)\, ds, \ 0<\alpha<1, \ \eta \geq 0.$$

with $w \in W^{1,1}(0,t)$ and $t>0$.
The original porous-elastic system is given by the following equations (see **[2]** and **[3]**) :

$$(1) \qquad \begin{cases} \rho u_{tt} \ = \ T_x, \\ J\phi_{tt} \ = \ H_x + G. \end{cases}$$

where T is the stress, H is the equilibrated stress, and G is the equilibrated body force. The constitutive equations are:

$$(2) \qquad T = \mu u_x + b\phi, \ H = \delta\phi_x \ \text{ and } \ G = -bu_x - a\phi - \tau\partial_t^{\alpha,\eta}\phi.$$

Key Words and Phrases: *Porous-elastic system, Fractional derivative damping, Asymptotic stability.*

Then substituting (2) into (1), we obtain the porous elastic system (P).
We note that when $\mu = a = b$, the porous system reduces to well-known Timoshenko system. The system is completed with the following homogeneous boundary conditions

$$u(x,t) = \phi_x(x,t) = 0, \quad x = 0, L.$$

and the following initial conditions

$$\begin{cases} u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), \\ \phi(x,0) = \phi_0(x),\ \phi_t(x,0) = \phi_1(x),\ x \in (0,L). \end{cases}$$

where the initial data $(u_0, u_1, \phi_0, \phi_1)$ belong to a suitable Sobolev space.
In **[5]** the author considered the following porous elastic system with frictional damping in the second equation

$$\begin{cases} \rho u_{tt}(x,t) - \mu u_{xx}(x,t) - b\phi_x(x,t) = 0 & \text{in } (0,L) \times \mathbb{R}^+ \\ J\phi_{tt}(x,t) - \delta\phi_{xx}(x,t) + bu_x(x,t) + a\phi(x,t) + \tau\phi_t(x,t) = 0 & \text{in } (0,L) \times \mathbb{R}^+ \end{cases}$$

and he established the slow decay of the solutions.

2 Main results

Our main results are the global existence and strong stability of solutions, when $\eta \geq 0$. The existence of solutions of the porous elastic system (P) can be obtained by means of semi-group theory. The second main result of this work is the following theorem.

Theorem 2.1. *Assume $\eta \geq 0$. The C_0-semigroup $(e^{t\mathcal{A}})_{t\geq 0}$ associated to the system (P) is strongly stable.*

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DECAY RATES FOR THE ENERGY OF A SINGULAR NONLOCAL VISCOELASTIC SYSTEM

Draifia Ala Eddine*

Laboratory of mathematics, Informatics and Systems (LAMIS), Larbi Tebessi University, 12002 Tebessa, Algeria
E-mail: draifia1991@gmail.com

Abstract
This work deals with decay rates for the energy of an initial boundary value problem with a nonlocal boundary condition for a system of nonlinear singular viscoelastic equations. We prove the decay rates for the energy of a singular one-dimensional viscoelastic system with a nonlinear source term and nonlocal boundary condition of relaxation kernels described by the inequality $g_i'(t) \leq -H(g_i(t))$, $(i = 1, 2)$ for all $t \geq 0$, with H convex.

1 Introduction

In this paper, we study the decay rates for the energy of the following system

$$\begin{cases} u_{tt} - \dfrac{1}{x}(xu_x)_x + \int_0^t g_1(t-s)\dfrac{1}{x}(xu_x(x,s))_x\,ds = |v|^{q+1}|u|^{p-1}u, \text{ in } Q, \\ \\ v_{tt} - \dfrac{1}{x}(xv_x)_x + \int_0^t g_2(t-s)\dfrac{1}{x}(xv_x(x,s))_x\,ds = |u|^{p+1}|v|^{q-1}v, \text{ in } Q, \end{cases} \tag{1.1}$$

with initial data

$$\begin{cases} u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in (0,\alpha), \\ \\ v(x,0) = v_0(x), \quad v_t(x,0) = v_1(x), \quad x \in (0,\alpha), \end{cases} \tag{1.2}$$

and nonlocal boundary condition

$$u(\alpha,t) = v(\alpha,t) = 0, \ \int_0^\alpha xu(x,t)dx = \int_0^\alpha xv(x,t)dx = 0, \tag{1.3}$$

where $Q := (0,\alpha) \times (0,T)$, $\alpha < \infty$, $T < \infty$, $p, q > 1$. It is assumed that the kernels g_1 and g_2 satisfy certain conditions to be specified later, and $u_0(x)$, $v_0(x)$, $u_1(x)$ and $v_1(x)$ are given functions.

Key Words and Phrases: *Exponential decay, Polynomial decay, viscoelastic damping, viscoelastic damping, intrinsic decay rates.*

2 Decay of Solutions

The functions g_1 and g_2 are $C^1(\mathbb{R}_+,\mathbb{R}_+)$; they satisfy the following assumptions
$(\mathbf{A1})$ $g_1(0)>0,\ g_2(0)>0$ and

$$1-\int_0^\infty g_i(s)ds=l_i>0,\quad (i=1,2)\,. \tag{2.1}$$

$(\mathbf{A2})$ $g_i'(t)\le -H(g_i(t)),\ (i=1,2)$ for all $t\ge 0$, where $H\in C^1(\mathbb{R}_+)$ with $H(0)=0$ is a given strictly increasing and convex function. Moreover

$$H\in C^2(0,\infty)\ \text{ and }\ \lim\inf_{x\to 0^+}\left\{x^2H''(x)-xH'(x)+H(x)\right\}\ge 0. \tag{2.2}$$

We define the corresponding energy functional by

$$\begin{aligned}
E(t)\ :\ =&\left(\frac{p+1}{2}\right)\|u_t\|^2_{L^2_\rho(0,\alpha)}+\left(\frac{q+1}{2}\right)\|v_t\|^2_{L^2_\rho(0,\alpha)}\\
&+\left(\frac{p+1}{2}\right)\left(1-\int_0^t g_1(s)ds\right)\|u_x\|^2_{L^2_\rho(0,\alpha)}\\
&+\left(\frac{q+1}{2}\right)\left(1-\int_0^t g_2(s)ds\right)\|v_x\|^2_{L^2_\rho(0,\alpha)}-\int_0^\alpha x\,|u|^{p+1}\,|v|^{q+1}\,dx\\
&+\left(\frac{p+1}{2}\right)\int_0^\alpha (g_1\circ u_x)(t)\,dx+\left(\frac{q+1}{2}\right)\int_0^\alpha (g_2\circ v_x)(t)\,dx.
\end{aligned} \tag{2.3}$$

Lemma 2.1. *Let us assume that* $(\mathbf{A1})-(\mathbf{A2})$ *holds. Then, there exists a positive constant* $T_0>0$ *such that*

$$E((n+1)T)+\tilde{H}\left(C_{15}^{-1}E((n+1)T)\right)\le E(nT)\,,\quad n=1,2,3... \tag{2.4}$$

for all $T>T_0$ *and all* $n\in N$.

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Some pseudo-differential Operators on generalized Herz-type Triebel-Lizorkin Spaces

Aissa Djeriou

Mailing address: Department of Mathematics,
University of M'Sila, M'Sila, Algeria
E-mail: aissa.djeriou@univ-msila.dz

Abstract
In this work, we study the continuity of pseudo-differential operators on Herz-type Triebel-Lizorkin spaces $K_q^{\alpha,p}F_\beta^{v_\mu}(\mathbb{R}^n)$, under some parameters $\beta, p, q,\ \alpha$ and $v:\ [0,\infty) \to]0,\infty)$ be a function such that the inequality $v(ts) \geq c\, t^{-\mu} v(s)$ holds for $0 < t, s \leq 1$ and some real μ.

1 Introduction

In this work, we will be interested by the generalized Herz-type Triebel-Lizorkin spaces $K_q^{\alpha,p}F_\beta^{v_\mu}(\mathbb{R}^n)$, this new class of function spaces is defined as the set of all tempered distributions f, such that

$$\|f\|_{K_q^{\alpha,p}F_\beta^{v_\mu}} = \left\| \Big(\sum_{j=0} \left(v_\mu(2^{-j}) \left| \mathcal{F}^{-1}\left(\phi_j \mathcal{F} f\right) \right| \right)^\beta \Big)^{1/\beta} \right\|_{K_q^{\alpha,p}} < +\infty,$$

where $_\mu \in \mathbb{R}$, $0 < \beta, p, q \leq \infty$ and $\alpha > -n/q$. It is well known that Herz spaces play an impotant role in Harmonic Analysis. After they have been introduced in [2], the theory of these spaces had a remarkable development in part due to its usefulness in applications. For any multi-indices α and γ, $m \in \mathbb{R}$, $0 \leq \delta, \rho \leq 1$, the symbol $a(x,\xi)$ belongs to Hörmander's class $S^m_{\rho,\delta}$ if

$$|\partial_\xi^\alpha \partial_x^\gamma a(x,\xi)| \leq C_{\alpha,\beta}(1+|\xi|)^{m+\rho|\alpha|-\delta|\gamma|}, \quad x,\xi \in \mathbb{R}^n.$$

The exploration of classes of smooth symbols, in particular, the classes $S^m_{\rho,\delta}$, appears to be predominan in the pseudo-differential operators (ps.d.o.) literature. However, as diverse problems in Analysis and PDEs demand, the case in which the symbol has mild or no regularity in x has received considerable attention, see, for instance, [3, 4]
We denote by $S^m_{1,\delta}(\omega, N)$ the collection of such a, such that the inequalities

$$\begin{aligned} |\, \partial_\xi^\alpha \partial_x^\gamma a(x,\xi)\, | &\leq c_1 \left(1+|\xi|\right)^{m-|\alpha|+\delta|\gamma|}, & (1.1) \\ |\, \partial_\xi^\alpha \partial_x^\gamma a(x+h,\xi) - \partial_\xi^\alpha \partial_x^\gamma a(x,\xi)\, | &\leq c_2\, \omega(|h||\xi|^\delta) \left(1+|\xi|\right)^{m-|\alpha|+\delta|\gamma|}, & (1.2) \end{aligned}$$

Key Words and Phrases: *Pseudo-differential operators, Herz Space, Herz-Type Triebel-Lizorkin Space*

where ω is a positive nondecreasing function, vanishing near the origin and concave on $\mathbb{R}^+$ (so-called a *modulus of continuity*), $\delta \in [0,1[$, $m \in \mathbb{R}$ and for any multi-indices α and γ with $|\gamma| \leq N$.
The main aim of this paper is to study on $K_q^{\alpha,p} F_\beta^s (\mathbb{R}^n)$ the boundedness of some ps.d.o. T_a which is defined by the formula

$$T_a(f)(x) = (2\pi)^{-n} \int_{\mathbb{R}^n} e^{\mathrm{i}x\cdot\xi} a(x,\xi) \widehat{f}(\xi)\, d\xi\,, \qquad (f \in \mathcal{S}(\mathbb{R}^n)\,, \quad x \in \mathbb{R}^n)\,.$$

where $a \in S^m_{1,\delta}(\omega, N)$.

2 Main results

Here, we present some results

Theorem 2.1. *Let $\mu > N$. Then every ps.d.o T_a of symbol $a \in S^m_{1,\delta}(\omega, N)$ is a bounded operator from $K_q^{\alpha,p} F_\beta^{v_\mu+m}(\mathbb{R}^n)$ to $K_q^{\alpha,p} F_\beta^{v_\mu}(\mathbb{R}^n)$.*

Theorem 2.2. *Suppose*

$$\Big(\sum_{j=1}^{\infty} \Big(2^{(\mu-N)j}\, \omega(2^{-(1-\delta)j}) \Big)^{\beta} \Big)^{1/\beta} = +\infty$$

Then there exist a ps.d.o T_a of symbol $a \in S^m_{1,\delta}(\omega, N)$ and a function $g \in K_q^{\alpha,p} F_\beta^{v_\mu+m}(\mathbb{R}^n)$ such that $T_a(g) \notin K_q^{\alpha,p} F_\beta^{v_\mu}(\mathbb{R}^n)$.

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On the Fourier operators in realized homogeneous Besov and Triebel-Lizorkin spaces

Mohamed Benallia*

Mailing address: Department of mathematics, École Normale Supérieure de BouSaâda, 28024 Djebel Messaâd-M'Sila, Algeria
E-mail: benalliam@yahoo.fr

Abstract
In this paper, via the decomposition of Littlewood-Paley and the notion of realizations, we present study some properties and embeddings on the Fourier operators for the realized homogeneous Besov spaces $\dot{\widetilde{B}}^s_{p,q}(\mathbb{R}^n)$ and the realized homogeneous Triebel-Lizorkin spaces $\dot{\widetilde{F}}^s_{p,q}(\mathbb{R}^n)$ into the Lebesgue spaces. The homogeneous Besov spaces $\dot{B}^s_{p,q}(\mathbb{R}^n)$ and the homogeneous Triebel-Lizorkin spaces $\dot{F}^s_{p,q}(\mathbb{R}^n)$ play an important role in certain studies, as the boundedness of pseudodifferential operators, the Navier-Stokes equations e.g. [1], etc.

1 Introduction

In [4, chap. 6] it has been studied the continous embeddings of the Fourier operators $\mathcal{F}$ (or $\mathcal{F}^{-1}$) on homogeneous spaces $\dot{B}^s_{p,q}(\mathbb{R}^n)$ and $\dot{F}^s_{p,q}(\mathbb{R}^n)$. So, as these spaces are defined by distributions modulo polynomials, since $\|f\|_{\dot{B}^s_{p,q}} = \|f\|_{\dot{F}^s_{p,q}} = 0$ if and only if, f is a polynomial on $\mathbb{R}^n$, then it will be interesting to show the acting of $\mathcal{F}$ on *realized* spaces of $\dot{B}^s_{p,q}(\mathbb{R}^n)$ denoting by $\dot{\widetilde{B}}^s_{p,q}(\mathbb{R}^n)$. This presents the result of the paper; similar "similarly" for *realized* of homogeneous Triebel-Lizorkin spaces $\dot{\widetilde{F}}^s_{p,q}(\mathbb{R}^n)$. We set $\dot{\widetilde{A}}^s_{p,q}(\mathbb{R}^n)$ for either $\dot{\widetilde{B}}^s_{p,q}(\mathbb{R}^n)$ or $\dot{\widetilde{F}}^s_{p,q}(\mathbb{R}^n)$, respectively.
So use first need the following notation: All function spaces occurring in this work are defined on Euclidean space $\mathbb{R}^n$, then we omit $\mathbb{R}^n$ in notations. As usual, $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{Z}$ the integers, and $\mathbb{R}$ the real numbers. We put $a_+ := \max(0, a)$, $\forall a \in \mathbb{R}^n$. The symbol $\hookrightarrow$ indicates a continuous embedding. For $0 < p \leq \infty$ we denote by $\|\cdot\|_p$ the quasi-norm of L_p. For $f \in L_1$, we denoted by $\widehat{f}(\xi) := \int_{\mathbb{R}^n} \mathrm{e}^{-\mathrm{i}x\cdot\xi} f(x)\mathrm{d}x$ the Fourier transform and by $\mathcal{F}^{-1}f(x) := (2\pi)^{-n}\widehat{f}(-x)$ the inverse Fourier transform. The operators $\mathcal{F}$ and $\mathcal{F}^{-1}$ are extended to the whole $\mathcal{S}'$ in the usual way. For all $j \in \mathbb{Z}$, we denote by $\Omega_j := \{\xi \in \mathbb{R}^n : 1/2 \cdot 2^j \leq |\xi| \leq 3/2 \cdot 2^j\}$. For $k \in \mathbb{N}_0 \cup \{\infty\}$, we denote by $\mathcal{P}_k$ the set of all polynomials on $\mathbb{R}^n$ of degree $< k$ (in particular $\mathcal{P}_0 = \{0\}$, $\mathcal{P}_1 = \{c\}$, ..., $\mathcal{P}_\infty$ the set of all polynomials on $\mathbb{R}^n$). $\mathcal{S}_k$ will be used for the set of all $\varphi \in \mathcal{S}$ such that $\langle u, \varphi\rangle = 0$ $(\forall u \in \mathcal{P}_k)$. The topological dual of $\mathcal{S}_k$ is $\mathcal{S}'_k$. If $f \in \mathcal{S}'$, then $[f]_{\mathcal{P}}$ denotes the equivalence class of f modulo $\mathcal{P}_\infty$. The mapping which takes any $[f]_{\mathcal{P}}$ to the restriction of f to $\mathcal{S}_k$ turns is an isomorphism from $\mathcal{S}'/\mathcal{P}_k$ onto $\mathcal{S}'_k$.

Key Words and Phrases: *Fourier operators, homogeneous Besov spaces, homogeneous Triebel-Lizorkin spaces, realizations.*

The basic definitions of $\dot{B}^s_{p,q}$ and $\dot{F}^s_{p,q}$ are given by the Littlewood-Paley decompositions, for this reason we fix ρ a C^∞, radial function such that $0 \leq \rho \leq 1$, with $\rho(\xi) = 1$ if $|\xi| \leq 1$ and $\rho(\xi) = 0$ if $|\xi| \geq 3/2$. We put $\gamma(\xi) := \rho(\xi) - \rho(2\xi)$ which is supported by $1/2 \leq |\xi| \leq 3/2$, and the following identities hold
$\sum_{j\in\mathbb{Z}} \gamma(2^j\xi) = 1 \quad (\forall \xi \in \mathbb{R}^n \setminus \{0\}), \quad \rho(2^{-k}\xi) + \sum_{j\geq k+1} \gamma(2^{-j}\xi) = 1 \quad (\forall k \in \mathbb{Z}, \forall \xi \in \mathbb{R}^n).$
We define the pseudodifferential operators $(S_j)_{j\in\mathbb{Z}}$ and $(Q_j)_{j\in\mathbb{Z}}$ by $\widehat{S_j f} := \rho(2^{-j}\cdot)\widehat{f}$ and $\widehat{Q_j f} := \gamma(2^{-j}\cdot)\widehat{f}$. The operators Q_j and S_j take values in the space of analytical functions of exponential type, see Paley-Wiener theorem, see, e.g., [5, rem. 2.3.1/2, p. 45]. They are defined on $\mathcal{S}'$ and $\mathcal{S}'_\infty$, respectively, since $Q_j f(x) = 0$ if and only if, $f \in \mathcal{P}_\infty$. We also have $f = \sum_{j\in\mathbb{Z}} Q_j f$ in $\mathcal{S}_\infty$ ($\mathcal{S}'_\infty$, $f = S_k f + \sum_{j>k} Q_j f$ in $\mathcal{S}$ ($\mathcal{S}'$. For brevity, we make use of the following conventions: If $f \in \mathcal{S}'$, then $[f]_\mathcal{P} \in \mathcal{S}'_\infty$. If $f \in \mathcal{S}'_\infty$ we define $Q_j f := Q_j f_1$ for all $f_1 \in \mathcal{S}'$ such that $[f_1]_\mathcal{P} = f$.

Definition 1.1. ([4, 5]) Let $s \in \mathbb{R}$, $0 < p, q \leq \infty$ (with $p < \infty$ in the F-case). The homogeneous spaces $\dot{A}^s_{p,q}$ is the set of $f \in \mathcal{S}'_\infty$ such that $\|f\|_{\dot{B}^s_{p,q}} := \Big(\sum_{j\in\mathbb{Z}} 2^{jsq} \|Q_j f\|_p^q \Big)^{1/q} < \infty$ in $\dot{B}^s_{p,q}$-case, and $\|f\|_{\dot{F}^s_{p,q}} := \Big\| \Big(\sum_{j\in\mathbb{Z}} 2^{jsq} |Q_j f|^q \Big)^{1/q} \Big\|_p < \infty$ in $\dot{F}^s_{p,q}$-case.

Definition 1.2. ([3]) Let E be a vector subspace of $\mathcal{S}'_\infty$ endowed with a quasi-norm such that $E \hookrightarrow \mathcal{S}'_\infty$ holds. A realization of E in $\mathcal{S}'$ is a continuous linear mapping $\sigma : E \to \mathcal{S}'$ such that $[\sigma(f)]_\mathcal{P} = f$ for all $f \in E$. The image set $\sigma(E)$ is called the realized space of E.

2 Main results

Theorem 2.1. *Let $1 \leq p < 2$. Then $\mathcal{F}^{-1}$ is bounded form $\tilde{\dot{B}}^0_{\infty,1-p/2} \cap \tilde{\dot{A}}^{n/p}_{p,\infty}$ into L_1, for all $\widehat{f} \in \tilde{\dot{B}}^0_{\infty,1-p/2} \cap \tilde{\dot{A}}^{n/p}_{p,\infty}$.*
En particular, $\mathcal{F}^{-1}$ is bounded form $\tilde{\dot{B}}^0_{\infty,1-p/2} \cap \dot{W}^n_1$ into L_1, for all $\widehat{f} \in \tilde{\dot{B}}^0_{\infty,1-p/2} \cap \dot{W}^n_1$.

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Well-posedness and general decay of Moore–Gibson–Thompson system with past history

Sami Loucif*

Mailing address: Laboratory of Mathematics, Informatics and systems (LAMIS), University of Larbi Tebessi, Tebessa, Algeria
E-mail: conferences2022conferences@gmail.com

Abstract
In this work, we consider a one-dimensional system of Moore–Gibson–Thompson in the case of homogeneous material with past history. We estabilished the well-posedness by using the semi group theory and we showed that the dissipation given by this complementary controls guarantees the general stabilty..

1 Introduction

Moore–Gibson–thompson (MGT) equation is based on the modeling of high amplitude sound waves. There has been quite a bit of work in this area of research due to a wide range of applications such as medical and industrial use of high intensity ultrasound in lithotripsy, heat therapy, ultrasonic cleaning, etc.
Lacheheb and al. in [2] they establish general decay estimates of the solution for the Cauchy problem of a Moore–Gibson–Thompson equation with a viscoelastic term

$$\begin{cases} u_{ttt} + \alpha u_{tt} - \beta \Delta u_t - \gamma \Delta u + \int_0^t g\left(t-s\right) \Delta u\left(s\right) ds = 0,\ x \in \mathbb{R}^n,\ t > 0 \\ u\left(x,0\right) = u_0\left(x\right),\ u_t\left(x,0\right) = u_1\left(x\right),\ u_{tt}\left(x,0\right) = u_2\left(x\right), \end{cases}$$

where u_0, u_1, u_2 are given functions and the parameters α, β, γ are strictly positive constants. The convolution term $\int_0^t g\left(t-s\right) \Delta u\left(s\right) ds$ reflects the memory effect of the viscoelastic materials. By using the energy method in the Fourier space, they first discuss the well-posedness and they demonstrated the general stability, Finally, they give some illustrative examples. In [3] Braik et al. they consider the Moore-Gibson-Thompson equation with distributed delay

$$\begin{aligned} & u_{ttt} + \left(\alpha + \mu\right) u_{tt} + \left(\beta A + \alpha \mu I\right) u_t + \gamma A u \\ & +_{\tau_1}^{\tau_2} \sigma\left(s\right) \left(u_{tt} + \alpha u_t\right)\left(x, t-s\right) ds \ = \ 0,\ \text{in } \Omega \times \mathbb{R}^+, \end{aligned}$$

with the initial conditions

$$\begin{cases} u\left(x,0\right) = u_0\left(x\right),\ u_t\left(x,0\right) = u_1\left(x\right),\ u_{tt}\left(x,0\right) = u_2\left(x\right),\ in\ \Omega, \\ \left(u_{tt} + \alpha u_t\right)\left(x, -\tau\right) = f_0\left(x-\tau\right),\ in\ \Omega \times \left(0, \tau_2\right) \end{cases}$$

Key Words and Phrases: *Moore-Gibson-Thompson equation, past history. semi group theory. Lyapunov method, General decay.*

where α, β, γ are strictly positive constants and A is a positive self-adjoint operator. Under an appropriate assumptions and a smallness conditions on the parameters $\alpha, \beta, \gamma, \mu$ they prove this problem is well-posed, also they prove the exponential stability by using Lyapunov functionals. In [4] Quintanilla study the exponential stability for a one-dimensional Moore–Gibson–Thompson system in the case of homogeneous material
In the present work, we consider the following Moore–Gibson–Thompson system in the case of homogeneous material with past history

$$\begin{cases} \rho u_{tt} = \mu u_{xx} - \beta\left(\theta_{tx} + \tau\theta_{ttx}\right) - \int_0^\infty g\left(s\right) u_{xx}\left(t-s\right) ds, \text{ in } (0,1)\times(0,\infty), \\ c\theta_{tt} + c\tau\theta_{ttt} = -\beta u_{tx} + \left(\kappa\theta_t + \kappa^*\theta\right)_{xx}, \text{ in } (0,1)\times(0,\infty), \\ u\left(x,0\right) = u_0\left(x\right), \ u_t\left(x,0\right) = u_1\left(x\right), \ x\in(0,1), \\ \theta\left(x,0\right) = \theta_0\left(x\right), \ \theta_t\left(x,0\right) = \theta_1\left(x\right), \ \theta_{tt}\left(x,0\right) = \theta_2\left(x\right), x\in(0,1), \\ u\left(0,t\right) = u\left(1,t\right) = \theta_x\left(0,t\right) = \theta_x\left(1,t\right) = 0, \ t>0, \end{cases} \tag{1.1}$$

where $c > 0$, $\rho > 0$, $\mu > 0$, $k^* > 0$, $k > k^*\tau$, $\beta \neq 0$, $\tau > 0$. and g is the relaxation function, which satisfies some hypotheses.

2 Main results

This work is organized as follows. In the section 2, we introduced some transformations and assumptions needed to prove the main result. In the section 3, we used the semi group method to prove the well-posedness of problem (1.1). In section 4, we considered several lemmas that helps us to construct the Lyapunov functional. In section 5, we proved our general stability result.

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General decay rate of translational Euler-Bernoulli beam with memory

Amirouche Berkani*
Faculty of Sciences and Applied Sciences, University Akli Mohand Oulhadj of Bouira, Bouira, 10000, Algeria.
E-mail: aberkanid@gmail.com

Abstract
The aim of this talk is to study the existence and asymptotic behaviour of solutions for viscoelastic translational Euler-Bernoulli beam. First, we establish the well-posedness using the Galerkin method and, then investigate the arbitrary decay of solutions under a suitable control force, for a large class of kernels with weaker conditions used usually in viscoelasticity.

1 Introduction

We consider a viscoelastic cantilevered Euler-Bernoulli beam, it is fixed to a base in a translational motion at one end and to a tip mass at its free end. The problem may be written as follows

$$\begin{cases} m\ddot{S}(t) + \rho \displaystyle\int_0^L \Big(\ddot{S}(t) + u_{tt}(x,t)\Big)dx + m_E\Big(\ddot{S}(t) + u_{tt}(L,t)\Big) = f(t), \quad t \geq 0, \\ \rho\Big(\ddot{S}(t) + u_{tt}(x,t)\Big) + EIu_{xxxx}(x,t) - EI\displaystyle\int_0^t g(t-s)u_{xxxx}(s)ds \\ +Cu_{xt}(x,t) + Du_{xxxxt}(x,t) = 0, \quad \forall (x,t) \in (0,L)\times[0,\infty), \end{cases} \tag{1.1}$$

with the boundary conditions and the initial data

$$\begin{cases} u(0,t) = u_x(0,t) = 0, \quad t \geq 0, \\ Du_{xxxt}(L,t) + EIu_{xxx}(L,t) - EI\displaystyle\int_0^t g(t-s)u_{xxx}(L,s)ds = m_E\Big(u_{tt}(L,t) + \ddot{S}(t)\Big), \quad t \geq 0, \\ Du_{xxt}(L,t) + EIu_{xx}(L,t) - EI\displaystyle\int_0^t g(t-s)u_{xx}(L,s)ds = -Ju_{xtt}(L,t), \quad t \geq 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad S(0) = S_0, \quad \dot{S}(0) = S_1, \quad x \in (0,L). \end{cases} \tag{1.2}$$

Here the dot "." denotes the derivative with respect to the time t, $S(t)$ is the base motion, $u(x,t)$ is the beam transversal displacement with respect to the base, $f(t)$ is an external force acting on the base,
ρ is the linear density of the beam, L is the length of the beam, EI is the bending stiffness of the beam, m is the mass of the translational base, m_E is the mass with rotational J attached at the free end of the beam, C is the structural damping coefficient of the beam and D is the Kelvin-Voigt damping coefficient of the beam.
The convolution term in the second equation of (1.1) represents the memory term or the dependence on the history and the kernel involved there is the relaxation function.

Key Words and Phrases: *Existence of solution; arbitrary decay; translational Euler-Bernoulli beam; memory term; relaxation function; viscoelasticity.*

2 Main results

First, our assumptions on the kernel $g(t)$ are the following:

(G1) The kernel g is a continuously differentiable nonnegative function satisfying $0 < \kappa =: \int_0^{+\infty} g(s)ds < 1$.

(G2) $g'(t) \leq 0$ for almost all $t \geq 0$.

(G3) There exists a nondecreasing function $\gamma(t) > 0$ such that $\gamma'(t)/\gamma(t) =: \xi(t)$ is a decreasing function and $\int_0^{\infty} g(s)\gamma(s)ds < +\infty$.

We define the energy functional of the problem (1.1)–(1.2) by

$$2\mathcal{E}(t) = m\omega_t^2(0,t) + \rho\|\omega_t\|_2^2 + EI\left(1 - \int_0^t g(s)ds\right)\|\omega_{xx}\|_2^2 + m_E\omega_t^2(L,t) + J\omega_{xt}^2(L,t), \quad t \geq 0 \tag{2.1}$$

where $\|.\|_2$ is the norm in $L^2(0,L)$ and $\omega(x,t) = S(t) + u(x,t), \quad t \geq 0$.
Our result reads as follows.

Theorem 2.1. *Under the hypotheses* **(G1)**–**(G3)** *and the control force $f(t)$, if $\mathcal{R}_g$ is sufficiently small. Then, there exist positive constants Λ and ν such that*

$$\mathcal{E}(t) \leq \Lambda e^{-\nu \int_0^t \xi(s)ds}, \quad t \geq 0$$

if $\lim_{t\longrightarrow\infty} \xi(t) = 0$ *and*

$$\mathcal{E}(t) \leq \Lambda e^{-\nu t}, \quad t \geq 0$$

if $\lim_{t\longrightarrow\infty} \xi(t) \neq 0$.

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Modeling of the reinforcement of a Kirchhoff-Love plate with a thin layer of varying thickness.

Mokhtari Hanifa

Mailing address: Mathematics, Mouloud Mammeri University of Tizi-Ouzou, Algeria
E-mail: hanifamokhtari@yahoo.fr

Abstract
In this paper, we deal with the asymptotic modeling of the behavior of an elastic plate reinforced with a thin layer of varying thickness $\delta(x) = \delta f(x)$, where δ is a small positive parameter.
More precisely, an extension of the results obtained in a previous work, where the case of a layer with constant thickness was studied, is given. More general approximate boundary conditions are derived, valid for a larger class of layers, having a thickness variation as a function of geometry coordinates. Optimal Error estimates between the exact and the approximate solutions of the reinforced problem are proved.

The Kirchhoff-Love problem for a reinforced plate with layer of varying thickness

We consider a bi-dimensional elastic plate occupying the set $\Omega_+ =]0,1[\times]0,1[$, with boundary $\partial\Omega_+ = \Sigma \cup \Gamma_+$, where $\Sigma =]0,1[\times \left\{0\right\}$. The plate is clamped on the portion Γ_+ and reinforced by a thin layer on the part Σ. This last one occupies the set $\Omega_-^\delta = \left\{ (x,y) \in \mathbb{R}^2,\ 0 < x < 1,\ -\delta f(x) < y < 0 \right\}$, where δ is a small positive parameter and the function $x \longmapsto f(x)$ is supposed to be sufficiently derivable with respect to the variable x. The boundary of Ω_-^δ is given by $\partial\Omega_-^\delta = \Sigma_-^\delta \cup \Sigma \cup \Gamma_-^\delta$, where : $\Sigma_-^\delta = \left\{ (x,y) \in \mathbb{R}^2,\ 0 < x < 1,\ y = \delta f(x) \right\}$.
We denote by $\Omega^\delta = \Omega_+ \cup \Sigma \cup \Omega_-^\delta$ the complete domain occupied by the whole structure.
The Kirchhoff-Love model for this structure is given by :

$$\begin{cases} D_+\Delta^2 w_+ = g_+ & \text{in } \ \Omega_+, \\ D\Delta^2 w = 0 & \text{in } \ \Omega_-^\delta, \\ w = 0, \quad \partial_n w = 0 & \text{on } \ \Gamma_+ \cup \Gamma_-^\delta, \\ [[T(w)]] = h_1, \quad [[M(w)]] = h_2 & \text{on } \ \Sigma, \\ [[w]] = 0, \quad [[\partial_n w]] = 0 & \text{on } \ \Sigma, \\ T_-(w) = k_1, \quad M_-(w_-) = k_2 & \text{on } \ \Sigma_-^\delta. \end{cases}$$

The function w models the transverse displacement of the structure (w_+ and w denote respectively the restriction of w to Ω_+ and Ω_-^δ). We denote by $n = (n_1, n_2)$ the unit normal to Σ oriented outwardly of Ω_+ and by $[[]]$ the jump through Σ.
Furthermore, the indices "+" and "-" stand for the restriction to Ω_+ and Ω_-^δ, respectively. The

Key Words and Phrases: *Asymptotic modeling, Asymptotic expansion method, Reinforcement, Approximate boundary conditions, Kirchhoff-Love plate, Thin layer with variable thickness.*

trace operators T and M designate respectively the shear forces and the bending moment and are defined as follows :

$$\begin{aligned}
M &= D\Big[\Delta + (1-\nu)\left(2n_1n_2\partial_{xy} - n_1^2\partial_y^2 - n_2^2\partial_x^2\right)\Big],\\
T &= D\Big[\partial_n\Delta + (1-\nu)\,\partial_\tau\Big(\left(n_1^2 - n_2^2\right)\partial_{xy} + n_1n_2\left(\partial_y^2 - \partial_x^2\right)\Big)\Big],
\end{aligned}$$

where ∂_τ represents the tangential derivative. The coefficient $D = \dfrac{E}{(1-\nu^2)}$ is the flexural rigidity of the plate having a Young modulus E and a Poisson's ratio ν. We assume that $E > 0$, $0 < \nu < \dfrac{1}{2}$ and that these coefficients are piecewise constant : $E = E_+$ in Ω_+ and E_-^δ in Ω_-^δ ; $\nu = \nu_+$ in Ω_+ and $\nu = \nu_-$ in Ω_-^δ. Consequently, we set $D = D_+$ in Ω_+ and D_-^δ in Ω_-^δ. In the following study, only the layer's Young modulus E_-^δ may depend on the parameter δ.

In this work, our objectif is to construct an asymptotic expansion of the solution w in powers of δ, of the form :

$$w_+^\delta = w_+^0 + \delta w_+^1 + \delta^2 w_+^2 + \cdots + \delta^n w_+^n, \tag{0.1}$$

$$w_-^\delta = w_-^0 + \delta w_-^1 + \delta^2 w_-^2 + \cdots + \delta^n w_-^n, \tag{0.2}$$

where the terms w_+^n and w_-^n do not depend on the parameter δ.

Conclusion

In this work, we have derived approximate models for the problem of reinforcement of an elastic Kirchhoff-Love plate with a thin layer of varying thickness. We have used the asymptotic expansion method to handle the case where the rigidity of the layer doesn't depend on . Indeed, naturally, the limit behavior in this situation corresponds to a model into which the effect of the layer is ignored. However, one of the merits of the asymptotic expansion method is the possibility of identifying a more precise model that incorporates this effect. In this fashion, we have derived approximate boundary conditions for this structure and given optimal error estimate.

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On reaction-diffusion systems with control of mass

Messaoud Bitam
Mailing address: Department of mathematics, University Of Farhat Abbas , Setif, Algeria
E-mail: messaoud.bitam@univ-setif.dz

Abstract
In this talk we are interested in reaction-diffusion systems with control of mass , we give some insights about global existence of solutions for this systems and as a typical example quadratic systems with polynomial growth .We state a lemma which gives the existence of classical local solutions then we state the main result of this talk in which we state the existence of global solutions for such systems.

1 Introduction

Let's consider the family of $m \times m$ reaction-diffusion systems satisfying the two main following properties:
- the nonnegativity of the solutions is preserved for all time .(P)
- the total mass of the components is a priori bounded on all finite intervals. (M)
More precisely, let us introduce the general system

$$\begin{cases} \forall i = 1, \dots, m, \\ \partial_t u_i - d_i \Delta u_i = f_i(u_1, \dots, u_m) \text{ on } (0,T) \times \Omega, \\ \alpha_i \frac{\partial u_i}{\partial n} + (1-\alpha_i) u_i = \beta_i \text{ on } (0,T) \times \partial\Omega, \\ u_i(0,\cdot) = u_{i0}, \end{cases}$$

where the $f_i : \mathbb{R}^m \to \mathbb{R}$ are C^1 functions of $u = (u_1, \dots, u_m)$, and for all $i = 1 \dots m, d_i \in (0,\infty), \alpha_i \in [0,1], \beta_i \in C^2([0,T] \times \bar{\Omega}), \beta_i \geq 0$. We denote $\Sigma_T = (0,T) \times \partial\Omega$. By classical solution on $[0,T)$, we mean that, at least

$$\begin{cases} u \in C\left([0,T); L^1(\Omega)^m\right) \cap L^\infty([0,T-\tau] \times \Omega)^m, \forall \tau \in (0,T), \\ \forall k,l = 1 \dots N, \forall p \in [1,\infty), \partial_t u, \partial_{x_k} u, \partial_{x_k x_l} u \in L^P((\tau, T-\tau) \times \Omega)^m, \\ \text{ and equations above are satisfied a.e.} \end{cases}$$

Let us first recall the classical local existence result under the above assumptions :

Lemma 1.1. *Assume $u_0 \in L^\infty(\Omega)^m$. Then, there exist $T > 0$ and a unique classical solution of (1.4) on $[0,T)$. If T^* denotes the greatest of these T 's, then*

$$\left[\sup_{t \in [0,T^*), 1 \leq i \leq m} \|u_i(t)\|_{L^\infty(\Omega)} < +\infty \right] \Rightarrow [T^* = +\infty].$$

If, moreover, the nonlinearity $(f_i)_{1 \leq i \leq m}$ is quasi-positive , then

$$[\forall i = 1, \dots, m, u_{i0} \geq 0] \Rightarrow [\forall i = 1, \dots, m, \forall t \in [0,T^*), u_i(t) \geq 0].$$

Nonnegativity of the solutions is preserved if (and only if) the nonlinearity $f = (f_1, \dots, f_m)$ is quasi-positive which means that (P) $\forall r \in [0,+\infty)^m, \forall i = 1 \dots m,$ $f_i(r_1, \dots, r_{i-1}, 0, r_{i+1}, \dots, r_m) \geq 0$, where we denote $r = (r_1, \cdots, r_m)$.

Key Words and Phrases: *Reaction-diffusion systems, Global existence, control of mass, Polynomial growth*

2 Main results

Let us consider the following 2×2 system

$$\begin{cases} \partial_t u - d_1 \Delta u = f(u,v) \\ \partial_t v - d_2 \Delta v = g(u,v) \\ u(0,\cdot) = u_0(\cdot) \geq 0, v(0,\cdot) = v_0(\cdot) \geq 0 \\ \text{ with either : } \frac{\partial u}{\partial n} = \beta_1, \frac{\partial v}{\partial n} = \beta_2 \text{ on } (0,+\infty) \times \partial\Omega \\ \text{ or } : u = \beta_1, v = \beta_2 \text{ on } (0,+\infty) \times \partial\Omega \end{cases}$$

where $d_1, d_2 \in (0,+\infty), \beta_1, \beta_2 \in [0,+\infty)$ and $f, g : [0,+\infty)^2 \to \mathbb{R}$ are C^1. For $u_0, v_0 \in L^\infty(\Omega)$ with $u_0, v_0 \geq 0$, existence of classical nonnegative bounded solutions holds on some maximal interval $[0, T^*)$ (see Lemma 1.1). Then, we have the first following global existence result :

Theorem 2.1. *Asume (P)* +(M) *holds for the* 2×2 *system . Assume moreover that* $u_0, v_0 \in L^\infty(\Omega), u_0, v_0 \geq 0$ *and, for some* $U, C \geq 0$

$$\forall u \geq U, \forall v \geq 0, f(u,v) \leq C[1 + u + v].$$
$$\exists r \geq 1; \quad \forall u, v \geq 0, |g(u,v)| \leq C\left[1 + u^r + v^r\right].$$

Then, $T^* = +\infty$.

As an application let's consider the next system :

$$\begin{cases} \partial_t u - d_1 \Delta u = -uh(v) \\ \partial_t v - d_2 \Delta v = uh(v) \end{cases}$$

with $h \geq 0$ and homogeneous Neumann boundary conditions. Then, the hypothesises of theorem (2.1) are satisfied with $C = 0$. By maximum principle we have

$$\forall t \in [0, T^*), \quad \|u(t)\|_{L^\infty(\Omega)} \leq \|u_0\|_{L^\infty(\Omega)}.$$

Now, it is not obvious that v is bounded. Theorem 2.1 claims that this is the case if moreover h grows at most like a polynomial as $v \to +\infty$.

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Uniform regularity and vanishing viscosity limit for the full viscous MHD system with critical axisymmetric initial data

Youssouf Maafa*

University of Batna -2- ,LEDPA
E-mail: y.maafa@univ-batna2.dz

Abstract
In this paper we establish the global well-posedness for the three dimensional full viscous (MHD) equations with axisymmetric initial data belonging to critical Besov spaces, and we obtain uniform estimates with respect to the viscosity. Furthermore, we study the strong convergence when the viscosity goes to zero in the resolution spaces.

1 Introduction

In this paper, we consider full viscous Magnetohydrodynamics (MHD) of an incompressible flow equations :

$$\begin{cases} \partial_t v_\mu + v_\mu \cdot \nabla v_\mu - \mu\Delta v_\mu + \nabla p_\mu = B_\mu \cdot \nabla B_\mu & \text{if } (t,x) \in \mathbb{R}_+ \times \mathbb{R}^3, \\ \partial_t B_\mu + v_\mu \cdot \nabla B_\mu - \kappa\Delta B_\mu = B_\mu \cdot \nabla v_\mu & \text{if } (t,x) \in \mathbb{R}_+ \times \mathbb{R}^3, \\ \mathrm{div} v_\mu = 0, \quad \mathrm{div} B_\mu = 0, \\ (v_\mu, B_\mu)_{|t=0} = (v_0^\mu, B_0^\mu). \end{cases} \tag{MHD$_{\mu,\kappa}$}$$

where $v = (v^1, v^2, v^3)$ is the velocity vector field of the fluid particles and $B = (B^1, B^2, B^3)$ is the magnetic field. The pressure p is a scalar function that can be recovered from the velocity and the magnetic field by following elliptic equation:

$$p \equiv -\sum_{i,j=1}^{2} \mathcal{R}_i\mathcal{R}_j(v^i v^j) + \sum_{i=1}^{2} \mathcal{R}_i\mathcal{R}_j(B^i B^j),$$

where $\mathcal{R}_i = \frac{\partial_i}{\sqrt{-\Delta}}$ stands to Riesz's operator. The positive parameters μ and κ represent the viscosity and resistivity of the fluid, respectively.
For $\mu = 0, \kappa = 1$ the system (MHD$_{\mu,\kappa}$) takes the form

$$\begin{cases} \partial_t v + v \cdot \nabla v + \nabla p = B \cdot \nabla B & \text{if } (t,x) \in \mathbb{R}_+ \times \mathbb{R}^3, \\ \partial_t B + v \cdot \nabla B - \Delta B = B \cdot \nabla v & \text{if } (t,x) \in \mathbb{R}_+ \times \mathbb{R}^3, \\ \mathrm{div} v = 0, \quad \mathrm{div} B = 0, \\ (v, B)_{|t=0} = (v_0, B_0) \end{cases} \tag{MHD$_0$}$$

Key Words and Phrases: *Full viscous MHD system, global well-posedness, axisymmetric solutions, critical Besov spaces, inviscid limit.*

If $B \equiv 0$, then the system ($\mathrm{MHD}_{\mu,\kappa}$) is reduced to the classical incompressible Navier-Stokes equations denoted by (NS_μ) and reads as follows.

$$\begin{cases} \partial_t v + v \cdot \nabla v + \nabla p - \mu \Delta v = 0 \quad \text{if } (t,x) \in \mathbb{R}_+ \times \mathbb{R}^3, \\ \mathrm{div} v = 0, \\ v_{|t=0} = v_0. \end{cases} \tag{NS_μ}$$

When the viscous forces vanish ($\mu = 0$), we get the classical incompressible Euler equation :

$$\begin{cases} \partial_t v + v \cdot \nabla v + \nabla p = 0 \quad \text{if } (t,x) \in \mathbb{R}_+ \times \mathbb{R}^3, \\ \mathrm{div} v = 0, \\ v_{|t=0} = v_0. \end{cases} \tag{E}$$

Abidi, Hmidi and Keraani [1] propagate globally in time the regularity $\mathscr{B}^{\frac{3}{p}+1}_{p,1}(\mathbb{R}^3)$ for Euler equations, the key of the proof is a new decomposition of the vorticity $\omega \triangleq \mathrm{curl}\, v$. T.Hmidi and the M.Zerguine [4] used the previous result to obtain the same result as in [1] studied the inviscid limit.Recently, Z. Hassainia [3] generalized the result of Abidi, Hmidi and Keraani [1] for system (MHD_0) by exploited the structure

$$v(t,x) = v^r(t,r,z)\vec{e}_r + v^z(t,r,z)\vec{e}_z, \quad B = B^\theta \vec{e}_\theta. \tag{1.1}$$

Our aim is to generalize the result of T.Hmidi and the M.Zerguine for the viscous and resistive (MHD) system by the following Theorem:

2 Main results

Here are the main result of this paper

Theorem 2.1. *Let (v_μ, B_μ) and (v, B) be the solutions of* ($\mathrm{MHD}_{\mu,\kappa}$) *and* ($\mathrm{MHD}_0$) *systems respectively with the same initial data, which satisfying the same conditions as in* [3, Theorem 1]. *Then for every $p \in (2, \infty)$, we have:*

$$\|v_\mu - v\|_{L^\infty_t \mathscr{B}^0_{p,1}} + \|B_\mu - B\|_{L^\infty_t \mathscr{B}^{-1}_{p,1} \cap L^1_t \mathscr{B}^1_{p,1}} \leq \Big((\mu t)^{\frac{1}{5}+\frac{3}{5p}} + (\mu t)^{\frac{3}{5p}} + (\mu t)^{\frac{3}{\max(p,6)}} \Big) \Phi_6(t). \tag{2.1}$$

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Youssouf Maafa*

University of Batna -2- ,LEDPA
E-mail: y.maafa@univ-batna2.dz

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where $\mathcal{R}_i = \frac{\partial_i}{\sqrt{-\Delta}}$ stands to Riesz's operator. The positive parameters μ and κ represent the viscosity and resistivity of the fluid, respectively.
For $\mu = 0, \kappa = 1$ the system (MHD$_{\mu,\kappa}$) takes the form

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Problème d'interaction fluide-structure en contact étudié par la méthode de domaine fictif avec pénalisation

Othman Yakhlef*
Mailing address: Department of mathematics , Mohamed Seddik Ben Yahia University, Jijel, Algeria ,
E-mail: othmeny@gmail.com

Abstract
Dans ce travail, nous avons proposé un algorithme de point fixe afin de résoudre un problème d'interaction fluide-structure avec la contrainte supplémentaire que les déplacements de la structure sont limités par un obstacle rigide. Une approche de domaine fictif avec pénalisation est utilisée pour les équations de fluide. Les forces surfacique du fluide agissant sur la structure sont calculées en utilisant la solution du fluide dans le domaine de la structure. Un problème d'optimisation sous contraintes convexes est résolu afin d'obtenir les déplacements de la structure. Les résultats numériques sont présentés.

1 Introduction

L'interaction fluide-structure (IFS) s'intéresse au comportement d'un système constitué par des entités mécaniques considérées comme distinctes : une structure mobile (rigide ou déformable) et un fluide (en écoulement ou au repos) autour ou à l'intérieur de la structure. L'évolution de chacune des deux entités dépendant de celle de l'autre. Plus précisément, le mouvement de la structure est influencé par l'écoulement du fluide à travers les efforts transmis à l'interface, et réciproquement, le mouvement de la structure influence l'écoulement du fluide par les déplacements de l'interface qui entraîne le fluide dans son mouvement, un phénomène de couplage apparat.
Les phénomènes d'interaction fluide-structure font partie de la vaste classe des problèmes multi-physiques. Un grand nombre de situations font apparatre ces phénomènes [1, 5]. Les applications en bio-mécanique font, en général, intervenir un liquide et une structure déformable : écoulements sanguins dans les vaisseaux, déformation des globules rouges dans les capillaires. Dans le domaine du génie nucléaire, l'usure d'un faisceau tubulaire d'un échangeur de chaleur, par instabilité sous écoulement, peut prendre peine quelques secondes ; cet effet de couplage est pris en compte de facon primordiale pour des raisons de sûreté des installations de production d'énergie. La compréhension des effets de vibrations induites par écoulement a initié de nombreuses campagnes expérimentales et justifie aujourd'hui le développement de méthodes de calcul numérique en couplage fluide-structure. Dans le domaine du génie civil, nous citons fréquemment l'exemple de destruction du pont de Tacoma dont la compréhension a donné lieu à une littérature scientifique abondante et qui illustre l'importance des effets d'interaction fluide-structure.
Le but principal de ce travail est d'établir une procédure numérique pour un problème d'interaction fluide-structure en contact étudié par la méthode de domaine fictif avec pénalisation.

Key Words and Phrases: *Domaine fictif, interaction fluide-structure, vitesse de structure.*

2 Main results

Nous avons effectué le comportement d'une structure élastique immergé dans un fluide visqueux incompressible. Nous avons utilisé l'équation de Stokes pour modéliser le mouvement d'écoulement

$$\begin{cases} -\mu\Delta\mathbf{u} + \nabla p &= \mathbf{f} \quad \text{dans } \Omega, \\ \text{div } \mathbf{u} &= 0 \quad \text{dans } \Omega, \\ \mathbf{u} &= 0 \quad \text{sur } \partial\Omega. \end{cases} \tag{2.1}$$

Le vecteur $\mathbf{u}$ représente le champ de vitesse du fluide, la fonction scalaire p est la pression qui lui est associée. On considère encore le champ de force $\mathbf{f}$ agissant sur le système et le coefficient de viscosité dynamique μ.
et le déplacement de la structure avec les équations d'élasticité linéaire.

$$-\nabla \cdot \sigma(U) = f \qquad \text{dans } \Omega \tag{2.2}$$
$$U = 0 \qquad \text{sur } \Gamma_D \tag{2.3}$$
$$\sigma(U)n = h \qquad \text{sur } \Gamma_N \tag{2.4}$$

où U est le vecteur déplacement et le tenseur de contraintes $\sigma(U) = \mu(\nabla U + (\nabla U)^T)$, avec des conditions par rapport à l'obstacle sur la frontière de la structure.
Des exemples numériques ont été donnés où les calculs numériques ont été élaborés via le logiciel "FreeFem++".

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On Local Existence and Blow-up Solutions for a Time-Space Fractional Variable Order Superdiffusion Equation with Exponential Nonlinearity

Khaoula Bouguetof

Department of Mathematics; University of Larbi Tebessi, Tebessa, Algeria
E-mail: khaoula.bouguetof@gmail.com

Abstract
The aim purpose of the present work is the generalization of semi-linear wave equation. This latter sheds light the order of derivative that changes with time and nonlocal nonlinearity of exponential growth. Our findings strongly indicated the existence and the blowup solutions that were carefully studied in details. [3]
keywords Blow-up; Caputo derivative; local existence; variable fractional order.

1 Introduction

In this paper we consider the following Cauchy problem

$$\mathbf{D}_{0|t}^{\alpha(t)} u + (-\Delta)^s u = I_{0|t}^{1-\gamma} \mathrm{e}^{\mathrm{u}}, \quad \mathrm{t} > 0, \quad \mathrm{x} \in \Omega, \tag{1.1}$$

with the initial conditions

$$u(0,x) = u_0,\ u_t(0,x) = u_1(x), \qquad x \in \Omega, \tag{1.2}$$

supplemented with the boundary condition

$$u = 0, \quad t > 0, \quad x \in \mathbb{R}^N \setminus \Omega, \tag{1.3}$$

where $\Omega \subset \mathbb{R}^N, \alpha : \mathbb{R}_+ \to (1,2)$, $0 < s, \gamma < 1$ and $\mathbf{D}_{0|t}^{\alpha(t)}$ is the Caputo derivative of variable fractional order $\alpha(t)$. The term $I_{0|t}^{1-\gamma} \mathrm{e}^{\mathrm{u}}$ defined as

$$I_{0|t}^{1-\gamma} \mathrm{e}^{\mathrm{u}} = \frac{1}{\Gamma(\gamma)} \int_0^{\mathrm{t}} (\mathrm{t} - \tau)^{-\gamma} \mathrm{e}^{\mathrm{u}(\tau,\mathrm{x})} \mathrm{d}\tau.$$

The fractional Laplacien operator is defined as

$$(-\Delta)^s u(t,x) = \frac{C(N,s)}{2} \int_{\mathbb{R}^N} \frac{2u(t,x) - u(t,x+y) - u(t,x-y)}{|y|^{N+2s}} dy,$$

where $C(N,s)$ is a positive normalizing constant depending on N and s.
In recent years, fractional variable order differentiel equations have interested for real life phenomena. According to [2], fractional variable order is the best operators to describe the effect of memory, which change with time or spatial location.

2 The local Cauchy problem

In this section, we apply the Banach fixed point theorem to prove the local existence of a unique mild solution of our problem (1.1) − (1.3), when $\alpha(t) \equiv \alpha$ is a constant function. Firstly, we recall the following definition

Definition 2.1. Let $1 < \alpha < 2$ and $u_0, u_1 \in L^2_{loc}(\mathbb{R}^N)$. A function $u \in C([0,T], L^2_{loc}(\mathbb{R}^N))$ is a mild solution to problem (1.1) − (1.3) if

$$u(t,x) = P_\alpha(t)u_0 + I^1_{0|t} P_\alpha(t)u_1 + \int_0^t (t-s)^{\alpha-1} S_\alpha(t-s) I^{1-\gamma}_{0|s} \mathrm{e}^{\mathrm{u}} \mathrm{ds},$$

where $P_\alpha(t)$ and $S_\alpha(t)$ are defined as in (**??**) and (**??**) respectively.

We are now in a position to stat and prove our main result in this section.

Theorem 2.1. *Let $1 < \alpha < 2$ and $u_0, u_1 \in L^2_{loc}(\mathbb{R}^N)$. Then there exists $T > 0$ such that problem (1.1) − (1.3) has a unique mild solution $u \in C([0,T], L^2_{loc}(\mathbb{R}^N))$.*

Theorem 2.2. *Assume that*

$$\int_{\mathbb{R}^{\mathbb{N}}} u_0 dx > 0, \int_{\mathbb{R}^{\mathbb{N}}} u_1 dx > 0.$$

Then any solution to (1.1)-(1.3) blows up in a finite time.

Proof. We assume the contrary. □

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PART D

Ordinary Differentials equations

Existence of solution of a class of factional differential equations with non-instantaneous impulses Using Darbo's fixed point theorem with measure of noncompactness.

Benzahi Ahlem*
Mailing address: Department of Mathematics, Abdelhafid Boussouf University Center, Mila, Algeria
E-mail: a.benzahi@centre-univ-mila.dz

Abada Nadjet
Mailing address: Department of Mathematics, Ecole Normale Supérieure El Katiba Assia Djebar, Consrantine, Algeria
E-mail: n65abada@yahoo.fr

Arar Nouria
Mailing address: Department of Mathematics, Frères Mentouri Constantine University, Consrantine, Algeria
E-mail: arar.nouria@umc.edu.dz

Abstract
In the present work, I established the existence of solutions of the initial value problem for linear implicit fractional differential equations with non instantaneous impulses under the Caputo Fabrizio fracional derivative. The result of existence is obtained by applying the Darbo's fixed point theorem associated with the technique of Kuratowski measure of non compactness. The same initial value problem for linear implicit fractional differential equations with non instantaneous impulses under the Caputo Fabrizio fracional derivative was already studied with Monch's fixed point theorem and Kuratowski measure of non compactness. Finally, I presented an example to illustrate the applicability of the found result.

1 Introduction

Fractional calculus is a branch of classical mathematics, which deals with the generalization of operations of differentiation and integration to fractional order. Although fractional derivation theory is a subject almost as old as classical calculus as we know it today, its origins date back to the late 17th century, when Newton and Leibniz developed the foundations differential and integral calculus. In particular, Leibniz introduced the symbol $\frac{d^n f}{dt^n}$ when he announced in a letter to the Hospital , the Hospital replied " What does mean $\frac{d^n f}{dt^n}$ if $n = \frac{1}{2}$"? This letter from Hopital , written in 1695, is today accepted as the first incident of what we call fractional derivation.
Differential equations with instantaneous impulsesare frequently used to describe mathematical simulations of real-world phenomena that experience rapid changes of state.

Key Words and Phrases: *Caputo fractional derivative, Non-instantaneous impulse, Fixed point theorems, Measure of non compactness*

2 Main results

In this section , we will study the existence of the following initial value problems ($0 < \alpha < 1$).

$$\begin{cases} {}^{CF}D^{\alpha}_{s_i}u(t) = f(t, u(t)), t \in (s_i, t_{i+1}], i = 0, ..., m, \\ u(t) = g_i(t, u(t_i^-)), t \in (t_i, s_i], i = 1, ..., m, \\ u(0) = u_0 \in E, \end{cases} \tag{2.1}$$

where $I_i = (s_i, t_{i+1}], J_i = (t_i, s_i], f : I_i \times E \to E,\ g_i : J_i \times E \to E, i = 1 \cdots m$, are given continuous functions, $I = [0, T]$, $0 = s_0 < t_1 \leq s_1 \leq t_2 < \cdots < t_m \leq s_m \leq t_{m+1} = T$.

Theorem 2.1 (Darbo's fixed point Theorem)**.** *Let D be a non-empty, closed, bounded and convex subset of a Banach space E and let N be a continuous mapping of D into itself such that for any non-empty subset C of D,*

$$\mu(N(C)) \leq k\mu(C) \tag{2.2}$$

Let us list some conditions on the functions involved in this IVP .

(H1) There exists a continuous function $G \in C(I_i, \mathbb{R}_+), i = 0 \cdots m$, such that

$$\|f(t, u)\| \leq G(t)\|u\|,\ u \in E,\ t \in I_i \ \text{ with} G^* = \sup_{t \in \mathcal{I}} G(t),$$

(H2) There exists a continuous function $H_i \in C(J_i, \mathbb{R}_+), i = 1 \cdots m$, such that

$$\|g_k(t, u)\| \leq H_i(t),\ u \in E,\ t \in J_i \ \text{ with} H^* = \max_{i=0...m} (\sup_{t \in J_i} H_i(t)).$$

(H3) For each bounded set $D \in E$ and for each $t \in I_i, i = 0 \cdots m$, we have

$$\mu(f(t, D)) \leq G(t)\mu(D),\ t \in I_i.$$

(H4) For each bounded set $D \in E$ and for each $t \in J_i, i = 1 \cdots m$, we have

$$\mu(g_k(t, D)) \leq H_i(t)\mu(D),\ t \in J_i.$$

Theorem 2.2. *Assume that assumptions $(H1) - (H4)$ hold. If*

$$k = \max\{H^*, (a_\alpha + Tb_\alpha)G^*\} < 1, \tag{2.3}$$

then the IVP (**??**) *has at least one solution on I.*

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Neutral delay Mackey-Glass model

Marwa Khemis*

LAMAHIS laboratory, Department of Mathematics, University of 20 August 1955, Skikda, Algeria
E-mail: khemismarwa08@gmail.com

Ahlème Bouakkaz

LAMAHIS laboratory, Department of Mathematics, University of 20 August 1955, Skikda, Algeria
E-mail: ahlemkholode@yahoo.com

Abstract
In this work, we propose a neutral Mackey-Glass model with two delays which can be describe the production of blood cells in the bone marrow. With the help of the Krasoneselskii's and Banach fixed point theorems along with the Green's functions method, we establish some sufficient conditions that ensure the existence and uniqueness of positive periodic solutions for the proposed model.

1 Introduction

This work investigates the existence and uniqueness of positive periodic solutions for a first-order neutral delay differential equation which models the process leading to the formation and regulation of red blood cells in the bone marrow. More precisely, the present work deals with an iterative neutral hematopoiesis model with two delays, the first one represents the proliferating phase which is assumed to depend on time and the density of mature cells while the second denotes a transit time required to release mature cells into the circulating bloodstream.
As we know, there are just very few authors who have studied neutral hematopoiesis models that incorporate multiple variable delays. Motivated by this fact, we use the Krasnoselskii's and Banach fixed point theorems combined with the Green's functions method for establishing some easily verifiable sufficient conditions that guarantee the existence and uniqueness of a positive periodic solution of the considered equation. The main idea of this work consists to convert our proposed model into an equivalent integral equation where the kernel is a Green's function. Then we construct an appropriate integral operator that can be considered as a sum of two operators and next employing the aforementioned theorems as well as some useful properties of the obtained kernel for establishing our new results on the existence of unique positive periodic solution.

Key Words and Phrases: *Neutral differential equation, Green's functions, fixed point theory , Mackey-Glass model*

2 Main results

Let $\mathbb{X}$ be the Banach space of all T-periodic continuous functions equipped with the supremum norm and for $\alpha, \beta, M > 0$, let Ω a closed, convex and bounded subset of $\mathbb{X}$ such that $\Omega = \{x \in \mathbb{X}, \alpha \leq x(t) \leq \beta, |x(t_2) - x(t_1)| \leq M |t_2 - t_1|, \forall t_1, t_2 \in \mathbb{R}\}$,
Throughout this work, we adopt the following notations:

$$A = \frac{\exp\left(-\int_0^T a(u)du\right)}{\exp\left(\int_0^T a(u)du\right) - 1}, B = \frac{\exp\left(\int_0^T a(u)du\right)}{\exp\left(\int_0^T a(u)du\right) - 1}, b_1 = \inf_{t\in[0,T]} b(t), \ b_2 = \sup_{t\in[0,T]} b(t), \ a_2 = \sup_{t\in[0,T]} a(t),$$

Moreover, we need the following conditions:

$$TBb_2 \leq \beta(1-c), \tag{2.1}$$
$$\frac{TAb_1}{1+\beta} - cTBa_2\beta \geq \alpha(1-c), \tag{2.2}$$
$$B(b_2 + ca_2\beta)(2 + Ta_2) \leq M - cM(M+1), \tag{2.3}$$
$$TB(b_2M + b_2 + ca_2) + c < 1, \tag{2.4}$$

The conversion of the studied equation into an equivalent integral one, allows us to define two operators S_1 and S_2 where $S_1, S_2 : \Omega \to \mathbb{X}$

$$(S_1x)(t) = \int_t^{t+T} G(t,s)\left\{\frac{b(s)}{1 + x^{[2]}(s)} - ca(s)x(s - \tau(s))\right\} ds, \tag{2.5}$$
$$(S_2x)(t) = cx(t - \tau(t)), \tag{2.6}$$

where the kernel is a Green's function given by $G(t,s) = \frac{\exp\left(\int_t^s a(u)du\right)}{\exp\left(\int_0^T a(u)du\right)-1}$.

Theorem 2.1. *If conditions* (2.1) − (2.3) *hold, then our equation has at least one positive periodic solution in* Ω.

Theorem 2.2. *If conditions* (2.1) − (2.4) *hold, then our equation has a unique positive periodic solution in* Ω.

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Existence, uniqueness and Ulam Hyers stability results for Hadamard fractional differential equations of variable order

Amar Benkerrouche *

Department of Mathématiques, Ziane Achour University, Djelfa, Algeria
E-mail: amar.benkerrouche3@gmail.com

Abstract
In this manuscript, we examine both the existence, uniqueness and the stability of solutions to the boundary value problem of Hadamard fractional differential equations of variable order. All results in this study are established using Krasnoselskii fixed-point theorem and the Banach contraction principle. Further, the Ulam-Hyers stability of the given problem is examined.

1 Introduction

The fractional calculus of variable fractional order is a generalization of constant order and many studies have been done on the existence of solutions to fractional constant-order problems, on the contrary, few papers deal with the existence of solutions to problems via variable order. Therefore, all our results in this work are novel and worthwhile.
In this paper we will study the following boundary value problem for the Hadamard fractional differential equation of variable order

$$\begin{cases} {}^H D_{1^+}^{u(t)} x(t) = f(t, x(t), {}^H I_{1^+}^{u(t)} x(t)), \ t \in J, \\ x(1) = x(T) = 0, \end{cases} \tag{1.1}$$

where $J = [1, T]$, $1 < T < \infty$, $u(t) : J \to (1, 2]$ is the variable order of the fractional derivatives, $f : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a continuous function and the left Hadamard fractional integral of variable-order $u(t)$ for function $x(t)$ is (see, for example, [1], [2])

$$ {}^H I_{1^+}^{u(t)} x(t) = \frac{1}{\Gamma(u(t))} \int_1^t (log\frac{t}{s})^{u(t)-1} \frac{x(s)}{s} ds, \ \ t \in J, \tag{1.2}$$

where $\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$ is the Gamma function and the left Hadamard fractional derivative of variable-order $u(t)$ for function $x(t)$ is (see, for example, [1], [2])

$$ {}^H D_{1^+}^{u(t)} x(t) = \frac{t^2}{\Gamma(2-u(t))} \frac{d^2}{dt^2} \int_1^t (log\frac{t}{s})^{1-u(t)} \frac{x(s)}{s} ds, \ \ \ t \in J. \tag{1.3}$$

We notice that, if the order $u(t)$ is a constant function u, then the Hadamard variable order fractional derivative (1.3) and integral (1.2) are the usual Hadamard fractional

Key Words and Phrases: *derivatives and integrals of variable-order, boundary value problem, piecewise constant functions, Hadamard derivative*

derivative and integral, respectively(see [1, 2]).
Remark For general functions $u(t)$, $v(t)$, we notice that the semigroup property doesn't hold, i.e:

$$^{H}I_{a^+}^{u(t)}(^{H}I_{a^+}^{v(t)})h(t) \neq^{H} I_{a^+}^{u(t)+v(t)}h(t).$$

Lemma 1.1. *([3, 4]) If* $u : J \to (1,2]$ *be a continuous function, then for* $h \in C_\delta(J,\mathbb{R}) = \{h(t) \in C(J,\mathbb{R}),\ (logt)^\delta h(t) \in C(J,\mathbb{R})\}$, $0 \leq \delta \leq \min_{t\in J} |(u(t))|$ *the variable order fractional integral* $^{H}I_{1^+}^{u(t)}h(t)$ *exists for any points on* J.

Lemma 1.2. *([3, 4]) Let* $u : J \to (1,2]$ *be a continuous function, then* $^{H}I_{1^+}^{u(t)}h(t) \in C(J,\mathbb{R})$ *for* $h \in C(J,\mathbb{R})$.

2 Main results

In this work, we introduced an abstract variable–order boundary value problem of Hadamard fractional differential equations of variable order, where the function $u(t)$: $[1,T] \to (1,2]$ stands for the variable order of the given system. First, we reviewed some important specifications of Hadamard variable–order operators and by an example, we showed that the semi-group property is not valid for variable–order Hadamard integrals. Then by defining a partition based on the generalized intervals, we introduced a piecewise constant function $u(t)$ and converted the given variable–order Hadamard fractional differential equations 1.1 to an equivalent standard Hadamard boundary value problem of the fractional constant order. By using the standard fixed–point theorems, we established the existence and uniqueness of solutions. Finally, the Ulam–Hyers stability of its possible solutions was checked.

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Fractional-order problem coupled with a second-order Moreau's sweeping process

S. TAMOUZA*

Mailing address: Mathematics, Mohammed Seddik Benyahia , Jijel, Algeria
E-mail: tamouzasomia@gmail.com

F. ALIOUANE

Mailing address: Mathematics, Mohammed Seddik Benyahia, Jijel, Algeria
E-mail: faliouane@gmail.com

D. AZZAM-LAOUIR

Mailing address: Mathematics, Mohammed Seddik Benyahia, Jijel, Algeria
E-mail: laouir.dalila@gmail.com

Abstract
In this paper, by applying Schauder's fixed point theorem, we investigate the existence of solutions for a fractional-order multi-point boundary-value problem coupled with a second-order perturbed time and state-dependent Moreau's sweeping process.

1 Introduction

We consider the following problem

$$(\mathcal{P})\begin{cases} D^q x(t) \ = \ v(t) \quad \forall t \in \ [1,T], \\ x(1)=0, \quad D^r x(T) = \sum\limits_{i=1}^{n} \lambda_i D^r x(\mu_i), \\ -\ddot{v}(t) \in \ N_{C(t,x(t),D^{q-1}x(t),v(t))}(\dot{v}(t)) + g(t,x(t),v(t),\dot{v}(t)) \quad a.e.\ t \in [1,T], \\ \dot{v}(t) \ \in \ C(t,x(t),D^{q-1}x(t),v(t)) \quad \forall t \in [1,T], \\ v(1)=0, \ \dot{v}(1)=u_0, \end{cases}$$

where D^q is the Hadamard fractional derivative of order $q \in (1,2],\ r \in (0,1),\ \mu_i \in (1,T)$, $\lambda_i \in \mathbb{R}, i = \overline{1,n},\ n \geq 2,\ g : [1,T] \times H \times H \times H \to\ H$ is Lebesgue measurable on $[1,T]$ and continuous on $H \times H \times H$ and $C : [1,T] \times H \times H \times H \rightrightarrows H$ is a set-valued map with nonempty, convex and closed values.

Key Words and Phrases: *Differential inclusion, convex set, fixed point, Hadamard fractional derivative, normal cone.*

2 Main results

Suppose that

$(\mathcal{H}_1^C)$ For any bounded subset $K \subset H \times H \times H$, the set $C\big(J \times K\big)$ is ball-compact and there exists an integrably bounded set-valued map $F : J \rightrightarrows H$ with nonempty, convex and compact values such that $C(t,x,y,u) \subset F(t), \forall t \in J, \forall x,y,u \in H$.

$(\mathcal{H}_2^C)$ There are an absolutely continuous function $a : J \to H$, which is nonnegative and nondecreasing, and a nonnegative real constant λ such that for all $t, s \in J$ and $x, x', x'' y, y', y'', u \in H$,

$$|d_{C(t,x,x',x'')}(u) - d_{C(s,y,y',y'')}(u)| \leq |a(t) - a(s)| + \lambda\big(\|x-y\| + \|x'-y'\| + \|x''-y''\|\big).$$

$(\mathcal{H}_1^g)$

$$\|g(t,x,u,v)\| \leq c(1 + \|x\| + \|u\| + \|v\|) \quad \forall\, (t,x,u,v) \in J \times H \times H \times H.$$

The uniqueness of the solution requires the following additional assumptions.

$(\mathcal{H}_g^2)$ For all $t \in J,\ x_i, y_i, z_i \in H,\ i = 1,2$, and for some nonnegative function $k(\cdot) \in \mathbf{L}^1(J,\mathbb{R})$ one has

$$\|g(t,x_1,y_1,z_1) - g(t,x_2,y_2,z_2)\| \leq k(t)\big(\|x_1 - x_2\| + \|y_1 - y_2\| + \|z_1 - z_2\|\big).$$

$(\mathcal{H}_g^C)$ The normal cone of $C(t,x,y,u)$ is hypomonotone in the following sense: for a given $L > 0$, there exists a nonnegative real function $\alpha_L \in \mathbf{L}^1(J,\mathbb{R})$ such that if

$$a_i \in N_{C(t,x_i,y_i,u_i)}(b_i) \quad \text{for } a_i \in H,\ x_i, y_i, u_i, b_i \in L\overline{\mathbf{B}},\ i = 1,2,$$

then $\langle a_1 - a_2, b_1 - b_2\rangle \geq -\alpha_L(t)\|b_1 - b_2\|(\|x_1 - x_2\| + \|y_1 - y_2\| + \|u_1 - u_2\|)$.

Theorem 2.1. *Assume that $(\mathcal{H}_1^C), (\mathcal{H}_2^C), (\mathcal{H}_1^g), (\mathcal{H}_2^g)$ and $(\mathcal{H}_g^C)$ hold true. Then, for any $u_0 \in H$ such that $u_0 \in\ C(1,0,0,0)$, there exists a unique absolutely continuous solution $(x,u,v) : J \to H \times H \times H$ of the problem $(\mathcal{P})$.*

With the estimate $\|\ddot{v}(t)\| \leq M(t)$ *a.e.* $t \in J$, for some nonnegative function $M(\cdot) \in \mathbf{L}^1(J,\mathbb{R})$.

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Investigate fractional differential equations using the topological degree method

Iatime Khadidja *

Mailing address: Department of Mathematics, Echahid Hamma Lakhdar University of El-Oued, Algeria.
E-mail: iatime-khadidja@univ-eloued.dz

Azouzi Mounira

Mailing address: Department of Mathematics, Echahid Hamma Lakhdar University of El-Oued, Algeria.
E-mail: mouniramath2020@gmail.com

Abstract
In this paper we study the existence of solutions to multi-point boundary value problem of fractional differential equations at resonance, involving the Generalized Proportional Fractional derivative(GPF derivatives). the concerned results are obtained via extansion of Mawhin's continuation theorem. An illustrative example is presented.

1 Introduction

The aim of this paper is to study the existance of solutions for a class of fractional differential equations by using the extension of Mawhin's continuation theorem, More specifically, we consider the following generalized proportional fractional differential equation, with multi-point boundary conditions of the form:

$$^{c}\mathfrak{D}_0^{\alpha,\rho}u(t) = f(t, u(t), {}^{c}\mathfrak{D}_0^{\alpha-1,\rho}u(t)) \quad 0 < t < 1 \tag{1.1}$$

$$u(0) = 0, \tag{1.2}$$

$$^{c}\mathfrak{D}_0^{\alpha-1,\rho}u(1) = \sum_{i=1}^{i=m} \sigma_i \, {}^{c}\mathfrak{D}_0^{\alpha-1,\rho}u(\eta_i) \tag{1.3}$$

where $^{c}\mathfrak{D}_0^{\alpha,\rho}$ denote the generalized proportional fractional derivative of Caputo type of order $\alpha \in (1, 2], \rho \in (0, 1]$, $0 < \eta_i < 1$, $\sigma_i \in \mathbb{R}$, $\sum\limits_{i=1}^{i=m} \sigma_i = 1,\ m \in \mathbb{N}^*$,and $f : [0, 1] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a given continuous function.
To investigate the problem, we use the condition

Key Words and Phrases: *multi-point boundary value problem,the Generalized Proportional Fractional derivative, resonance case, nonlinear fractional differential equation.*

$$\sum_{i=1}^{i=m} \sigma_i \eta_i^{2-\alpha} e^{-\delta(1-\eta_i)} = 1 \tag{1.4}$$

where $\delta = \frac{\rho-1}{\rho}$.
Next, we present the notations and nomenclatures with regard to the coincidence degree, see ([3]).

Definition 1.1 ([3]). Let X, Y be two real Banach spaces, Ω be an open bounded subset of X, and $L : dom(L) \subset X \to Y$ is a linear operator, $N : X \to Y$ is nonlinear mapping. If L is a closed set of Y and $\dim\ker(L) = co\dim(L) < +\infty$, then L is called a Fredholm operator of index zero. In this case there exist two linear continuous projectors $P : X \to X$, $Q : Y \to Y$ such that $P = \ker L$, and $\ker Q = L$ and we can write $X = \ker(L) \oplus \ker(P)$, $Y = (L) \oplus (Q)$. It follows that $L_P = L_{|dom(L)\cap\ker P} : dom(L) \cap \ker P \to L$ is invertible. We denote the inverse of L_P by K_P. If $dom(L)\cap\bar{\Omega} \neq \emptyset$, the mapping N will be called L-compact on Ω if $QN(\bar{\Omega})$ is bounded and $K(I-Q)N : \bar{\Omega} \to X$ is compact.

Theorem 1. *Let X, Y be two real Banach spaces, $L : dom(L) \subset X \to Y$ be a Fredholm operator of index zero and $N : X \to Y$ be an L-compact mapping on Ω. Assume that the following conditions are satisfied:*
(1). $Lu \neq \lambda N u$ for all $(u, \lambda) \in (dom(L) \backslash \ker L) \cap \partial\Omega \times (0, 1)$,
(2). $QNu \neq 0$ for all $x \in \ker L \cap \partial\Omega$,
(3). $\deg(QN_{|\ker L}, \Omega \cap \ker L, 0) \neq 0$.
Then the equation $Lu = Nu$ has at least one solution in $dom\ L \cap \bar{\Omega}$.

2 Main results

Theorem 2. *Suppose that there exists:*

(C_1) *There exists a L^1-Carathéodory function $\Phi : [0,1] \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ which is non decreasing with respect to the last two variables such that*

$$|f(t, x, y)| \leq \Phi(t, |x|, |y|),$$

for all $(x; y) \in \mathbb{R}^2$ and $t \in [0, 1]$.

(C_2) *A real $M_0 > 0$, such that if we have $\left|{}^c\mathfrak{D}_0^{\alpha-1,\rho} u(t)\right| > M_0$ for all $t \in [0, 1]$, then*

$$I_1 f(t, u(t), {}^c\mathfrak{D}_0^{\frac{1}{4},\frac{1}{2}} u(t)) - I_2 f(t, u(t), {}^c\mathfrak{D}_0^{\alpha-1,\rho} u(t)) \neq 0.$$

(C_3) *A real $M_1 > 0$, such that for $|c| > M_1$, then either*

$$c\left(I_1 N(cte^{\delta t}) - I_2 N(cte^{\delta t})\right) > 0, \tag{2.1}$$

or

$$c(I_1 N(cte^{\delta t}) - I_2 N(cte^{\delta t})) < 0, \tag{2.2}$$

then the fractional BVPs (1.1)-(1.2)-(1.3) has at least one solution in $dom\,(L) \subset X$*, provided that*

$$\int_0^1 \Phi\,(t,r,r)\,dt \leq \frac{\rho^{\alpha}\Gamma(\alpha)e^{2\delta}}{\rho\Gamma(\alpha)(L_1+L_2)+e^{\delta}(1+\rho^{\alpha-1}\Gamma(\alpha))}r+\beta. \tag{2.3}$$

Where β *is a positive constant.*

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Some Precise Estimations of [p,q]-Order of Solutions of Linear Differential Equations

Mansouria SAIDANI *
Department of mathematics, Laboratory of Pure and Applied Mathematics, University of Mostaganem (UMAB), B. P. 227 Mostaganem-(Algeria)
E-mail: mansouria.saidani@univ-mosta.dz

Benharrat BELAIDI
Department of mathematics, Laboratory of Pure and Applied Mathematics, University of Mostaganem (UMAB), B. P. 227 Mostaganem-(Algeria)
E-mail: benharrat.belaidi@univ-mosta.dz

Abstract
In this work, we investigate the $[p,q]$-order of meromorphic solutions to higher order non homogeneous linear differential equations in which the coefficients are meromorphic functions of finite $[p,q]$-order and the second member is a meromorphic function of infinite $[p,q]$-order.
We get some results about the $[p,q]$-order and the $[p,q]$-convergence exponent of the solutions for such equations.

1 Introduction

In the recent years, many authors have studied the complex linear differential equations

$$f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_1(z)f' + A_0(z)f = 0 \tag{1.1}$$

and

$$f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_1(z)f' + A_0(z)f = F(z). \tag{1.2}$$

In [1], Belaidi considered the growth of meromorphic solutions of equations (1.1) and (1.2) with meromorphic coefficients of finite iterated $p-$order and obtained some results which improve and generalize some previous results. After that, in [2] the authors have studied the growth of solutions of the equations (1.1) and (1.2) when $A_s(z)$ dominates all other coefficients and they got some results about $\rho_{p+1}(f)$.
Thus, the following question arises: Can we give information on the properties of solutions of equations

$$A_k(z)f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_1(z)f' + A_0(z)f = 0 \tag{1.3}$$

and

$$A_k(z)f^{(k)} + A_{k-1}(z)f^{(k-1)} + \cdots + A_1(z)f' + A_0(z)f = F(z) \tag{1.4}$$

when the coefficients $A_j(j = 0, 1, ..., k), F(z)$ are of $[p,q]$-order? In this work, we proceed this way and we obtain the following results.

Key Words and Phrases: *linear differential equations; meromorphic functions; [p; q]-order; [p; q]-exponent of convergence of zeros.*

2 Main results

Theorem 2.1. *Let $H \subset (1, +\infty)$ be a set with a positive upper logarithmic density (or $m_l(H) = +\infty$) and let A_j $(j = 0, 1, ..., k)$ with $A_k \not\equiv 0$ be meromorphic functions with finite $[p, q]-$ order. If there exist a positive constant $\sigma > 0$ and an integer s, $0 \leq s \leq k$, such that for sufficiently small $\varepsilon > 0$, we have $|A_s(z)| \geq \exp_{p+1}\left((\sigma - \varepsilon)\log_q r\right)$ as $|z| = r \in H$, $r \to +\infty$ and $\rho = \max\left\{\rho_{[p,q]}(A_j)\ (j \neq s)\right\} < \sigma$, then every non-transcendental meromorphic solution $f \not\equiv 0$ of (1.3) is a polynomial with $\deg f \leq s - 1$ and every transcendental meromorphic solution f of (1.3) with $\lambda_{[p,q]}\left(\frac{1}{f}\right) < \mu_{[p,q]}(f)$ satisfies*

$$\mu_{[p,q]}(f) = \rho_{[p,q]}(f) = +\infty$$

and

$$\sigma \leq \rho_{[p+1,q]}(f) \leq \rho_{[p,q]}(A_s).$$

Theorem 2.2. *Let $H \subset (1, +\infty)$ be a set with a positive upper logarithmic density (or $m_l(H) = +\infty$), with $A_k \not\equiv 0$ $(j = 0, 1, ..., k)$ and $F \not\equiv 0$ be meromorphic functions with finite $[p, q]-$ order. If there exist a positive constant $\sigma > 0$ and an integer s, $0 \leq s \leq k$, such that for sufficiently small $\varepsilon > 0$ we have $|A_s(z)| \geq \exp_{p+1}\left((\sigma - \varepsilon)\log_q r\right)$ as $|z| = r \in H$, $r \to +\infty$ and $\rho = \max\left\{\rho_{[p,q]}(A_j)\ (j \neq s), \rho_{[p,q]}(F)\right\} < \sigma$, then every non-transcendental meromorphic solution $f \not\equiv 0$ of (1.4) is a polynomial with $\deg f \leq s - 1$ and every transcendental meromorphic solution f of (1.4) with $\lambda_{[p,q]}\left(\frac{1}{f}\right) < \{\sigma, \mu_{[p,q]}(f)\}$ satisfies*

$$\overline{\lambda}_{[p,q]}(f) = \lambda_{[p,q]}(f) = \rho_{[p,q]}(f) = \mu_{[p,q]}(f) = +\infty$$

and

$$\sigma \leq \overline{\lambda}_{[p+1,q]}(f) = \lambda_{[p+1,q]}(f) = \rho_{[p+1,q]}(f) \leq \rho_{[p,q]}(A_s).$$

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Existence results for Hilfer-Katugampola-type fractional implicit differential equations with nonlocal conditions

Abada Nadjet*
Laboratoire MAD, Departement Informatique et Mathematiques, ENS Constantine, Algerie
E-mail: abada.nadjat@ensc.dz

Benzahi Ahlem
Laboratoire MAD, ENS Constantine, Centre Universitaire Mila
E-mail: a.benzahi@centre-univ-mila.dz

Abstract
This work contains a new discussion for Hilfer-Katugampola-type fractional derivative. We establish an existence results of Hilfer-Katugampola-type fractional derivative for implicit differential equations with the help of Schaefers fixed point theorem. Further, the examples are given to illustrate our main results.

1 Introduction

The aim of this article is to study the implicit differential equation with nonlocal condition involving Hilfer-Katugampola-type fractional derivative of the following form

$$({}^\rho D_{a^+}^{\alpha,\beta} y)(t) = f(t, y(t), ({}^\rho D_{a^+}^{\alpha,\beta} y)(t)), \quad \forall t \in (a, T], a > 0 \tag{1.1}$$

$$y(T) = c \in \mathbb{R} \tag{1.2}$$

Where ${}^\rho D_{a^+}^{\alpha,\beta}$ is the Hilfer-Katugampola-type fractional derivative of order $\alpha \in (0, 1)$ and type $\beta \in [0, 1]$ and $f : (a, T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a given function.

2 Main results

In this part, we present notations and definitions that we will use throughout this paper. Let $0 < a < T$, $J = [a, T]$. By $C(J, \mathbb{R})$ we denote the Banach space of all continuous functions from J into $\mathbb{R}$ with the norm:

$$\|y\|_\infty = \sup \big\{ |y(t)| : t \in J \big\}.$$

Let $\rho > 0$ et $0 \leq \gamma < 1$. We consider the weighted spaces of continuous functions:

$$C_{\gamma,\rho}(J) = \Big\{ y : (a, T] \to \mathbb{R} \text{ tel que } \left(\frac{t^\rho - a^\rho}{\rho}\right)^\gamma y(t) \in C(J, \mathbb{R}) \Big\}$$

$$\|y\|_{C_{\gamma,\rho}(J)} = \sup_{t \in J} \left| \left(\frac{t^\rho - a^\rho}{\rho} y(t) \right) \right|.$$

Key Words and Phrases: *Hilfer-Katugampola-type fractional derivative, implicit differential equation, Schaefers fixed point theorem, existence, uniqueness, Ulam stability*

Definition 2.1. Let $\alpha > 0, t > a, \rho > 0$ and $g \in C_{\gamma,\alpha}(J)$, The Katugampola fractional integral of order α is defined by:

$$ {}^{\rho}I_{a^+}^{\alpha} g(t) = \frac{1}{\Gamma(\alpha)} \int_a^t s^{\rho-1} \left(\frac{t^\rho - s^\rho}{\rho} \right)^{\alpha-1} g(s) ds. $$

Definition 2.2. Let $\alpha \in \mathbb{R}_+/\mathbb{N}, \rho > 0, t > a$ and $g \in C_{\gamma,\alpha}(J)$, The Katugampola fractional derivative of order α is defined by

$$ \begin{aligned} {}^{\rho}D_{a^+}^{\alpha} g(t) &= \delta_\rho^n \left({}^{\rho}I_{a^+}^{n-\alpha} g \right)(t) \\ &= \left(t^{1-\rho} \frac{d}{dt} \right)^n \frac{1}{\Gamma(n-\alpha)} \int_a^t s^{\rho-1} \left(\frac{t^\rho - s^\rho}{\rho} \right)^{n-\alpha-1} g(s) ds \end{aligned} \tag{2.1} $$

where $n = [\alpha] + 1$

suppose that the function $f : (a, T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous and satisfies the conditions
$[H1]$ For all $u, v \in C_{1-\gamma,\rho}(J)$:$f(., u(.), v(.)) \in C_{1-\gamma,\rho}^{\beta(1-\alpha)}(J)$
$[H2]$ Let $\phi, \psi, \chi \in C_{1-\gamma,\rho}(J)$.

$$ |f(t, u, v)| \leq \phi(t) + \psi(t)|u| + \chi(t)|v|, \forall t \in J \text{ et } \forall u, v \in \mathbb{R} $$

with

$$ \phi^* = \sup_{t \in J} \phi(t) \ , \ \psi^* = \sup_{t \in J} \psi(t) \ , \ \chi^* = \sup_{t \in J} \chi(t) < 1 $$

Theorem 2.1. *Assume (H1) and (H2) hold. If:*

$$ \frac{2\phi^*\Gamma(\gamma)}{(1-\chi^*)\Gamma(\alpha+\gamma)} \left(\frac{T^\rho - a^\rho}{\rho} \right)^\alpha < 1 \tag{2.2} $$

Th problem the (1.1)-(1.2) *have at least one solutio in* $C_{1-\gamma,\rho}^{\gamma}(J)$.

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Existence and Stability Analysis of Solution for Mathieu Fractional Differential Equations with Applications on Some Physical Phenomena

Tabouche Nora*

Mailing address: Department of Mathematics, 8 May 1945 University-Guelma , Guelma, Algeria
E-mail: tabouche.nora@univ-guelma.dz

Berhail Amel

Mailing address: Department of Mathematics, 8 May 1945 University-Guelma , Guelma, Algeria
E-mail: berhail.amel@univ-guelma.dz

Abstract
This paper deals with a class of nonlinear Mathieu fractional differential equations. The reported results discuss the existence, uniqueness and stability for the solution of proposed equation. We prove the main results by the aid of fixed point theorems and Ulam's approach. The paper is appended with two applications that describe the force of periodic pendulum and the motion of a particle in the plane. Graphical representations are used to illustrate the results.

1 Introduction

The Mathieu equation is an important equation of mathematical physics as it has many applications in fields of the physical sciences, such as optics, quantum mechanics, frequency modulation, alternating gradient focusing, mirror trap for neutral particles, inverted pendulum, vibrations in an elliptic drum, stability of a floating body and general relativity (Campbell 1950; Dingle and Muller 1964; McLachlan 1951; Marathe and Chatterjee 2006; Muller-Kirsten and Dingle 1962; Muller-Kirsten 2006; Buren et al. 2007). Mathieu equation was first introduced by the French mathematician Emile Leonard Mathieu (Mathieu 1868) who encountered them while studying vibrating elliptical drumheads. The equation is a second-order differential equation of the form

$$D^2z(t) + [a - 2b\cos(2t)]z(t) = 0 \tag{1.1}$$

where $D^2z := \frac{dz^2}{dt^2}$, a and b are real or complex constants. The solution of equation (1.1) is built in the form $z(t) = exp(i\sigma t)\eta(t)$, where η is a periodical function with period π and σ is so–called characteristic index depending on the values of a and b.

Key Words and Phrases: *Mathieu equations Á Caputo fractional differential equation, HU stability, Schauder's fixed point theorem, Banach contraction principle.*

In this work, we consider the existence, uniqueness and stability of solutions for the following fractional Mathieu equation

$$\begin{cases} {}^c\mathcal{D}^{\mu} z(t) + \Lambda(t) z(t) = \psi(t, z(t), {}^c\mathcal{D}^{\nu} z(t)), \mu \in (1,2], \nu \in (0,1], \\ z(0) = 0, \\ z'(0) = z_1 \end{cases} \tag{1.2}$$

where $\Lambda(t) = a - 2b\cos(2t)$, ${}^c\mathcal{D}^{\diamond}$, are Caputo's fractional derivative of order $\diamond \in \{\mu, \nu\}$ and $\psi : [0,T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a nonlinear continuous function.

2 Main results

Notations: $C_1 = \frac{T^{\mu}}{\Gamma(\mu+1)}[N + |a+2b|] + T,\ C_2 = \frac{MT^{\mu}}{\Gamma(\mu+1)},\ C_3 = \frac{T^{\mu-\nu}}{\Gamma(\mu-\nu+1)}[N + |a+2b|] + \frac{T^{1-\nu}}{\Gamma(2-\nu)},\ C_4 = \frac{MT^{\mu-\nu}}{\Gamma(\mu-\nu+1)}$.

Theorem 2.1. *Suppose that the function ψ satisfies the following condition*
(H1) there exists a positive constant L such that

$$|\psi(t,u,v)| \leq L,\ t \in [0,T],\ u,\ v \in \mathbb{R}.$$

(H2) There exist a constant $N > 0$, $M > 0$ such that

$$|\psi(t,u_1,v_1) - \psi(t,u_2,v_2)| \leq N \|u_1 - u_2\| + M \|v_1 - v_2\|,$$

for any $t \in [0,T]$, $u_1, v_1, u_2, v_2 \in \mathbb{R}$, and let $d = \max\limits_{0 \leq t \leq T} |\psi(t,0,0)|$. If

$$k = max\{C_1, C_2, C_3, C_4\} < 1, \tag{2.1}$$

then problem (1.2) has a unique solution in $\Im$.

Where, $\Im = \{z/z \in \mathcal{C},\ {}^c\mathcal{D}^{\nu} z \in \mathcal{C}\}$, is a Banach space equipped with the norm $\|z\|_{\Im} = \max\left\{\sup_{0\leq t\leq T}|z(t)|, \sup_{0\leq t\leq T}|{}^c\mathcal{D}^{\nu} z(t)|\right\}$.

Theorem 2.2. *Suppose that conditions (H1) and (2.1) hold. Then, the solution of (1.2) is UH and GUH stable.*

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On the Existence of Solutions for Fractional Coupled Differential Equations

Mebarki Khadidja*

Laboratory of Mathematics Modeling and Applications, University of Adrar
E-mail: kha.mebarki@univ-adrar.edu.dz

Boudaoui Ahmed

Laboratory of Mathematics Modeling and Applications, University of Adrar
E-mail: ahmedboudaoui@univ-adrar.edu.dz

Abstract
In this presentation, we employ coupled fixed points to give sufficient conditions to guarantee a solution to system of fractional differential equations. The proof of the existence theorem is based on a coupled fixed point theorem in $b-$fuzzy metric space endowed with directed graph.

1 Introduction

In 1987, Guo and Lakshmikantham [1] introduced the notion of coupled fixed point. By using this notion, Bhaskar and Lakshmikantham [3] gave sufficient conditions to solve some differential equations by introducing and proving many nice results for coupled fixed points.

2 Main results

Let $(X, M, *, G)$ stands to a complete b-fuzzy metric space with constant $s \geq 1$ (introduced by Sedghi and Shobe [2]) such that $a * a \geq a^2$ and $\lim_{t \to \infty} M(x, y, t) = 1$, endowed with directed graph G such that $V(G) = X$, $E(G) \supseteq \Delta$ and G has no parallel edges. The mapping $T \colon X \times X \to X$ be mapping Further, we endow the product space $X \times X$ by another graph denoted also by G, such that

$$((x, y), (u, v)) \in E(G) \Leftrightarrow (x, u) \in E(G) \text{ and } (v, y) \in E(G),$$

for any $(x, y), (u, v) \in X \times X$.
We denote by Ω the set of function $\varphi \colon \mathbb{R}^+ \to \mathbb{R}^+$ that meets all of the following criteria: φ is nondecreasing; for all $a \in \mathbb{R}^+$ and $t \in \mathbb{R}^+$ we have $\varphi(at) = a\varphi(t)$; $\sum_{i=0}^{\infty} \varphi^i(t)$ converges for all $t > 0$.

Key Words and Phrases: *Fractional differential equations, Coupled fixed point, $b-$fuzzy metric space, Directed graph.*

Definition 2.1. The mapping $T : X \times X \to X$ is called φ-fuzzy contraction if there exist $\varphi \in \Omega$ such that:

1. for all $x, y, u, v \in X$, T is edge preserving, i.e.,
$$((x, y), (u, v)) \in E(G) \text{ then } ((T(x, y), T(y, x)), (T(u, v), T(v, u))) \in E(G);$$
2. for all $(x, y), (u, v) \in X \times X$ such that $((x, y), (u, v)) \in E(G)$,
$$M(T(x, y), T(u, v), \varphi(t)) \geq M(x, u, st)^{\frac{1}{2}} * M(y, v, st)^{\frac{1}{2}}. \tag{2.1}$$

Theorem 2.1. *On $(X, M, *)$, suppose that T is continuous mapping and φ-fuzzy contraction mapping. If there exist $x_0, y_0 \in X$ such that $((x_0, y_0), (T(x_0, y_0), T(y_0, x_0))) \in E(G)$, then T possess a coupled fixed point; ie., $x = T(x, y)$ and $y = T(y, x)$.*

we will study the existence of a continuous solution for a system of fractional differential equations. Let us consider the following system

$$D^\alpha x(t) = f(t, x(t), y(t)), \quad D^\alpha y(t) = f(t, y(t), x(t)), \quad t \in J \tag{2.2}$$

$$x(0) = x_0 = y(0). \tag{2.3}$$

Where the symbol D^α denotes the Caputo fractional derivative of order $\alpha \in (0, 1)$, $J := [0, L]$, $f : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a given function satisfying some assumptions that will be specified later.

Assumption 2.1.
1. $f : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is contionous;
2. For all $x, y, u, v \in \mathbb{R}$, with $x \leq u$ and $v \leq y$ we have $f(t, x, y) \leq f(t, u, v)$, $\forall t \in J$;
3. For each $t \in J$, $x, y, u, v \in \mathbb{R}$, $x \leq u$ and $v \leq y$, we have
$$|f(t, x, y) - f(t, u, v)|^2 \leq \frac{1}{8}\left(|x - u|^2 + |y - v|^2\right);$$
4. We suppose that $K = \frac{L^{2\alpha-1}}{\Gamma(\alpha)^2} < 1$.

Theorem 2.2. *Consider the system* (2.2)-(2.3) *suppose that the Assumption 2.1 is satisfied. Assume that there exists $(u_0, v_0) \in X \times X$ such that $u_0(t) \leq x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha-1} f(s, u_0(s), v_0(s))ds$ and $v_0(t) \geq x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - s)^{\alpha-1} f(s, v_0(s), u_0(s))ds$, $t \in J$. Then, there exists a unique solution of the integral system* (2.2)-(2.3).

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Existence and relaxation of extreme solutions for evolution differential inclusions involving time dependent maximal monotone operators

A. Makhlouf*
Mailing address: LAOTI, University of Mohammed Seddik Benyahia-Jijel, Algeria
E-mail: amira.makhlouf18@gmail.com

D. Azzam-Laouir
Mailing address: D LAOTI, University of Mohammed Seddik Benyahia-Jijel, Algeria
E-mail: laouir.dalila@gmail.com

Abstract
In this paper, we establish the existence of extreme solutions for evolution problems involving time dependent maximal monotone operators in Hilbert spaces. Then we show that the set of extreme solutions is dense in the solution set of the original problem.

1 Introduction

Let H be a separable Hilbert space and $I = [0, T]$ $(T > 0)$ be an interval of $\mathbb{R}$. In this paper we consider a class of evolution differential inclutions of the forme

$$(\mathcal{P}_F) \quad \begin{cases} -\dot{u}(t) \in A(t)u(t) + F(t, u(t)), \ a.e. \ t \in I, \\ u(t) \in D(A(t)), \ a.e. \ t \in I, \\ u(0) = u_0 \in D(A(0)), \end{cases}$$

where $F : I \times H \rightrightarrows H$ is a set-valued map with compact values, and, for all $t \in I$, $A(t)$ is a maximal monotone operator of H and $D(A(t))$ its domain. The dependence $t \mapsto A(t)$ is absolutely continuous with respect to the pseudo-distance, $dis(\cdot, \cdot)$, introduced by Vladimirov [3], that is,

$$dis(A,\ B) = \sup\left\{ \frac{\langle y - \hat{y}, \hat{x} - x\rangle}{1 + \|y\| + \|\hat{y}\|} :\ (x, y) \in gph(A), (\hat{x}, \hat{y}) \in gph(B) \right\}, \tag{1.1}$$

where $A : D(A) \subset H \to 2^H$ and $B : D(B) \subset H \to 2^H$ are two maximal monotone operators.

We are interested in the problem of the existence of extreme solutions for the problem $(\mathcal{P}_F)$. By using this result, we prove under a suitable hypothesis on F, that the solutions set of problem

$$(\mathcal{P}_{ext(F)}) \quad \begin{cases} -\dot{u}(t) \in A(t)u(t) + ext(F(t, u(t))), \ a.e. \ t \in I \\ u(t) \in D(A(t)), \ a.e. \ t \in I, \\ u(0) = u_0 \in D(A(0)), \end{cases}$$

is dense in the solutions set of the problem $(\mathcal{P}_F)$ (relaxation theorem).

Key Words and Phrases: *Absolutely continuous variation, maximal monotone operators, pseudo-distance, extreme points, perturbation, relaxation, weak norm*

2 Main results

For the statement of our theorems of this work, we have to assume the following hypotheses.
$(\mathcal{H}_A^1)$ There exists a function $\beta \in W^{1,2}(I, \mathbb{R})$ which is nonnegative on $[0, T[$ and nondecreasing such that

$$dis(A(t), A(s)) \leq |\beta(t) - \beta(s)|, \quad \text{for all } t, s \in I.$$

$(\mathcal{H}_A^2)$ There exists $c \geq 0$ such that

$$\|A^0(t, x)\| \leq c(1 + \|x\|), \quad \text{for all } t \in I \text{ and } x \in D(A(t)).$$

$(\mathcal{H}_A^3)$ For every $t \in I$, $D(A(t))$ is relatively ball compact.
$(\mathcal{H}_F^1)$ F is $(\mathcal{L}(I) \otimes \mathcal{B}(H))$-measurable.
$(\mathcal{H}_F^2)$ There exists a nonnegative function $m \in \mathrm{L}^2(I, \mathbb{R})$ such that

$$F(t, x) \subset m(t)\overline{B}_H, \quad \text{for all } (t, x) \in I \times H.$$

$(\mathcal{H}_F^3)$ There exists a nonnegative function $k \in \mathrm{L}^1(I, \mathbb{R})$ such that

$$\mathcal{H}(F(t, x), F(t, y)) \leq k(t)\|x - y\|, \quad \text{for all } (t, x, y) \in I \times H \times H.$$

Now, we present our main result.

Theorem 2.1. *Let for every $t \in I$, $A(t) : D(A(t)) \subset H \to 2^H$ be a maximal monotone operator satisfying $(\mathcal{H}_A^1)$, $(\mathcal{H}_A^2)$ and $(\mathcal{H}_A^3)$. Let $F : I \times H \rightrightarrows H$ be a set-valued map with nonempty, convex and compact values satisfying $(\mathcal{H}_F^1)$, $(\mathcal{H}_F^2)$ and $(\mathcal{H}_F^3)$. Then the solution set $S(\mathcal{P}_{ext(F)})$ of the problem $(\mathcal{P}_{ext(F)})$ is nonempty and dense in the solution set $S(\mathcal{P}_F)$ of the problem $(\mathcal{P}_F)$ with respect to the topology of uniform convergence.*

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h-stability results for solutions of certain nonlinear perturbed systems

Abir Kicha*

Mailing address: Department of Mathematics, University of Jijel, Algeria
E-mail: abir.kicha@univ-jijel.dz

Hanen Damak and Mohamed Ali Hammami

Mailing address: Department of Mathematics, University of Sfax, Tunisia
E-mail: hanen.damak@yahoo.fr and MohamedAli.Hammami@fss.rnu.tn

Abstract
This paper is related to the study of the preservation of global uniform h-stabilizability of time-varying nonlinear control systems with a perturbation Lyapunov's theory. Some examples and simulations are given to illustrate the main results.

1 Introduction

The main purpose of developing stability theory is to examine the dynamic responses of a system to disturbances as time approaches infinity. It has been and still is the subject of intense investigations due to its intrinsic interest and its relevance to all practical systems in engineering, natural science and social science. To study the stability of nonlinear differential systems we generally refer to the results introduced by Lyapunov in 1892, which is called: Lyapunovs second method. It is one of the most well known techniques for studying the stability properties of dynamic systems. This method uses an auxiliary function, called Lyapunov function, which checks the stability behavior of a specific system without the need to generate system solutions. In this work, we investigated the global uniform h-stabilization for certain classes of nonlinear time-varying systems and we proved that the proposed controller guarantees the global uniform h-stability of the closed-loop system. The technique is based on the Lyapunov theory. We illustrated the applicability of the obtained results by numerical examples with simulations. Our original results generalize well known fundamental results: exponential stabilization for nonlinear time-varying systems.

Key Words and Phrases: *Global uniform boundedness, Lyapunov function, Uniform h-stability.*

2 Main results

Consider the linear time-varying system:

$$\dot{x} = A(t)x, \qquad x(t_0) = x_0, \qquad t \geq t_0 \geq 0, \tag{2.1}$$

where $A(\cdot)$ is $n \times n$ matrix whose entries values are continuous functions of $t \in \mathbb{R}_+$.
We state a converse theorem when the origin is a globally uniformly h-stable equilibrium point of the linear system (2.1), by defining a Lyapunov function that satisfies certain properties.

Theorem 2.1. *Let the origin be globally uniformly h-stable equilibrium point of system* (2.1). *Assume that $h \in \mathcal{H}$ with h' exists and continuous on $\mathbb{R}_+$. Suppose that $A(t)$ is continuous and bounded on $\mathbb{R}^n$. Let $Q(t)$ be a continuous, bounded, positive definite and symmetric matrix. Then, there is a continuously differentiable, bounded, positive and symmetric matrix $P(t)$, which is the solution of the Riccati equation:*

$$\dot{P}(t) = h'(t)h(t)^{-1}P(t) - A^T(t)P(t) - P(t)A(t) - Q(t). \tag{2.2}$$

Theorem 2.2. *Let the solutions of system* (2.1) *be globally uniformly h-stable with $h \in \mathcal{H}$ and h' exists continuous on $\mathbb{R}_+$. Suppose that $A(t)$ is continuous and bounded on $\mathbb{R}^n$. Thus, there exists a function $V(t,x)$ satisfying the following properties:*

(*i*) $\|x\|^2 \leq V(t,x) \leq (c_1+1)\|x\|^2, \qquad (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n,$

(*ii*) $\dot{V}(t,x) \leq h'(t)h(t)^{-1}V(t,x), \qquad (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n,$

where c_1 is a positive constant.

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Solitons in Several Space Dimensions with $P(x)$ variable exponent as soliton

Dellal Abdelkader*
Mailing address: Department of Mathematics and Computer Science, University of Tissemsilt, Tissemsilt, Algeria
E-mail: adellal@yahoo.fr

John R. Graef
Department of Mathematics, University of Tennessee at Chattanooga Chattanooga, TN 37403-2504, USA
E-mail: John-Graef@utc.edu

Abdelghani Ouahab
Mailing address: Laboratory of Mathematics, University of Sidi Bel Abbès, Sidi Bel Abbès, Algeria
E-mail: agh_ouahab@yahoo.fr

Abstract
In this paper we study a class of Lorentz invariant nonlinear field equations in several space dimensions. The main purpose is to obtain soliton-like solutions with $p(x)$ variable as exponent soliton. The fields are characterized by a topological invariant, which we call the charge. We prove the existence of a static solution which minimizes the energy among the configurations with nontrivial charge. Moreover, under some symmetry assumptions, we prove the existence of infinitely many solutions, which are constrained minima of the energy.

1 Introduction

The study of partial differential equation with $p(x)-$growth condition has received more and more attention in recent years. The specific attention accorded to such kinds problems is due to applications in mathematical physics. More precisely, such an equation is used in electrorheological fluid [5] and in elastic mechanics . They also have wide applications in different research fields.
The Lorentz invariant Lagrangian density proposed in [1] has the form

$$\alpha(\rho, s) = a\rho + b|\rho|^{\frac{s(\cdot)}{2}},\ a \geq 0,\ b > 0,\ s(0) > n. \tag{1.1}$$

$$s : \mathbb{R}^{n+1} \to \mathbb{R}.$$

We shall consider Lagrangian densities of the form

$$\mathcal{L}(\psi, \rho) = -\frac{1}{2}\alpha(\rho, s) - V(\psi), \tag{1.2}$$

in paper [1] we e proof an existence analysis of the finite-energy static solutions in more than one space dimension for a class of Lagrangian densities $\mathcal{L}$ which include the study in the paper [2] with variable exponents.

Key Words and Phrases: *Soliton, variational calculus, variable exponents, splitting lemma and compactness properties related to symmetry*

2 Main results

The aim of this paper is to carry out the existence of infinitely many solutions, which are constrained minima of the energy. More precisely, for every $N \in \mathbb{N}$ there exists a solution of charge N, in more than one space dimension for a class of Lagrangian densities $\mathcal{L}$ in the form of (1.1)-(1.2) with variable exponents as soliton.

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Principe de continuation de Manasevich et Mawhin pour des équations diffrentielles impulsives

Samia YOUCEFI*
P. O. Box 89, 22000 Sidi Bel Abbes: Ecole superieur en informatique, Sidi Bel Abbes, Algérie
E-mail: sa.youcefi@esi-sba.dz

Johnny Henderson
Texas 76798-7328 USA: Department of Mathematics, Baylor University, Waco, Texas USA
E-mail: Johnny Henderson@baylor.edu

Abdelghani Ouahab
P. O. Box 89, 22000 Sidi Bel Abbes: Department of Mathematics, Université Djillali Liabes, Sidi Bel Abbes, Algérie
E-mail: agh ouahab@yahoo.fr

Abstract
Dans ce travail on établit l'existence de solutions pour un problème diffrentiel impulsif avec phi-Laplacien et des conditions aux bords périodiques en utilisant le degré de coïncidence de Mawhin. Le résultat principal de cette étude est prouvé par le théorème de continuation de Manasevich et Mawhin. Le degré de coïncidence introduit par J.Mawhin en 1970 est un outil topologique très important pour prouver l'existence de solutions d'une grande variété de problmes aux limites non linaires. De nombreux chercheurs l'ont utilisé pour leurs investigations en particulier pour des équations différentielles avec conditions périodiques. L'objectif principal de la théorie du degré de coïncidence est de transformer le problème étudié en un problème équivalent de la forme

$$Lx = Nx$$

où L est un oprateur de Fredholm linaire pas nécessairement inversible et N est une perturbation non linéaire et prouver l'existence de solutions en utilisant la théorie du degré de Leray-Schauder.

1 Introduction

On étudie l'existence de solutions du problème suivant

$$(\phi(y'(t)))' = f(t, y(t), y'(t)), t \in J := [0, b], \quad t \neq t_k, \quad k = 1, \dots, m, \tag{1.1}$$

$$y(t_k^+) - y(t_k^-) = I_k(y(t_k^-)), \quad t = t_k, \quad k = 1, \dots, m, \tag{1.2}$$

$$y'(t_k^+) - y'(t_k^-) = \overline{I}_k(y(t_k^-)), \quad t = t_k, \quad k = 1, \dots, m, \tag{1.3}$$

$$y(0) = y(b), \quad y'(0) = y'(b), \tag{1.4}$$

où $0 < t_1 < t_2 < \cdots < t_m < b$, $f : [0, b] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$ est une fonction Carathéodory, $I_k, \overline{I}_k \in C(\mathbb{R}^n, \mathbb{R}^n)$ et $\phi : \mathbb{R}^n \to \mathbb{R}^n$ est un opérateur homeomorphisme monotone.

Key Words and Phrases: $\phi-$*Laplacien, degré de coïncidence, problème impulsif, conditions périodiques*

2 Main results

Theorem 2.1. *Supposons que Ω est un ensemble ouvert born dans $PC([0,b],\mathbb{R}^n)$ telles que les hypothses suivantes sont satisfaites:*

(H_1) *Pour tout $\lambda \in (0,1)$ le problème*

$$\begin{cases} (\phi(y'))' = \lambda f(t,y,y'), \\ \triangle y(t_k) = \lambda I_k(y(t_k)), \\ \triangle y'(t_k) = \lambda \overline{I}_k(y(t_k)), \\ y(0) = y(b), y'(0) = y'(b), \end{cases} \tag{2.1}$$

n'admet pas de solutions dans $\partial\Omega$.

(H_2) *l'équation*

$$\begin{array}{rcl} G_{c,d}(a) & := & \frac{1}{b}[\sum_{k=1}^{m} I_k(L_{k-1}(h)(t_k)) + \int_0^b \phi^{-1}[d \\ & + & \sum_{k=1}^{m} \overline{I}_k(l_{k-1}(h)(t_k))) + \int_0^s f(s,a,0)ds]dt] = 0, \end{array} \tag{2.2}$$

où $h = f(t,a,0)$ and $L_0(h)(t) = c + \int_0^t \phi^{-1}[(\phi(d) + \int_0^s f(\tau,a,0)d\tau]ds,\ t \in [0,t_1]$ n'admet pas de solutions dans $\partial\Omega \cap \mathbb{R}^n$.

(H_3) *le degré de Brouwer*

$$d_B[G, \Omega \cap \mathbb{R}^n, 0] \neq 0.$$

Alors le problème (1.1)-(1.4) admet une solution dans Ω.

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Generalization of Coincidence points theorem

Sarra Benarab*

Mailing address: Laboratoire de Mathèmatiques Appliquèes et de Modèlisation "LMAM", 08 May 1945 University, Guelma, Algeria
E-mail: benarab.sarraa@gmail.com

Wassim Merchela

Mailing address: Laboratoire de Mathèmatiques Appliquèes et de Modèlisation "LMAM", 08 May 1945 University, Guelma, Algeria
E-mail: merchela.wassim@gmail.com

Abstract
We consider the problem of coincidence point for mappings $\psi, \varphi : X \to Y$, i.e. the problem of existence of a solution for the equation $\psi(x) = \varphi(x)$, $x \in X$. In which the space X is a metric space with $\infty-$metric ρ (a metric with possibly infinite value) and Y a space with $\infty-$distance d satisfying the identity axiom. We provide conditions of the existence of coincidence points in terms of a covering set for the mapping ψ and a Lipschitz set for the mapping φ in the space $X \times Y$.

1 Introduction

Theorem on the existence of the coincidence point of the covering and Lipschitz mappings, acting in metric spaces, obtained A. V. Arutyunov in [1]. In works [2,3], the concept of covering was extended to spaces with different generalized metrics (distances) and obtained on such space generalizations of theorems of coincidence points. In [4], a statement about the existence of the coincidence point of the covering and Lipschitz mappings acting from a metric space into a set with a distance satisfying only axiom of identity was obtained. Here we assume that the metric and the distance in the considered spaces can take the value ∞; and weaken assumptions on covering and Lipschitz properties of mappings. Also, authors in [5] by using these assumptions formulated conditions ensuring the existence of solutions $x \in X$ to operator equations of form $F(x, x) = y$, $y \in Y$, with a mapping $F : X \times X \to Y$.

Key Words and Phrases: *metric, distance, coincidence point of two mappings, covering mapping*

2 Main results

Let X be metric space with metric $\rho : X \times X \to \overline{\mathbb{R}}_+$, where $\overline{\mathbb{R}}_+ = [0, +\infty]$. $B_X(x_0, r) = \{x \in X : \rho(x_0, x) \leq r\}$ — closed ball in X centered at a point $x_0 \in X$ of a radius $r \in \overline{\mathbb{R}}_+$. (naturally, we assume that $B_X(x_0, +\infty) = X$ for any $x_0 \in X$).
Also, we suppose that a set $Y \neq \emptyset$ is given, on which a distance is defined, which is a mapping $d : Y \times Y \to \overline{\mathbb{R}}_+$ satisfying only the following condition

$$\forall y_1, y_2 \in Y \quad d(y_1, y_2) = 0 \quad \Leftrightarrow \quad y_1 = y_2, \tag{2.1}$$

it is important that the mapping d may not possess the other properties of metrics. In the space Y we define the notion of convergence of a sequence $y_i \subset Y$ to an element $y \in Y$ as $i \to \infty$ by the relation

$$y_i \to y \Leftrightarrow d(y, y_i) \to 0.$$

We observe that under such convergence, the limit y is not necessary unique and a symmetric scalar sequence $d(y_i, y)$ not necessary converge to zero. Denote by $\operatorname{Lim} y_i = \{y | \, y_i \to y\}$ the set of all its limits.

Let us formulate the definition of the "weakened" closedness property, which, in contrast to closedness, is connected in the usual way with the continuity property of the mapping.

Definition 2.1. Mapping $f : X \to Y$ is *d-closed in point $x \in X$*, if from convergence to x sequence $\{x_i\} \subset X$ and exists $\operatorname{Lim} f(x_i) \neq \emptyset$ implies the inclusion $f(x) \in \operatorname{Lim} f(x_i)$. Mapping, d-closed at all points, we call *d-closed.*

It is obvious that a mapping that is continuous at a certain point is d-closed at this point. The reverse is not true.
We define weakened properties of covering and Lipschitz mapping $f : X \to Y$. Given a set $U \subset X$ and numbers $\alpha > 0$, $\beta \geq 0$. We define sets:

$$\begin{aligned} \operatorname{Cov}_\alpha[f; U] &:= \big\{(x, y) \in X \times Y \,| \\ &\qquad \exists u \in U \;\, f(u) = y, \;\, \rho(u, x) \leq \alpha^{-1} d_Y(y, f(x)), \;\, \rho(u, x) < \infty\big\}; \\ \operatorname{Lip}_\beta[f; U] &:= \big\{(x, y) \in X \times Y \,|\, \forall u \in U \;\, f(u) = y \Rightarrow d_Y(y, f(x)) \leq \beta \rho(u, x)\big\}; \end{aligned}$$

Given mappings $\psi, \varphi : X \to Y$. We recall that the coincidence point of mappings ψ, φ is the solution of the equation

$$\psi(x) = \varphi(x) \tag{2.2}$$

with an unknown $x \in X$. We formulate a statement on solvability of equation (2.2).

Theorem 2.1. *Let a metric space X be complete, and suppose that we are given $\alpha > \beta \geq 0$, $\varepsilon > 0$, $x_0 \in X$ such that $d\big(\varphi(x_0), \psi(x_0)\big) < \infty$. We define:*

$$R = (\alpha - \beta)^{-1} d\big(\varphi(x_0), \psi(x_0)\big), \quad U = B_X(x_0, R). \tag{2.3}$$

Assume that for each $x \in U$ the embeddings hold:

$$(x, \psi(x)) \in \operatorname{Lip}_\beta[\varphi; U], \quad (x, \varphi(x)) \in \operatorname{Cov}_\alpha[\psi; X];$$

and on the ball U the mapping ψ is closed, and the mapping φ — continuous. Then in

Remark 2.1. In theorem 2.1, the condition that the mapping $\psi : X \to Y$ is closed can be replaced by the less condition d-closed, but additionally we assume that

$$\forall \{x_i\} \subset U \ \ \forall x \in U \ \ \big(x_i \to x \ \text{ and } \ \varphi(x) \in \operatorname{Lim} \psi(x_i) \big) \ \Rightarrow \ \varphi(x) = \psi(x).$$

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Form of solutions of a higher-order system of difference equations

Imane Dekkar*

LMAM Laboratory and Department of Mathematics, Mohamed Seddik Ben Yahia University, Jijel, Algeria.
E-mail: imane_dek@hotmail.fr, dekkar.imane@univ-jijel.dz

Nouressadat Touafek

LMAM Laboratory and Department of Mathematics, Mohamed Seddik Ben Yahia University, Jijel, Algeria.
E-mail: ntouafek@gmail.com, touafek@univ-jijel.dz

Ibrahim Yalcinkaya

Department of Mathematics and Computer Sciences, faculty of Science, Necmettin Erbakan University, 42090, Meram Yeni Yol, Konya, Turkey.
E-mail: iyalcinkaya@erbakan.edu.tr

Abstract
In this work, we are interested to study a higher-order system of difference equations. Mainly, we give the closed-form of the solutions of this system and discuss the existence of periodic solutions, and finally, we determine the forbidden set of initial values.

1 Introduction

The study of difference equations has gained prominence in the recent decades and lot of works were been devoted to this subject by researchers, see for instance [1]-[3] and the references cited therein. Difference equations is a rapidly growing field and it has a great number of interesting applications in different disciplines. Obviously, one of the important questions to treat when studying nonlinear differences equations is to give the explicit formulas of their solutions, and this will be the main topic to investigate in the present work which treat the following system of difference equations

$$x_{n+1} = \frac{y_n x_{n-2} y_{n-3}}{a y_{n-3} x_{n-2} + b y_n y_{n-3} + c y_n x_{n-2}},\ y_{n+1} = \frac{y_{n-2} x_{n-1}}{\alpha y_{n-2} + \beta x_{n-1}},\ n = 0, 1, \cdots, \tag{1.1}$$

where the initial values x_{-2}, x_{-1}, x_0, y_{-3}, y_{-2}, y_{-1}, y_0 are nonzero complex numbers and the parameters a, b, c, α, β are real numbers.

Key Words and Phrases: *Rational difference equation, Form of solutions, Periodicity, Forbidden set.*

2 Main results

In the resolution of the System (1.1), we distinguish different cases: When $\alpha = 0$; when $\alpha \neq 0$ with $c\alpha - b\beta = 0$, $a\alpha + \beta + b \neq 0$ and $a\alpha + b \neq 0$; when $\alpha \neq 0$ with $c\alpha - b\beta \neq 0$ and $a\alpha + \beta + b = 0$; and finally, when $\alpha \neq 0$ with $(c\alpha - b\beta)(a\alpha + \beta + b) \neq 0$. So, from the series of theorems in which we give the form of solutions of System (1.1), we present for example the following theorem in the case $\alpha = 0$.

Theorem 2.1. *Assume that $\alpha = 0$. Then, for all $n = 0, 1, \cdots$, the solution of System (1.1) is given by*

$$y_{3n-2} = y_{-2}\beta^{-n},\ y_{3n-1} = y_{-1}\beta^{-n},\ y_{3n} = y_0\beta^{-n},$$

$$x_{3n+1} = \frac{y_0x_{-2}y_{-3}}{(ay_{-3}x_{-2} + by_0y_{-3} + cy_0x_{-2})\,b^n + (a\beta + c)x_{-2}y_{-3}\sum_{i=1}^{n}\beta^{n-i}b^{i-1}},\ b \neq \beta,$$

$$x_{3n+1} = \frac{y_0x_{-2}y_{-3}\beta^{1-n}}{\beta y_0(by_{-3} + cx_{-2}) + (a\beta + (a\beta + c)n)x_{-2}y_{-3}},\ b = \beta,$$

$$x_{3n-1} = \frac{x_{-1}y_{-2}}{y_{-2}b^n + (a\beta + c)x_{-1}\sum_{i=1}^{n}\beta^{n-i}b^{i-1}},\ b \neq \beta,$$

$$x_{3n-1} = \frac{x_{-1}y_{-2}\beta^{1-n}}{\beta y_{-2} + (a\beta + c)x_{-1}n},\ b = \beta,$$

$$x_{3n} = \frac{x_0y_{-1}}{y_{-1}b^n + (a\beta + c)x_0\sum_{i=1}^{n}\beta^{n-i}b^{i-1}},\ b \neq \beta,$$

$$x_{3n} = \frac{x_0y_{-1}\beta^{1-n}}{\beta y_{-1} + (a\beta + c)x_0n},\ b = \beta.$$

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Existence Results for a System of Random Difference Equations

Mohammed Asseddik SLIMANI*

Mailing address: Department of Mathematics and Informatics, University Center of Naama, Algeria
E-mail: slimani.m@cuniv-naama.dz

Abdelghani OUAHAB

Mailing address: Department of Mathematics and Informatics, University of Sidi Bel-Abbès, Algeria
E-mail: agh_ouahab@yahoo.fr

Abstract
In this work we present some existences and uniqueness results of solution for random difference equations with initial and boundary conditions by using some random fixed point theorems in generalized Banach spaces.

1 Introduction

In this work, we are interested in investigating the nonlinear initial value problems of random difference equations of first order for different aspects of the solutions under suitable conditions.
We study the following systems :

$$\begin{cases} \Delta x(\omega, k) & = f(k, x(\omega, k), y(\omega, k), \omega), \ k \in \mathbb{N}(0, b), \\ \Delta y(\omega, k) & = g(k, x(\omega, k), y(\omega, k), \omega), \ k \in \mathbb{N}(0, b), \\ x(\omega, 0) & = x_0(\omega), \ \omega \in \Omega, \\ y(\omega, 0) & = y_0(\omega), \ \omega \in \Omega, \end{cases} \tag{1.1}$$

where $f, g : \mathbb{N}(0, b) \times \mathbb{R} \times \mathbb{R} \times \Omega \to \mathbb{R}$, $(\Omega, \mathcal{A})$ is measurable space and $x_0, y_0 : \Omega \to \mathbb{R}$ are a random variable.

2 Main results

Let us introduce the following hypothesis:

(H_1) $f, g : \mathbb{N}(0, b) \times \mathbb{R} \times \mathbb{R} \times \Omega \to \mathbb{R}$ be two Carathéodory functions,

(H_2) There exist random variables $p_1, p_2, p_3, p_4 : \Omega \to \mathbb{R}$ such that

Key Words and Phrases: *generalized metric space, random fixed point, random difference equations*

$$\begin{cases} |f(k,x,y,\omega) - f(k,\overline{x},\overline{y},\omega)| \leq p_1(\omega)|x-\overline{x}| + p_2(\omega)|y-\overline{y}| \\ |g(k,x,y,\omega) - g(k,\overline{x},\overline{y},\omega)| \leq p_3(\omega)|x-\overline{x}| + p_4(\omega)|y-\overline{y}|, \end{cases}$$

for all $x, y, \overline{x}, \overline{y} \in \mathbb{R}$.

(H_3) There exist $a_1, a_2 : \Omega \to \mathbb{R}$ such that

$$\begin{cases} |f(k,x,y,\omega)| \leq a_1(k,\omega)(|x|+|y|), \ k \in \mathbb{N}(0; b-1), \ x, y \in \mathbb{R}, \\ |g(k,x,y,\omega)| \leq a_2(k,\omega)(|x|+|y|), \ k \in \mathbb{N}(0; b-1), \ x, y \in \mathbb{R}. \end{cases}$$

Theorem 2.1. *Assume that (H_1) and (H_2) are satisfied and the matrix*

$$M(\omega) = \begin{pmatrix} p_1(\omega) & p_2(\omega) \\ p_3(\omega) & p_4(\omega) \end{pmatrix} \in M_{2\times 2}(\Omega).$$

If $M(\omega)$ converges to zero. Then the problem (1.1) has unique random solution.

Theorem 2.2. *Assume that (H_1) and (H_3) are satisfied.*
Then the problem (1.1) has at least one random solution.

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System of fractional boundary value problem with $p-$laplacian and advanced arguments

MAHDJOUBA AMINA*

Mailing address: Laboratory of Mathematics, Sidi-Bel-Abbès University, PoBox 89, 22000 Sidi-Bel-Abbès, Algeria
E-mail: math_amina@yahoo.fr

J.J. Nieto and A. Ouahab

Mailing address: Laboratory of Mathematics, Sidi-Bel-Abbès University, PoBox 89, 22000 Sidi-Bel-Abbès, Algeria
E-mail: juanjose.nieto.roig@usc.es

A. Ouahab

Mailing address: Departamento de Estatística, Análise Matemática, e Optimización, Instituto de Matemáticas, Universidade de Santiago de Compostela, Santiago de Compostela, 15782, Spain
E-mail: agh_ouahab@yahoo.fr

Abstract
In this paper, we discuses the existence and multiplicity of positive solutions for system of fractional differential equations with boundary condition and advanced arguments. The existence result proved via Lear-Schauder's fixed point theorem type in vector Banach space. Further, by using new fixed point theorem order Banach space, we study the multiplicity of positive. Finally, some example are given to illustrate the result.

1 Introduction

Fractional calculus and differential equations have now proved to be important tools modeling many real world phenomena like chemistry and physics. For the description of hereditary properties of fractional calculus.
Recently, there are some papers deal with the existence and multiplicity of solution (or positive solution) of nonlinear initial fractional differential equation by the use of techniques of nonlinear analysis.

In this paper, we are concerned with the existence and multiplicity of positive solutions of the following problem:

Key Words and Phrases: *Fractional differential equations,p-Laplacian operator,Cone, Fixed point theorem, Positive solutions, Multiplicity of solutions*

$$\begin{cases} (\varphi_p(D^{\alpha}_{0+}u(t)))' + a_1(t)f(u(\theta_1(t)), v(\theta_2(t))) = 0, & 0 < t < 1, \\ (\varphi_{\tilde{p}}(D^{\alpha}_{0+}v(t)))' + a_2(t)g(u(\theta_1(t)), v(\theta_2(t))) = 0, & 0 < t < 1, \\ D^{\alpha}_{0+}u(0) = u(0) = u'(0) = 0, D^{\beta}_{0+}u(1) = \gamma D^{\beta}_{0+}u(\eta), \\ D^{\alpha}_{0+}v(0) = v(0) = v'(0) = 0, \quad D^{\beta}_{0+}v(1) = \gamma D^{\beta}_{0+}v(\eta), \end{cases} \tag{1.1}$$

where $\eta \in (0,1), \gamma \in (0, \frac{1}{\eta^{\alpha-\beta-1}}), D^{\alpha}_{0+}, D^{\beta}_{0+}$, are the standard Riemann-Liouville fractional derivatives with $\alpha \in (2,3), \beta \in (1,2)$ such that $\alpha \geq \beta + 1$, p-Laplacian operator is defined as $\varphi_p(s) = |s|^{p-2}s,\ p > 1$, and the functions $f, g \in C(\mathbb{R}^2, \mathbb{R})$.

For establish the existence and multiple positive solutions of problem (1.1), let us list the following assumptions:

(H_1) $a_i \in L^1[0,1]$ is nonnegative and $a_i(t) \not\equiv 0$ on any subinterval of $[0,1]$, for $i = 1, 2$.

(H_2) The advanced argument $\theta \in C((0,1),(0,1])$ and $0 \leq \theta(t) \leq 1, \forall t \in (0,1)$.

This work is organized as follows: In section 2, we introduce all the background material used in this paper such as fractional calculus analysis and some results from fixed point theory. In Sections 3, 4, the existence and multiplicity results of solutions for system of fractional p-Laplace differential equation (1.1) is discussed by using the fixed point theorems in generalized Banach space. We end the paper by an example to illustrate our main results.

2 Main results

Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}^+$ be the set of nonnegative real numbers. Denote by $C([0,1])$ the Banach space of all continuous functions from $[0,1]$ into $\mathbb{R}$ with the norm

$$\|u\| = \max\{|u(t)| : t \in [0,1]\}$$

Define the cone P in $C([0,1]^2)$ as $P = \{u \in C([0,1]) : u(t) \geq 0, t \in [0,1]\}$. Let $q > 1$ and $\tilde{q} > 1$ satisfy the relation $\frac{1}{p} + \frac{1}{q} = 1, \frac{1}{\tilde{p}} + \frac{1}{\tilde{q}} = 1$, where $p, \tilde{p}$ are given by (1.1).
To prove the existence of solutions to (1.1), we need the following auxiliary Lemma.

Lemma 2.1. *Given $h_1, h_2 \in C[0,1], \eta \in (0,1), \gamma \in (0, \frac{1}{\eta^{\alpha-\beta-1}})$ and $\alpha \geq \beta + 1$, the unique solution of C boundary value problem for a coupled system*

$$(\varphi_p(D^{\alpha}_{0+}u(t)))' + h_1(t) = 0, \quad 0 < t < 1, \tag{2.1}$$

$$(\varphi_{\tilde{p}}(D^{\alpha}_{0+}v(t)))' + h_2(t) = 0, \quad 0 < t < 1, \tag{2.2}$$

$$D^{\alpha}_{0+}u(0) = u(0) = u'(0) = 0, \quad D^{\beta}_{0+}u(1) = \gamma D^{\beta}_{0+}u(\eta), \tag{2.3}$$

$$D^{\alpha}_{0+}v(0) = v(0) = v'(0) = 0, \quad D^{\beta}_{0+}v(1) = \gamma D^{\beta}_{0+}v(\eta), \tag{2.4}$$

are

$$\begin{aligned} u(t) &= {}_0^1 G_1(t,s)\varphi_q\left({}_0^s h_1(\tau)d\tau\right)ds \\ &+\tfrac{\gamma t^{\alpha-1}}{1-\gamma\eta^{\alpha-\beta-1}}{}_0^1 G_2(\eta,s)\varphi_q\left({}_0^s h_1(\tau)d\tau\right)ds, \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} v(t) &= {}_0^1 G_1(t,s)\varphi_{\tilde q}\left({}_0^s h_2(\tau)d\tau\right)ds \\ &+\tfrac{\gamma t^{\alpha-1}}{1-\gamma\eta^{\alpha-\beta-1}}{}_0^1 G_2(\eta,s)\varphi_{\tilde q}\left({}_0^s h_2(\tau)d\tau\right)ds, \end{aligned} \tag{2.6}$$

where

$$G_1(t,s)=\begin{cases} \frac{t^{\alpha-1}(1-s)^{\alpha-\beta-1}-(t-s)^{\alpha-1}}{\Gamma(\alpha)} & 0\le s\le t\le 1,\\ \frac{t^{\alpha-1}(1-s)^{\alpha-\beta-1}}{\Gamma(\alpha)} & 0\le t\le s\le 1,\end{cases}$$

$$G_2(\eta,s)=\begin{cases} \frac{[\eta(1-s)]^{\alpha-\beta-1}-(\eta-s)^{\alpha-\beta-1}}{\Gamma(\alpha)} & 0\le s\le \eta\le 1,\\ \frac{[\eta(1-s)]^{\alpha-\beta-1}}{\Gamma(\alpha)} & 0\le \eta\le s\le 1.\end{cases}$$

Theorem 2.1. *Assume (H_1)-(H_2), and that the following condition holds:*

(H_3) There exist functions $p, q, h, \breve{p}, \breve{q}$, and $\bar{h} \in L^1([0,1],\mathbb{R}^+)$ and constants $\alpha_1, \alpha_2, \alpha_3$, and $\alpha_4 \in [0,1)$ such that

$$|f(u,v)| \le p(t)|u|^{\alpha_1} + q(t)|v|^{\alpha_2} + h(t) \quad \text{for each } t\in[0,1] \text{ and } u,v\in\mathbb{R}$$

and

$$|g(u,v)| \le \breve{p}(t)|u|^{\alpha_3} + \breve{q}(t)|v|^{\alpha_4} + \breve{h}(t) \quad \text{for each } t\in[0,1] \text{ and } u,v\in\mathbb{R}.$$

If $\alpha_1 p, \alpha_2 p, \alpha_3 q$, and $\alpha_4 q \in [0,1)$. Then the system (1.1) has at least one solution.

Theorem 2.2. *Assume that there exist $\alpha_i, \beta_i > 0$ with $\alpha_i \neq \beta_i, i = 1,2$, such that*

$$\begin{array}{lcl} B\Gamma_1^{q-1} \le \alpha_1 & , & A\gamma_1^{q-1} \ge \beta_1, \\ B\Gamma_2^{q-1} \le \alpha_2 & , & A\gamma_2^{q-1} \ge \beta_2. \end{array} \tag{2.7}$$

Then (1.1) has a positive solution $u = (u_1, u_2)$ with $r_i \le \|u_i\| \le R_i, i = 1,2$, where $r_i = \min\{\alpha_i,\beta_i\}$, $R_i = \max\{\alpha_i,\beta_i\}$. Moreover, corresponding orbit of u is included in the rectangle $[\rho r_1, R_1] \times [\rho r_2, R_2]$.

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Growth and oscillation of solutions of linear differential equations in a punctured disc

Said Mazouz*

Mailing address: Department of mathematics, Abdelhamid Ibn Badis University, Mostaganem, Algeria
E-mail: said.mazouz.etu@univ-mosta.dz

Saada Hamouda

Mailing address: Department of science and technology, Abdelhamid Ibn Badis University, Mostaganem, Algeria
E-mail: saada.hamouda@univ-mosta.dz

Abstract
In this paper, we investigate the local growth and oscillation, near the singular point $z = 0$, of solutions to the differential equation

$$f'' + \left(A(z)\exp\left\{\frac{a}{z^n}\right\} + A_0(z)\right)f' + \left(B(z)\exp\left\{\frac{b}{z^n}\right\} + B_0(z)\right)f = H(z),$$

where $A(z), A_0(z), B(z), B_0(z), H(z)$ are analytic functions in

$$D(0,R) = \{z \in \mathbb{C} : 0 < |z| < R\}$$

and a, b are non-zero complex constants.

1 Introduction

The idea to study the growth of solutions of the linear differential equations near a finite singular point by using the Nevanlinna theory has began by the paper [2]; then after some publications have followed, see [1, 3]; (for the fundamental results, the definitions and the standard notations of the Nevanlinna theory see [5]). The principal tools used in these investigations is the estimates of the logarithmic derivative $\left|\frac{f^{(k)}(z)}{f(z)}\right|$ for a meromorphic function f in $\overline{\mathbb{C}} \setminus \{z_0\}$, $\left(\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}\right)$. A question was asked in [2, 3] about if we can get similar estimates near z_0 of $\left|\frac{f^{(k)}(z)}{f(z)}\right|$ where f is a meromorphic function in a region of the form $D_{z_0}(0,R) = \{z \in \mathbb{C} : 0 < |z - z_0| < R\}$. This question was answered in [4]. In this work, we will give some applications of these estimates on a punctured disc.

Key Words and Phrases: *Linear differential equations, growth and oscillation of solutions, finite singular point, Nevanlinna theory.*

2 Main results

Theorem 2.1. *Let $A(z) \not\equiv 0, A_0(z), B(z) \not\equiv 0, B_0(z), F(z)$ be analytic functions in $D(0,R)$ such that*

$$\max\{\sigma(A_0,0), \sigma(B_0,0), \sigma(A,0), \sigma(B,0), \sigma(F,0)\} < n, \ n \in \mathbb{N} \setminus \{0\};$$

let a, b be complex constants such that $ab \neq 0$ and $a = cb, \ c < 0$. Then, every solution $f(z) \not\equiv 0$ of the differential equation

$$f'' + \left(A(z)\exp\left\{\frac{a}{z^n}\right\} + A_0(z)\right)f' + \left(B(z)\exp\left\{\frac{b}{z^n}\right\} + B_0(z)\right)f = F(z), \qquad (2.1)$$

satisfies $\sigma(f,0) = \infty$. Further, if $F(z) \not\equiv 0$, we have

$$\bar{\lambda}(f,0) = \lambda(f,0) = \sigma(f,0) = +\infty, \ \bar{\lambda}_2(f,0) = \lambda_2(f,0) = \sigma_2(f,0) \leq n.$$

Theorem 2.2. *Let $A(z) \not\equiv 0, B(z) \not\equiv 0, F(z) \not\equiv 0$ be analytic functions in $D(0,R)$ such that $\max\{\rho(A,0), \rho(B,0), \rho(F,0)\} < n$, $n \in \mathbb{N} \setminus \{0\}$ and $P(z) \not\equiv 0, Q(z) \not\equiv 0$ are polynomials. Let a, b be complex numbers such that $ab \neq 0, a \neq b$. Then, every solution f of the differential equations*

$$f'' + P(z)\exp\{\frac{a}{z^n}\}f' + B(z)\exp\{\frac{b}{z^n}\}f = F(z)\exp\{\frac{a}{z^n}\}, \qquad (2.2)$$

$$f'' + A(z)\exp\{\frac{a}{z^n}\}f' + Q(z)\exp\{\frac{b}{z^n}\}f = F(z)\exp\{\frac{b}{z^n}\} \qquad (2.3)$$

satisfies

$$\bar{\lambda}(f,0) = \lambda(f,0) = \sigma(f,0) = +\infty, \ \bar{\lambda}_2(f,0) = \lambda_2(f,0) = \sigma_2(f,0) \leq n$$

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Study the existence for multi-term boundary value problem with fixed point theorems in Banach space

Elhabib Hadjadj*
Mailing address: Laboratoire de la Théorie des Opérateurs et EDP, University of Eloued, Eloued, Algeria
E-mail: hadjadj-elhabib@univ-eloued.dz

Safia Meftah
Mailing address: Laboratoire de la Théorie des Opérateurs et EDP, University of Eloued, Eloued, Algeria
E-mail: safia-meftah@ univ-eloued.dz

Abdallah Bensayah
Mailing address: Laboratoire de Mathématiques Appliquées, Université Kasdi Merbah, BP511, Ouargla,30000, Algeria
E-mail: bensayahabd@gmail.com

Abstract
Herein, we study the existence for convex multi-term problem by using some notions and properties on set-valued maps together with classical fixed point theorems, in particular we investigated the convex multi-term by Leray-Schauder theorem. Finally, some examples are given to illustrate our main results.

1 Introduction

Fractional differential inclusions generalize ordinary differential inclusions to arbitrary non-integer orders. They appear naturally in various fields like physics, engineering, biophysics, chemistry, biology, economics, control theory, etc.
Recently, many works have been published on fractional differential inclusions by authors applying fixed point theory to prove some existence theorems. A lot of articles have been published in this direction. We are concerned with the study some results of existence to the following semilinear fractional differential inclusions for multi-term boundary value problem

$$\begin{cases} \boldsymbol{D}^{\rho}_{0^+}\boldsymbol{u}(\boldsymbol{t}) \in \mathcal{H}(\boldsymbol{t}, \boldsymbol{u}(\boldsymbol{t}), \boldsymbol{I}^{\delta_1}_{0^+}\boldsymbol{u}(\boldsymbol{t}), \dots, \boldsymbol{I}^{\delta_n}_{0^+}\boldsymbol{u}(\boldsymbol{t})), \quad \boldsymbol{t} \in [0,1], \\ \boldsymbol{u}(0) = 0, \\ \boldsymbol{D}^{\nu}_{0^+}\boldsymbol{u}(1) = p\boldsymbol{I}^{\mu}_{0^+} g(1, \boldsymbol{u}(1)), \end{cases} \tag{1.1}$$

with $1 < \rho < 2$, $p, \mu \geq 0$, $0 < \delta_1 < \cdots < \delta_n$, $g : [0,1] \times \mathbb{R} \to \mathbb{R}$ be a continuous function, $\mathcal{H} : [0,1] \times \mathbb{R}^{n+1} \to \mathcal{P}(\mathbb{R})$ is a set-valued function, $\boldsymbol{D}^{\rho}_{0^+}$ Riemann-Liouville fractional derivative of order ρ and , $\boldsymbol{I}^{\delta_1}_{0^+}, \dots, \boldsymbol{I}^{\delta_n}_{0^+}$ are Riemann-Liouville fractional integral of order $\delta_1, \dots, \delta_n$ respectively, and $\mathcal{P}(\mathbb{R})$ will be defined later.

Key Words and Phrases: ...., ..., ..., ...

2 Main results

We assume that the multivalued $\mathcal{H}$ realize these three conditions

(H1) $\mathcal{H} : [0,1] \times \mathbb{R}^{n+1} \to \mathcal{P}_{cp,c}(\mathbb{R})$ is a L^1-Carathéodory set-valued map.

(H2) There are $m_1, m_2 \in L^\infty([0,1], \mathbb{R}^+)$ and $\psi, \varphi_1, \ldots, \varphi_n : [0,+\infty) \to (0,+\infty)$ continuous, nondecreasing such that

$$\begin{aligned}\|\mathcal{H}(\boldsymbol{t}, x, x_1, \ldots, x_n)\| &= \sup\{|v| : v \in \mathcal{H}(\boldsymbol{t}, x, x_1, \ldots, x_n)\} \\ &\leq m_1(\boldsymbol{t})\psi(|x|) + m_2(\boldsymbol{t}) \sum_{j=1}^{n} \varphi_j(|x_j|),\end{aligned}$$

(H3) There exist $L > 0$ such that

$$\frac{L}{P_L + \left(\psi(L)\|m_1\|_{L^\infty} + \sum_{j=1}^{n} \varphi_j\left(\frac{L}{\Gamma(\delta_j+1)}\right)\|m_2\|_{L^\infty}\right)\mathcal{Q}} > 1,$$

where

$$\mathcal{Q} = \frac{1}{\Gamma(\rho+1)} + \frac{1}{(\rho-\nu)\Gamma(\rho)},$$

and

$$P_L = \frac{\alpha_L p \Gamma(\rho-\nu)}{\Gamma(\mu+1)\Gamma(\rho)}, \tag{2.1}$$

with
$\alpha_L = \sup\{|g(\boldsymbol{t}, x)|, (\boldsymbol{t}, x) \in [0,1] \times [-L, L]\}$.

Theorem 2.1. *If* $(H1) - (H3)$ *hold. Then there exists a solution for the problem* (1.1).

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On periodic solutions for a first-order differential inclusion type

Aya Bouabsa*

Mailing address: Laboratoire LMPA, Department of mathematics, Jijel University, Jijel, Algeria
E-mail: aya.bouabsa@univ-jijel.dz

Soumia Saidi

Mailing address: Laboratoire LMPA, Department of mathematics, Jijel University, Jijel, Algeria
E-mail: soumiasaidi44@gmail.com

Abstract
We are interested in the current work in the study of global stability solutions to a class of perturbed differential inclusions. We show the existence of at least one global solution for Lipschitz single-valued perturbation of a differential inclusion governed by time-dependent subdifferential operators, we also establish the uniqueness, the periodicity, and the exponential stability of the solution.

1 Introduction

In this paper we are going to study the stability of arbitrary global periodic solution of

$$-\dot{x}(t) \in \partial\varphi(t, x(t)) + f(t, x(t)) \quad \text{for all } t \in \mathbb{R}. \tag{1.1}$$

Such that $\partial\varphi(t,\cdot)$ is the subdifferential of a time-dependent proper lower semicontinuous (lsc) convex function $\varphi(t,\cdot)$ of $\mathbb{R}^n$ into $[0,+\infty]$, where $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a Lipschitz single-valued function satisfies such conditions. our study is mainly motivated by [5], the authors are established the existence and the stability of a periodic global solution when the normal cone of r-prox regular moving subset $C(t)$ of $\mathbb{R}^n$ is considered instead of $\partial\varphi(t,\cdot)$. we also used some results from [3] which it confirmed the existence and uniqueness of (1.1) for all t in a compact of $\mathbb{R}$ with a initial condition. The present paper is a new contribution since we deal with subdifferential operator which has a nature different of the normal cone. It is attracting to some applications such as crowd motion model, when the normal cone of moving subset is considered instead of the subdifferential operator. For the proof of our existence theorem, we use some result from [3] and show that the weaker monotonocity is enough to get a global exponentially stable solution, hence the uniqueness. The outline of the paper is as follows, In section 2 we introduce some notation and preliminaries. In section 3 we are going to proof our main result (Theorem (2.3)), which gives conditions for global asymptotic stability of a periodic solution of (1.1).

Key Words and Phrases: *Differential inclusion, stability, solution.*

2 Main results

Let $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ be a map such that
(i) for some $L_f > 0$ and for all $(t, s) \in \mathbb{R} \times \mathbb{R}$, there exists $\eta > 0$ such that, $x, y \in \bar{B}_{\mathbb{R}^n}[0, \eta]$

$$||f(t, x) - f(s, y)|| \leq L_f ||x - y||$$

(ii) for some fixed $\alpha > 0$,

$$\langle f(t, x_1) - f(t, x_2), x_1 - x_2 \rangle \geq \alpha \|x_1 - x_2\|^2 \quad \text{for all} \quad t \in \mathbb{R} \quad x_1, x_2 \in \mathbb{R}^n.$$

Theorem 2.1. *let $\varphi : \mathbb{R} \times \mathbb{R}^n \to [0, +\infty]$ be a map satisfying (H_1)-(H_2). Let $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ be a single-valued function satisfying (i), and there exists $M_f > 0$, such that*

$$||f(t, x)|| \leq M_f \quad \text{a.e. } t \in \mathbb{R}, \quad x \in \cup_{t \in \mathbb{R}} dom\, \varphi(t, \cdot). \tag{2.1}$$

Then, (1.1) *has at least one solution defined on the entire* $\mathbb{R}$.

Theorem 2.2. *Let the conditions of theorem* (2.1) *hold true. Assume that* (ii) *holds. Then,* (1.1) *has a unique solution x defined on $\mathbb{R}$. Furthermore the global solution x is globally exponentially stable.*

Theorem 2.3. *The unique global solution x which comes from theorem* (2.2) *is T-periodic, if both maps $t \mapsto \mathrm{dom}\varphi(t, .)$ and $t \mapsto f(t, x)$ are T-periodic.*

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Multiplicity of solutions for fractional p-Laplacian Problem involving critical exponent

Djamel Abid*

Mailing address: Department of Mathematics and Computer Science , University Laarbi tebessi tebessa, Cheria, Algeria

E-mail: djamelabid0312@gmail.com

Abstract

This paper is devoted to the study of the existence and multiplicity of nontrivial solutions for a p-fractional laplacian equation with homogeneous Dirichlet boundary conditions. Using Nehari Manifold and fibering maps, we obtain multiple solutions for critical cases to the following problem.

$$\begin{cases} (-\Delta)_p^s u(x) = \int_\Omega f(x) u^{p_s^*-2}u + \lambda g(x,u) \text{ in } \Omega, u > 0, \\ \\ u = 0 \text{ on } \mathbb{R}^n \setminus \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ $(n > ps)$, is a bounded smooth domain, $s \in (0,1)$ λ is positive parameter,f : $\Omega \longrightarrow \mathbb{R}^+$ is a positive continuous function $g : \bar{\Omega} \times [0,\infty) \longrightarrow \mathbb{R}$, is continuou homogenous function of degree $r-1$ and $0 < r-1 < p < p_s^*$ where $p_s^* = \frac{pn}{n-ps}$ is the critical Sobolev exponent. Associated to **??** we have

$$J_\lambda(u) = \frac{1}{p}\|u\|^p - \frac{1}{p_s^*}\int_\Omega f(x)|u|^{p_s^*}dx - \lambda \int_\Omega G(x,u)dx \tag{0.1}$$

In order to obtain multiplicity of solutions, we split $\mathcal{N}_{\lambda,\mu}$ into the following three parts

$$\begin{aligned} \mathcal{N}_{\lambda,\mu}^+ &= \{u \in \mathcal{N}_{\lambda,\mu} : \varphi_u''(1) > 0\} = \{u \in X_0 : \varphi_u'(1) = 0 \text{ and } \varphi_u''(1) > 0\}, \\ \mathcal{N}_{\lambda,\mu}^- &= \{u \in \mathcal{N}_{\lambda,\mu} : \varphi_u''(1) < 0\} = \{u \in X_0 : \varphi_u'(1) = 0 \text{ and } \varphi_u''(1) < 0\}, \\ \mathcal{N}_{\lambda,\mu}^0 &= \{u \in \mathcal{N}_{\lambda,\mu} : \varphi_u''(1) = 0\} = \{u \in X_0 : \varphi_u'(1) = 0 \text{ and } \varphi_u''(1) = 0\}. \end{aligned}$$

where $\phi_u(t) = J_\lambda(tu)$.

1 Introduction

Due to the loss of compacity of the embedding of our space X_0 in some L^p spaces we use the Palais-Smale condition for the sequence to converge and then finding our results which is.

Theorem 1.1. *Let $s \in (0,1), n > ps$, and $0 < r < p < p_s^*$ then there exist $\lambda^* > 0$, such that for $\lambda \in (0, \lambda^*)$ our problem has at leat two nontrivial solutions.*

Key Words and Phrases: *Nehari manifold, fibering maps, critical grouth condition, Palais-Smale condition.*

2 Main results

First we prove that the functional energy associated to our problem is coercive on the Nehari manifold, and then using the Palais-Smale condition with tehnics of funcational analysis such as Brezis lieb lemma we obtain our results

Proof. we have hat the funcational J_λ is coercive and bounded from bellow on the Nehari manifold $\mathcal{N}_\lambda$, and by the decmoposition of the Nehari we can prove that $\mathcal{N}_\lambda^0 = \emptyset$ so we have that $\mathcal{N}_\lambda = \mathcal{N}_\lambda^- \cup \mathcal{N}_\lambda^+$, also frome the fibering maps $\phi_u(t) = J_\lambda(tu)$, and $\psi_u(t) = t^{1-r}\phi_u''(t) + \lambda r \int_\Omega G(x,u)dx$ there exist t_1, t_2 such that $t_1 u \in \mathcal{N}_\lambda^+$ and $t_2 u \in \mathcal{N}_\lambda^-$ Also there exist $M = M(p, p_s^*, r, \gamma, |\Omega|)$, where $|g(x,u)| \leq \gamma|u|^r$ such that every Palais-Male sequence $\{u_n\}$ for J_λ at level $c < \frac{s}{n}(p_s^*\gamma)^{-\frac{n}{sp_s^*}} S_p^{\frac{n}{sp}} - M\lambda^{\frac{p}{P-R-1}}$. where S_p is the fractional Sobolev constant. Has a convergent subsequence,
Our result Using the above we get that there exists two sequences $\{u_n^+\}, \{u_n^-\} \in X_0$, such that

$$J_\lambda(u_n^+) \longrightarrow \inf_{u \in \mathcal{N}_\lambda^+} J_\lambda(u), \quad J_\lambda(u_n^-) \longrightarrow \inf_{u \in \mathcal{N}_\lambda^-} J_\lambda(u).$$

as $n \to \infty$. Using some convergence criteria and the fact that $\mathcal{N}_\lambda^- \cap \mathcal{N}_\lambda^+ = \emptyset$. Then problem **??** has two dintict positive solutions. □

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Existence of solution for differential inclusion in the nonconvex case with delay

Hanane Chouial*
Department of mathematics, LMPA Laboratory University of Jijel
E-mail: Hananechouial@yahoo.com

Mustapha Fateh Yarou
Department of mathematics, LMPA Laboratory University of Jijel
E-mail: mfyarou@yahoo.com

Abstract
Our aim in this paper is to present a reduction method that solves first order functional differential inclusion in the nonconvex case. This approach is based on a discretization of the time interval, a construction of approximate solutions by reducing the problem to a problem without delay and an application of known results in this case. We generalise earlier results, the right hand side of the inclusion has nonconvex values and satisfies a linear growth condition instead to be integrably bounded. The lack of convexity is replaced by the topological properties of decomposable sets, that represents a good alternative in the absence of convexity.

1 Introduction

Let $\tau,\ T \ \geq 0$, be two non-negative real numbers, $\mathcal{C}_0 := \mathcal{C}_{\mathbb{R}^n}([-\tau,0])$ is the Banach space of all continuous mappings from $[-\tau,\ 0]$ to $\mathbb{R}^n$ equipped with the norm of uniform convergence and $F : [0,T] \times \mathcal{C}_0 \rightharpoonup \mathbb{R}^n$ be a set-valued mapping with nonempty closed values. In this work, we study the existence of solutions for the following differential inclusion with delay

$$(\mathcal{DP}) \qquad \begin{cases} \dot{x}(t) \in F(t, \mathcal{T}(t)x) & \text{a.e. } t \in [0,T]; \\ x(t) = \varphi(t) & t \in [-\tau, 0]; \end{cases}$$

where $\varphi \in \mathcal{C}_0$ and $\mathcal{T}(t) : \mathcal{C}_T \longrightarrow \mathcal{C}_0$ defined by $\mathcal{T}(t)x(s) = x(t+s), \forall s \in [-\tau,\ 0\ ]$. In [2], Fryszkowski proved an existence result for $(\mathcal{DP})$ when F is a set-valued mapping with nonconvex values, measurable, integrably bounded and lower semicontinuous in x. The proof of this theorem is based on the construction of a continuous selection for a class of nonconvex set-valued mapping. The existence of such selection is proved in [1]. In [3], Fryszkowski and Gorniewicz proved an existence result for differential inclusion of the form

$$(\mathcal{P}) \qquad \begin{cases} \dot{x}(t) \in F(t, x(t)) & \text{a.e. } \ t \in [0,T]; \\ x(0) = x_0, \end{cases}$$

where F is a set-valued mapping measurable in (t,x) and lower semicontinuous in x with nonconvex values, satisfying a linear growth condition.

Key Words and Phrases: *Lower semicontinuous, nonconvex differential inclusion, reduction, delay, linear growth condition.*

2 Main results

In this section, we begin by the following result for the undelayed problem due to Fryszkowski and Gorniewicz (see [3]).

Theorem 2.1. *Let $G : [0,T] \times \mathbb{R}^n \rightharpoonup \mathbb{R}^n$ be a set-valued mapping with nonempty closed values satisfying*

(i) G is $\mathcal{L} \otimes \mathcal{B}(\mathbb{R}^n)$ measurable;

(ii) for every $t \in [0,T]$, $G(t,\cdot)$ is Lower semicontinuous in $\mathbb{R}^n$;

(iii) there exists an integrable function $\rho : [0,T] \longrightarrow \mathbb{R}^+$ such that

$$|y| \leq (\ 1 + |x|\ )\, \rho(t), \text{ for every } y \in G(t,x) \text{ and } (t,x) \in [0,T] \times \mathbb{R}^n.$$

Then, $\forall\, x_0 \in \mathbb{R}^n$, the problem

$$\begin{cases} \dot{x}(t) \in G(t, x(t)) & a.e. \text{ on } [0,T]; \\ x(0) = x_0; \end{cases} \tag{2.1}$$

admits at least one solution $x : [0,T] \to \mathbb{R}^n$ absolutely continuous on $[0,T]$.

Now, we are able to give the existence result for the delayed problem

Theorem 2.2. *Let $F : [0,T] \times \mathcal{C}_0 \rightharpoonup \mathbb{R}^n$ be a set-valued mapping with nonempty closed values such that*

(i) F is $\mathcal{L} \otimes \mathcal{B}(\mathcal{C}_0)$ measurable;

(ii) for every $t \in [0,T]$, $F(t,\cdot)$ is Lower semicontinuous in $\mathcal{C}_0$;

(iii) for every $(t,\varphi) \in [0,T] \times \mathcal{C}_0$

$$\|F(t,\varphi)\| \leq (1 + |\varphi(0)|)\rho(t).$$

Then, $\forall\, \varphi \in \mathcal{C}_0$, the problem $(\mathcal{DP})$ admits at least one continuous solution $x : [-\tau, T] \to \mathbb{R}^n$, absolutely continuous on $[0,T]$.

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BVC solutions to a class of evolution problems

Soumia Saïdi*

LMPA Laboratory, Department of Mathematics, Mohammed Seddik Benyahia University, Jijel, Algeria
E-mail: soumiasaidi44@gmail.com

Abstract
In this work, we focus on the existence of solutions for a new class of perturbed state-dependent maximal monotone operators (in a real separable Hilbert space). The operators are bounded variation in time, and Lipschitz continuous in state. We proceed via a discretization approach to establish our main result. Examples are given.

1 Introduction

Differential inclusions governed by maximal monotone operators have been considered in the scientific literature, see eg., [2], [3] among others. We are motivated by the recent contributions [1], [4], concerning evolution problems involving time and state dependent maximal monotone operators.
We are interested in the existence of Bounded Variation Continuous (or shortly BVC) solutions, in the context of a real separable Hilbert space H, to the evolution problem described by

$$\begin{cases} -\dfrac{du}{dr}(t) \in B(t, x(t))u(t) + G(t, x(t), u(t)) \quad dr - \text{a.e. } t \in I := [0, T], \\ x(t) = x_0 + \int_0^t u(s)dr(s), \quad t \in I \\ u(t) \in \mathrm{D}\left(B(t, x(t))\right), \quad t \in I \\ u(0) = u_0 \in \mathrm{D}\left(B(0, x_0)\right),\ x(0) = x_0 \in H. \end{cases}$$

For any $(t, y) \in I \times H$, $B(t, y) : \mathrm{D}\left(B(t, y)\right) \subset H \rightrightarrows H$ is a maximal monotone operator, whose domain is denoted $\mathrm{D}\left(B(t, y)\right)$. The dependence $(t, y) \mapsto B(t, y)$ is bounded variation on I and Lipschitz continuous on H, in the sense of the pseudo-distance (see [5]). The perturbation G acts as external forces.

2 Main results

Recall that a function $u : I \to H$ is **BVC** if u is bounded variation and continuous.
We are concerned with the existence of BVC solutions to our evolution problem.

Key Words and Phrases: *Differential inclusion, maximal monotone operator, perturbation, solution, pseudo-distance.*

Theorem 2.1. *Let for any* $(t, y) \in I \times H$, $B(t, y) : \mathrm{D}(B(t, y)) \subset H \rightrightarrows H$ *is a maximal monotone operator satisfying appropriate assumptions. Let* $G : I \times H \times H \rightrightarrows H$ *be a set-valued map that verifies suitable conditions. We will prove that, for any* $(u_0, x_0) \in \mathrm{D}(B(0, x_0)) \times H$, *there exists a BVC solution* $(u, x) : I \to H \times H$ *to the evolution problem above.*

Then, we add related examples with applications.

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Nonlinear Fractional Differential Problem of Van Der Pol-Duffing Jerk Type: Solvability and Stability Analysis

Abdelnebi Amira*

Department of Mathematics, University of Abdelhamid Ibn Badis , Mostaganem, Algeria
E-mail: amira.math27@gmail.com
Supervised by Prof Zoubir DAHMANI

Abstract
The subject of this talk is the existence, uniqueness and stability of solutions for a new sequential Van der Pol-Duffing (VdPD) Jerk fractional differential oscillator with Caputo-Hadamard derivatives. The arguments are based upon the Banach contraction principle, Krasnoselskii fixed point theorem and Ulam-Hyers stabilities. As applications, one illustrative example is included to show the applicability of our results.

1 Introduction

Over the past four decades, the dynamical behaviors of nonlinear differential equations have been intensively studied by many researchers. This interest is justified by the promising applications generated by these equations. Among the non linear equations, the VdPD oscillator is a very prominent and interesting model that has been extensively studied in the context of several specific problems such as, chaos, control, synchronization, vibration description and asymptotic perturbation in physics, engineering, electronics, biology, neurology and many other disciplines.
The mathematical model for the VdPD oscillator is governed by a two-dimensional nonlinear differential equation of the form:

$$\frac{d^2u}{dt^2} - \epsilon(1-u^2)\frac{du}{dt} + \alpha u - \beta u^3 = f\sin(\omega t), \tag{1.1}$$

where ω is the external frequency of the periodic signal and f stands for the amplitude of the external excitation. The parameters ϵ, α and β are the dimensionless damping coefficient, linear and cubic nonlinearity parameters, respectively.
The authors in [?] proposed a three-dimensional problem for the autonomous VdPD oscillator obtained by a transformation of the autonomous two-dimensional VdPD oscillator into a Jerk device with $f = 0$ and $\alpha = 1$ in the previous equation (??), which they presented as follows:

$$\frac{d^3u}{dt^3} + \frac{d^2u}{dt^2} - \epsilon(1-u^2)\frac{du}{dt} + u - \beta u^3 = 0, \tag{1.2}$$

Key Words and Phrases: *Van der Pol-Duffing Jerk equation, Fixed point, Caputo-Hadamard fractional derivative, Ulam–Hyers stability*

where ϵ and β are positive parameters.
Recently, due to the frequent appearance of fractional derivatives in various applications in fluid mechanics, viscoelasticity, biology, physics and engineering, various kinds of VdPD Jerk equations of fractional order have attracted more and more attention.
In this work, we try to propose an appropriate fractional formulation for a three-dimensional problem of VdPD Jerk type.
So let us consider the following problem:

$$\begin{cases} D^{\alpha}(D^{2-\beta}+\lambda D^{\alpha})x(t)+k_1\ f_1(t,x(t),D^{\alpha}x(t))+k_2 f_2(t,x(t),J^{p}x(t))=h(t). \\ x(1)=0, \qquad D^{1-(\alpha-\beta)}D^{\alpha-\beta}x(1)=A^{*}\in\mathbb{R}, \qquad x(T)=0, \\ 0\leq\beta<\alpha\leq 1, \quad 0\leq\alpha+\beta<1, \quad 0<p, \quad t\in I, \end{cases} \tag{1.3}$$

where $D^{\alpha}, D^{2-\beta}$, are the Caputo-Hadamard fractional derivatives, J^{p} is the Hadamard fractional integral $I=[1,T]$, k_1, k_2 are real constants, the functions f_1, f_2 and h are continuous.
The motivation of our problem lies in using the Caputo-Hadamard approach in a sequential way, and the fact that this approach has many advantages over the usual Hadamard derivatives, so on the basis of these advantages, we have proposed the fractional problem associated with the (VdPL)-Jerk equation, by injecting the Caputo-Hadamard derivatives both sides of the equation, with boundary conditions, this consideration makes the considered problem more interesting, knowing that when $\alpha=1$ and $\beta=0$, we recover the type model (VdPL)-Jerk.

2 Main results

In this presentation, we prove three main theorems by applying the Banach contraction principle and Krasnoselskii fixed point theorem. One of them concerns the Ulam-Hyers stability of our problem. Finally, an example to illustrate the applicability of the main results.

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Existence Results for System of Sequential Fractional Differential Equations

Hammou BENMEHIDI

Mailing address: Department of Nature and Life Sciences, Ibn Khaldoun University, Tiaret, Algeria
E-mail: hammou.benmehidi@univ-tiaret.dz

Abstract
We are concerned with an extension of a coupled sequential differential system of fractional type. Using the well known Banach contraction principle, we establish new results for the existence and uniqueness of solutions.

1 Introduction

in this paper, we are concerned with extend the study of the work of S. Asawasamrit et al. [1], by considering the following sequential problem:

$$\begin{cases} {}^{C}D^{\alpha_1 H}D^{\beta_1}x(t) = f\left(t, x(t), y(t), {}^{H}D^{\alpha_2}y(t)\right), \ a \le t \le b, \\ {}^{H}D^{\beta_2 C}D^{\alpha_2}y(t) = g\left(t, x(t), {}^{H}D^{\beta_1}x(t), y(t)\right), \ a \le t \le b, \\ \gamma_1 x(a) + \gamma_2^{C}D^{\alpha_2}y(a) = \theta_1, \ \lambda_1 x(b) + \lambda_2^{C}D^{\alpha_2}y(b) = \theta_2, \\ \gamma_3 y(a) + \gamma_4^{H}D^{\beta_1}x(a) = \theta_3, \ \lambda_3 y(b) + \lambda_4^{H}D^{\beta_1}x(b) = \theta_4, \end{cases} \tag{1.1}$$

where, ${}^{C}D^{\alpha_i}, {}^{H}D^{\beta_i}$ are the Caputo and Hadamard fractional derivatives of orders α_i and β_i, respectively with, $0 < \alpha_i, \beta_i \le 1$, $i = \overline{1,2}$ and γ_i, λ_i, θ_i, $\left(i = \overline{1,4}\right)$ are real numbers such that γ_i, λ_i are no zero numbers, $a, b \in \mathbb{R}$ with $a > 0$, and $f, g : [a; b] \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ are two given functions.

2 Main results

We have to consider the hypothesis:
$(H1)$: Suppose that there exist some constants $l_{ij} > 0, i = \overline{1,2}, j = \overline{1,3}$ such that

$$\begin{aligned} |f(t, x_2, y_2, z_2) - f(t, x_1, y_1, z_1)| &\le l_{11}|x_2 - x_1| + l_{12}|y_2 - y_1| + l_{13}|z_2 - z_1|, \\ |g(t, x_2, y_2, z_2) - g(t, x_1, y_1, z_1)| &\le l_{21}|x_2 - x_1| + l_{22}|y_2 - y_1| + l_{23}|z_2 - z_1|, \end{aligned}$$

for each $t \in [a, b]$ and all $x_i, y_i, z_i \in \mathbb{R}$.

Key Words and Phrases: *Caputo derivative, Hadamard derivative, Fixed point*

Then, we have also to take into account to the expressions given by:

$$
\begin{aligned}
Q_1 \ &: = \frac{|\lambda_1|\, l_1}{|\Sigma|}\left(|\Lambda_3| + |\Lambda_4| \frac{\left(\log\left(\frac{b}{a}\right)\right)^{\beta_1}}{\Gamma(\beta_1+1)}\right)^{H} I^{\beta_1}\left({}^{RL}I^{\alpha_1}(1)\right)(b) \\
&\quad + \frac{|\lambda_4|\, l_1}{|\Sigma|}\left(|\Lambda_1| + |\Lambda_2| \frac{\left(\log\left(\frac{b}{a}\right)\right)^{\beta_1}}{\Gamma(\beta_1+1)}\right)^{RL} I^{\alpha_1}(1)(b) + l_1^{H} I^{\beta_1}\left({}^{RL}I^{\alpha_1}(1)\right)(b), \\
Q_2 \ &: = \frac{|\lambda_2|\, l_2}{|\Sigma|}\left(|\Lambda_3| + |\Lambda_4| \frac{\left(\log\left(\frac{b}{a}\right)\right)^{\beta_1}}{\Gamma(\beta_1+1)}\right)^{H} I^{\beta_2}(1)(b) \\
&\quad + \frac{|\lambda_3|\, l_2}{|\Sigma|}\left(|\Lambda_1| + |\Lambda_2| \frac{\left(\log\left(\frac{b}{a}\right)\right)^{\beta_1}}{\Gamma(\beta_1+1)}\right)^{RL} I^{\alpha_2}\left({}^{H}I^{\beta_2}(1)\right)(b), \\
Q_3 \ &: = \frac{|\lambda_1|\, l_1}{|\Sigma|}\left(\frac{|\gamma_4|}{|\gamma_3|}|\Lambda_4| + \frac{|\gamma_1|}{|\gamma_2|}|\Lambda_3| \frac{(b-a)^{\alpha_2}}{\Gamma(\alpha_2+1)}\right)^{H} I^{\beta_1}\left({}^{RL}I^{\alpha_1}(1)\right)(b) \\
&\quad + \frac{|\lambda_4|\, l_1}{|\Sigma|}\left(\frac{|\gamma_4|}{|\gamma_3|}|\Lambda_4| + \frac{|\gamma_1|}{|\gamma_2|}|\Lambda_3| \frac{(b-a)^{\alpha_2}}{\Gamma(\alpha_2+1)}\right)^{RL} I^{\alpha_1}(1)(b), \\
Q_4 \ &: = \frac{|\lambda_2|\, l_2}{|\Sigma|}\left(\frac{|\gamma_4|}{|\gamma_3|}|\Lambda_4| + \frac{|\gamma_1|}{|\gamma_2|}|\Lambda_3| \frac{(b-a)^{\alpha_2}}{\Gamma(\alpha_2+1)}\right)^{H} I^{\beta_2}(1)(b) \\
&\quad + \frac{|\lambda_3 l_2|}{|\Sigma|}\left(\frac{|\gamma_4|}{|\gamma_3|}|\Lambda_4| + \frac{|\gamma_1|}{|\gamma_2|}|\Lambda_3| \frac{(b-a)^{\alpha_2}}{\Gamma(\alpha_2+1)}\right)^{RL} I^{\alpha_2}\left({}^{H}I^{\beta_2}(1)\right)(b) + l_2^{RL} I^{\alpha_2}\left({}^{H}I^{\beta_2}(1)\right)(b), \\
M_1 \ &: = \frac{\left(\log \frac{b}{a}\right)^{1-\beta_1}}{\Gamma(2-\beta_1)}, \\
M_2 \ &: = \frac{\left(\log \frac{b}{a}\right)^{1-\alpha_2}}{\Gamma(2-\alpha_2)},
\end{aligned}
$$

where,

$$l_1 = \max(l_{11}, l_{12}, l_{13}), \quad l_2 = \max(l_{21}, l_{22}, l_{23}).$$

Theorem 2.1. *Under the condition* $(H1)$, *the problem* (1.1) *has a unique solution defined on* $[a,b]$, *provided that the quantity* $Q < 1$;

$$Q := \max\left\{\max\left((Q_1+Q_2), M_1(Q_1+Q_2)\right), \max\left((Q_3+Q_4), M_2(Q_3+Q_4)\right)\right\}.$$

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Existence, uniqueness and Ulam Hyers stability results for Hadamard fractional differential equations of variable order

Amar Benkerrouche *

Department of Mathématiques, Ziane Achour University, Djelfa, Algeria
E-mail: amar.benkerrouche3@gmail.com

Abstract
In this manuscript, we examine both the existence, uniqueness and the stability of solutions to the boundary value problem of Hadamard fractional differential equations of variable order. All results in this study are established using Krasnoselskii fixed-point theorem and the Banach contraction principle. Further, the Ulam-Hyers stability of the given problem is examined.

1 Introduction

The fractional calculus of variable fractional order is a generalization of constant order and many studies have been done on the existence of solutions to fractional constant-order problems, on the contrary, few papers deal with the existence of solutions to problems via variable order. Therefore, all our results in this work are novel and worthwhile.
In this paper we will study the following boundary value problem for the Hadamard fractional differential equation of variable order

$$\begin{cases} {}^{H}D_{1^+}^{u(t)}x(t) = f(t, x(t), {}^{H}I_{1^+}^{u(t)}x(t)),\ t \in J, \\ x(1) = x(T) = 0, \end{cases} \tag{1.1}$$

where $J = [1, T]$, $1 < T < \infty$, $u(t) : J \to (1, 2]$ is the variable order of the fractional derivatives, $f : J \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a continuous function and the left Hadamard fractional integral of variable-order $u(t)$ for function $x(t)$ is (see, for example, [1], [2])

$${}^{H}I_{1^+}^{u(t)}x(t) = \frac{1}{\Gamma(u(t))} \int_1^t (log\frac{t}{s})^{u(t)-1} \frac{x(s)}{s} ds,\ \ t \in J, \tag{1.2}$$

where $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$ is the Gamma function and the left Hadamard fractional derivative of variable-order $u(t)$ for function $x(t)$ is (see, for example, [1], [2])

$${}^{H}D_{1^+}^{u(t)}x(t) = \frac{t^2}{\Gamma(2-u(t))} \frac{d^2}{dt^2} \int_1^t (log\frac{t}{s})^{1-u(t)} \frac{x(s)}{s} ds, \quad t \in J. \tag{1.3}$$

We notice that, if the order $u(t)$ is a constant function u, then the Hadamard variable order fractional derivative (1.3) and integral (1.2) are the usual Hadamard fractional

Key Words and Phrases: *derivatives and integrals of variable-order, boundary value problem, piecewise constant functions, Hadamard derivative*

derivative and integral, respectively(see [1, 2]).
Remark For general functions $u(t)$, $v(t)$, we notice that the semigroup property doesn't hold, i.e:

$$^{H}I_{a^+}^{u(t)}(^{H}I_{a^+}^{v(t)})h(t) \neq^{H} I_{a^+}^{u(t)+v(t)}h(t).$$

Lemma 1.1. *([3, 4]) If $u : J \to (1, 2]$ be a continuous function, then for $h \in C_\delta(J, \mathbb{R}) = \{h(t) \in C(J, \mathbb{R}),\ (logt)^\delta h(t) \in C(J, \mathbb{R})\}$, $0 \leq \delta \leq \min_{t\in J} |(u(t))|$ the variable order fractional integral $^{H}I_{1^+}^{u(t)}h(t)$ exists for any points on J.*

Lemma 1.2. *([3, 4]) Let $u : J \to (1, 2]$ be a continuous function, then $^{H}I_{1^+}^{u(t)}h(t) \in C(J, \mathbb{R})$ for $h \in C(J, \mathbb{R})$.*

2 Main results

In this work, we introduced an abstract variable–order boundary value problem of Hadamard fractional differential equations of variable order, where the function $u(t)$: $[1, T] \to (1, 2]$ stands for the variable order of the given system. First, we reviewed some important specifications of Hadamard variable–order operators and by an example, we showed that the semi-group property is not valid for variable–order Hadamard integrals. Then by defining a partition based on the generalized intervals, we introduced a piecewise constant function $u(t)$ and converted the given variable–order Hadamard fractional differential equations 1.1 to an equivalent standard Hadamard boundary value problem of the fractional constant order. By using the standard fixed–point theorems, we established the existence and uniqueness of solutions. Finally, the Ulam–Hyers stability of its possible solutions was checked.

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PART E

Integro-differential equations

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Collocation Method for Some Classes of Two-Dimensional Partial Volterra Integro-Differential Equations

Cheima KHENNAOUI*

Mailing address: Department of Mathematics and Informatiques, University center AbdelHafid boussouf Mila, Algeria.
E-mail: khe.cheima@gmail.com

Azzeddine BELLOUR

Mailing address: Department of Mathematics, Ecole Normale Supérieure de Constantine, Algeria.
E-mail: bellourazze123@yahoo.com

Abstract
The numerical solution of two-dimensional linear partial Volterra integro-differential equations of second kind (2D-PVIDEs) subject to initial value conditions is provided in this work. The method is based on the use of Taylor polynomails in two-dimensional, and the approximate solution is obtained by using explicite schemes. Also, by means of an error estimation and convergence test, some examples to illustrate the accuracy and efficiency of the method are fulfilled.

1 Introduction

In this work, we will consider the following type integral equation. This includes two-dimensional linear partial Volterra integro-differential equation of second kind with desired order:

$$\begin{cases} \frac{\partial^{v+w}u(x,y)}{\partial x^v \partial y^w} = g(x,y) + \int_0^x \int_0^y K(x,y,t,s) \frac{\partial^{v+w}u(t,s)}{\partial x^v \partial y^w} dsdt, \quad (x,y) \in [0,1]\times[0,1] \\ Appropriate\ initial\ conditions. \end{cases} \tag{1.1}$$

Where u stands for the unknown function to be determined while the kernel K and the function g are known and assumed to be sufficiently smooth in order to guarantee the existence and uniqueness of the solution.

Generally, obtained results in theorical problems of different domains are as partial differential equations, integral and integro-differential equations which are usually difficult to solve analytically so it is required to obtain efficient numerical methods to find the approximate solution. The main aim of this work is to extend the Taylor collocation method of one dimensional Volterra integral and integro-differential equations to solve two-dimensional PVIDEs of $v+w$ order (1.1).

Key Words and Phrases: *Partial Integro-differential equations, Collocation method, Two-dimensional equations, Taylor polynomials.*

2 Main results

In this section, we develop the Taylor collocation method to approximate the solution of (1.1) in the real polynomial spline space

$S_{p,p}^{(-1)}(\Pi_{N,M}) = \{v : D \to \mathbb{R} : \ v_{n,m} = v|_{D_{n,m}} \in \pi_{p,p}, \ n = 0, ..., N-1; \ m = 0, 1, .., M-1\},$

This is the space of bivariate polynomial spline functions of degree (at most) p in x and y, its dimension is $NM(p+1)^2$, i.e., the same as the total number of the coefficients of the polynomials $v_{n,m}, n = 0, ..., N-1; m = 0, 1, .., M-1$. To find these coefficients, we use Taylor polynomial on each rectangle. $D_{n,m}$, $n = 0, ..., N-1$, $m = 1, ..., M-1$. We approximate u by the Taylor polynomial $v_{n,m}$ such that

$$v_{n,m}(x, y) = \sum_{i+j=0}^{p} \frac{1}{i!j!} \frac{\partial^{i+j} \hat{v}_{n,m}(x_n, y_m)}{\partial x^i \partial y^j} (x - x_n)^i (y - y_m)^j \ ; \quad (x, y) \in D_{n,m}, \tag{2.1}$$

and,

$$\begin{aligned}
\frac{\partial^{i+j+v+w} \hat{v}_{n,m}(x_n, y_m)}{\partial x^{i+v} \partial y^{j+w}} &= \partial_1^{(i)} \partial_2^{(j)} g(x_n, y_m, x_n, y_m) + \sum_{\xi=0}^{n-1} \sum_{\rho=0}^{m-1} \int_{x_\xi}^{x_{\xi+1}} \int_{y_\rho}^{y_{\rho+1}} \partial_1^{(i)} \partial_2^{(j)} K(x_n, y_m, t, s) v_{\xi,\rho}(t, s) ds dt \\
&+ \sum_{\xi=0}^{n-1} \sum_{r=0}^{j-1} \sum_{l=0}^{r} \binom{r}{l} \int_{x_\xi}^{x_{\xi+1}} \frac{\partial^i}{\partial x^i} \left[\frac{\partial^{r-l}}{\partial y^{r-l}} [\partial_2^{(j-1-r)} k(x, y, t, y)] \right]_{x=x_n, y=y_m} \frac{\partial^l v_{\xi,m}(t, y_m)}{\partial y^l} dt \\
&+ \sum_{\rho=0}^{m-1} \sum_{q=0}^{i-1} \sum_{\eta=0}^{q} \binom{q}{\eta} \int_{y_\rho}^{y_{\rho+1}} \frac{\partial^{q-\eta}}{\partial x^{q-\eta}} \left[\partial_1^{(i-1-q)} \partial_2^{(j)} k(x, y, x, s) \right]_{x=x_n, y=y_m} \frac{\partial^\eta v_{n,\rho}(x_n, s)}{\partial x^\eta} ds \\
&+ \sum_{r=0}^{j-1} \sum_{l=0}^{r} \sum_{q=0}^{i-1} \sum_{\eta=0}^{q} \binom{r}{l} \binom{q}{\eta} \frac{\partial^{q-\eta}}{\partial x^{q-\eta}} \left[\frac{\partial^{i-1-q}}{\partial x^{i-1-q}} \bigg|_{t=x} \left(\frac{\partial^{r-l}}{\partial y^{r-l}} \left[\partial_2^{(j-1-r)} k(x, y, t, y) \right] \right) \right]_{x=x_n, y=y_m} \frac{\partial^{l+\eta} \hat{v}_{n,m}(x_n, y_m)}{\partial x^\eta \partial y^l}.
\end{aligned}$$

for $n = 0, ..., N-1$ and $m = 1, ..., M-1$, and appropriate associated initial conditions.

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Numerical solution by using Lagrange collocation method for a class of weakly singular volterra integral equations

Kherchouche Khedidja*

Department of mathematics and informatics, Higher Normal School -Assia Djebar-, Constantine, Algeria.
E-mail: kherchouche.khadidja@ensc.dz

Bellour Azzeddine

Department of mathematics and informatics, Higher Normal School -Assia Djebar-, Constantine, Algeria.
E-mail: bellourazze123@yahoo.com

Abstract
In this work, an iterative collocation method based on the use of Lagrange polynomials is developed for the numerical solution of a class of nonlinear weakly singular Volterra integral equations. The error analysis of the proposed numerical method is studied theoretically. Numerical illustrations confirm our theoretical analysis.

1 Introduction

We consider the following nonlinear weakly singular Volterra integral equations:

$$x(t) = g(t) + \int_0^t p(t,s)k(t,s,x(s))ds, t \in I = [0,T], \tag{1.1}$$

where the functions g, k are sufficiently smooth and $p(t,s) = \frac{s^{\mu-1}}{t^{\mu}}$, $\mu > 1$.
Equations with this kind of kernel have a weak singularity at $t = 0$ and they are a particular case of the cordial equations, studied by G. Vainikko in [4, 1].
An existence and uniqueness result in $C^m([0,T])$ was obtained in [4].
The application of polynomial and spline collocation methods to cordial equations was studied in [4, 2] and [3], respectively.

Key Words and Phrases: *Nonlinear weakly singular Volterra integral equation, Collocation method, Iterative Method, Lagrange polynomials.*

2 Main results

We approximate the exact solution x by the function u of the real polynomial spline space $S_{m-1}^{(-1)}(I, \Pi_N)$ given by

$$u(t_n + \tau h) = \sum_{l=1}^{m} \lambda_l(\tau) u(t_{n,l}), \tau \in [0, 1], \tag{2.1}$$

such that $u(t_{n,j})$ satisfy a nonlinear system, that's why we will use an iterative collocation solution $u^q \in S_{m-1}^{(-1)}(I, \Pi_N), q \in \mathbb{N}$, to approximate the exact solution of (1.1) such that

$$u^q(t_n + \tau h) = \sum_{j=1}^{m} \lambda_j(\tau) u^q(t_{n,j}), \tau \in [0, 1] \tag{2.2}$$

for $j = 1, ..., m$, $n = 0, ..., N-1$. Where the coefficients $u^q(t_{n,j})$ are given by an iterative formula. such that the initial values $u^0(t_{n,j}) \in J$ (J is a bounded interval).

Theorem 2.1. *Let f, k be m times continuously differentiable on their respective domains. If $\frac{L\Gamma_m}{\mu} < \frac{1}{2}$, then the collocation solution u converges to the exact solution x, and the resulting error function $e := x - u$ satisfies:*

$$\|e\| \leq Ch^m,$$

where C is a finite constant independent of h.

Theorem 2.2. *Consider the iterative collocation solution $u^q, q \geq 1$. If $\frac{L\Gamma_m}{\mu} < \frac{1}{2}$, then for any initial condition $u^0(t_{n,j}) \in J$ (bounded interval), the iterative collocation solution $u^q, q \geq 1$ converges to the exact solution x. Moreover, the following error estimate holds*

$$\|u^q - x\| \leq d\rho^q + Ch^m$$

where d, C are finite constants independent of h and $\rho < 1$.

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The existence of solution for Higher order boundary value problem

Benhiouna Salah*

Mailing address: Department of Mathematics Enset Skikda azzaba ,Laboratoire de Mathmatiques appliques et didactique, ENSC , Constantine-Algeria
E-mail: benhiounasalah@yahoo.fr

Bellour Azzeddine

Mailing address:Laboratoire de Mathmatiques appliques et didactique, ENSC , Constantine-Algeria
E-mail: bellourazze123@yahoo.com

Abstract
In this paper, we use Schauder's Fixed point theorem and Generalization of Ascoli-Arzela Theorem to prove the existence of a solution for a boundary value problem of higher order.

1 Introduction

In this paper, we consider the following higher-order boundary value problem:

$$\begin{cases} u^{(n)} + f(t, u, u', ..., u^{(n-2)}) = 0, n \geq 2, t \in I = [0, 1], \\ u^{(i)}(0) = 0, 0 \leq i \leq n-3, \\ \alpha u^{(n-2)}(0) - \beta u^{(n-1)}(0) = 0, \\ \gamma u^{(n-2)}(1) + \delta u^{(n-1)}(1) = 0. \end{cases} \tag{1.1}$$

where n is a given positive integer, $\alpha, \gamma > 0, \beta, \delta \geq 0$, f is continuous and satisfies $|f(s, u_0, u_1, ..., u_{n-2})| \leq a(s) + \sum_{k=0}^{n-2} b_k |u_k|$ such that a is continuous on I and $b_k \in \mathbb{R}^+, k = 0, ..., n-2$.
Equation (1.1) and its particular forms have been studied by many authors (see for example [3, 2, 1] and the references therein).

Our main task in this paper in order to prove the compactness criteria and to use Schauder fixed point theorem in the space C^n and Ascoli-Arzela to prove the existences of a solution for the higher-order boundary value problem (1.1).

Key Words and Phrases: *Higher-order boundary value problem, Fixed point theorem, AscoliArzela theorem, integro-differential equation*

2 Application to the solution of a higher-order boundary value problem

In this section, we study the existence of a solution for the problem (1.1).
It is easy to check, (see [4]), that u is a solution of (1.1) in $C^n(I,\mathbb{R})$ if and only if u is a solution of the following integro-differential equation:

$$u(t) = \int_0^1 G(t,s) f(s,u,u',...,u^{(n-2)})ds, \tag{2.1}$$

in $C^{n-2}(I,\mathbb{R})$, such that $g(t,s) = \frac{\partial^{n-2}G(t,s)}{\partial t^{n-2}}$ is the Green's function

$$g(t,s) = \frac{1}{\alpha\gamma + \alpha\delta + \beta\gamma} \begin{cases} (\beta + \alpha s)[\delta + \gamma(1-t)], 0 \leq s \leq t, \\ (\beta + \alpha t)[\delta + \gamma(1-s)], t \leq s \leq 1. \end{cases} \tag{2.2}$$

Equation (2.1) will be studied under the following assumptions:

(i) $f \in C(I \times \mathbb{R}^{n-1}, \mathbb{R})$.

(ii) There exist a function $a \in C(I,\mathbb{R}^+)$ and constants $b_k \in \mathbb{R}^+ (k = 0,...,n-2)$ such that

$$|f(s,u_0,u_1,...,u_{n-2})| \leq a(s) + \sum_{k=0}^{n-2} b_k |u_k|$$

Theorem: If the hypotheses (i), (ii) hold, and if

$$r \sum_{i=0}^{n-2} \| \int_0^1 |\partial_1^{(i)} G(t,s)| ds \|_\infty < 1$$

such that $r = Max\{b_0,...,b_{n-2}\}$.
Then, the integro-differential equation (2.1) has a solution in $C^{n-2}(I,\mathbb{R})$

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Taylor Collocation Method for Solving Two-Dimensional Integral Equations of the First Kind

Fouzia Birem
Department of mathematics and computer science, University center of Mila
E-mail: birem.fo@gmail.com

Hafida Laib
Department of mathematics and computer science, University center of Mila
E-mail: hafida.laib@gmail.com

Abstract

This study presents a Taylor collocation method to solve a class of nonlinear two-dimensional Volterra integral equations (2D-VIEs) of the first kind. The nonlinear first kind equations are converted to linear second kind equations, then a convergent algorithm based on the use of Taylor polynomials is developed to construct a collocation solution for approximating the solution of 2D-VIEs of the second kind. Some numerical examples are included to illustrate the validity and accuracy of the presented method.

1 Introduction

The standard form of the nonlinear two-dimensional Volterra integral equations of the first kind with an unknown function w is given by

$$\int_0^x \int_0^y K(x,y,t,s)H(w(t,s))dsdt = f(x,y), \quad (x,y) \in D, \tag{1.1}$$

where $D := [0,a] \times [0,b] \subset \mathbb{R}^2$, f and K are given smooth functions on their corresponding domains, H is a given continuous inverse function, nonlinear in w. For solving equation (1.1), we set $u(t,s) = H(w(t,s))$, to obtain the linear equation

$$\int_0^x \int_0^y K(x,y,t,s)u(t,s)dsdt = f(x,y), \quad (x,y) \in D. \tag{1.2}$$

To get the approximate solution of u, we convert Volterra integral equations of the first kind (1.2) to Volterra integral equations of the second kind (1.3) by differentiating both sides of equation (1.2). The conversion technique works gives a linear two-dimensional Volterra integral equation of the form:

$$\begin{aligned} u(x,y) = g(x,y) &+ \int_0^x K_1(x,y,t)u(t,y)dt + \int_0^y K_2(x,y,s)u(x,s)ds \\ &+ \int_0^x \int_0^y K_3(x,y,t,s)u(t,s)dsdt, \quad (x,y) \in D, \end{aligned} \tag{1.3}$$

where the functions g, K_1, K_2 and K_3 are given (real-valued) smooth functions defined, and the approximate solution of equation (1.1) can be obtained as $H^{-1}(u(t,s)) = w(t,s)$.
We extend the collocation method to solve equation (1.1) and (1.3). Taylor's Theorem in two variables is used to solve the considered problem.

Key Words: *Volterra integral equations of the first kind; Taylor polynomials; Collocation method.*

2 Main results

Consider the linear 2D-VIE

$$u(x,y) = g(x,y) + \int_0^x \int_0^y st\cos(xy)u(x,y)dsdt$$

where $g(x,y) = \cos(x)\sin(y) - \cos(xy)\,(x\sin(x)+\cos(x)-1)\,(\sin(y)-y\cos(y))$ with the exact solution $u(x,y) = \cos(x)\sin(y)$.
Table 1 shows the absolute errors at some points with $p=3$ and $N=M=10,20$.

Table 1: Absolute errors

(x,y)	$N=M=10,p=3$	$N=M=20,p=3$	$N=M=10,p=4$
$(0.1,0.1)$	$5.86e-09$	$5.80e-10$	$4.0e-11$
$(0.2,0.2)$	$6.90e-08$	$7.30e-09$	$1.0e-10$
$(0.3,0.3)$	$3.01e-07$	$3.32e-08$	$1.50e-09$
$(0.4,0.4)$	$8.43e-07$	$9.47e-08$	$1.82e-08$
$(0.5,0.5)$	$1.80e-06$	$2.04e-07$	$8.19e-08$
$(0.6,0.6)$	$3.17e-06$	$3.60e-07$	$2.59e-07$
$(0.7,0.7)$	$4.76e-06$	$5.39e-07$	$6.33e-07$
$(0.8,0.8)$	$6.13e-06$	$6.89e-07$	$1.26e-06$
$(0.9,0.9)$	$6.66e-06$	$7.35e-07$	$2.10e-06$

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La résolution d'un système d'équations de Fredholm

Asma AYACHI*

Mathématique, Université 08 mai 1945, Guelma, Algérie
E-mail: ayachi.asma87@gmail.com

Hamza GUEBBAI

Mathématique, Université 08 mai 1945, Guelma, Algérie
E-mail: guebaihamza@yahoo.fr

Ammar KHELLAF

Ecole Nationale Polytechnique de Constantine, Algeria
E-mail: amarlasix@gmail.com

Abstract
Pour résoudre un système d'équations intégrales non linéaire de Fredholm de seconde ordre, on va appliquer une linéarisation numérique, obtenue à partir de la méthode de Newton-Kantorovich. Nous utilisons des itérations à pente fixe de rang fini pour construire une suite d'itération qui converge théoriquement vers la solution exacte. Après, nous proposons une nouvelle suite itérative plus adaptée à la programmation numérique, obtenu en appliquant la méthode de Nyström avec la règle de Trapèze et la méthode de Kantorovich avec les fonctions chapeaux de projection. On montre que cette nouvelle suite de rang fini converge parfaitement vers la solution exacte. Ensuite pour confirmer en pratique nos résultats théoriques, nous exhibons un exemple numérique.

1 Position de problème

Notre problème est de résoudre le système d'équations intégrales non linéaires de Fredholm du seconde ordre pour tout $t \in [a, b]$:

$$\begin{cases} x_1(t) &= \displaystyle\int_a^b k_1\left(t, s, x_1(s), \ldots, x_N(s)\right) ds + f_1(t), \\ x_2(t) &= \displaystyle\int_a^b k_2\left(t, s, x_1(s), \ldots, x_N(s)\right) ds + f_2(t), \\ \vdots & \vdots \qquad\qquad \vdots \\ x_N(t) &= \displaystyle\int_a^b k_N\left(t, s, x_1(s), \ldots, x_N(s)\right) ds + f_N(t), \end{cases}$$

on va considérer l'espace de Banach $E = \left(\mathsf{C}([a, b])\right)^N$ muni de la norme suivantes:

Key Words and Phrases: *Fredholm system of the second kind , Newton like method , Kantorovich method , Operators matrix.*

$$\forall x = (x_1, \cdots, x_N) \in E, \quad \|x\|_E = \sum_{i=1}^{N} \|x_i\|_{\mathsf{C}([a,b])}, \forall x_i \in \mathsf{C}([a,b]), \quad \|x_i\|_{\mathsf{C}([a,b])} = \max_{a \le t \le b} |x_i(t)|. \tag{1.1}$$

Avec $N \in N - \{0, 1\}$, on note par $\mathrm{BL}(E)$ et $\mathrm{BL}\left(\mathsf{C}([a,b])\right)$ l'espace de Banach des opérateurs linéaires borneés définis on E et $\mathsf{C}([a,b])$ respectivement équipé par les normes usuelles:

$$\begin{aligned} \forall S \in \mathrm{BL}(E), \quad \|S\| &= \sup_{\|x\|_E = 1} \|Sx\|_E, \\ \forall S \in \mathrm{BL}(\mathsf{C}([a,b])), \ \|S\| &= \sup_{\|x\|_{\mathsf{C}([a,b])} = 1} \|Sx\|_{\mathsf{C}([a,b])}. \end{aligned}$$

Notre but est de construire une approximation numérique pour la solution du problème suivant:
trouver $x \in E$, pour $f = (f_1, f_2, \cdots, f_N) \in E$ données, tel que:

$$G(x) := x - \mathsf{K}(x) - f = 0_E, \tag{1.2}$$

On va proposer un alternatif processeur numérique pour construire une suite itérative, on va utiliser l'itération à pente fixe pour la dérivée de Fréchet.
On suppose que l'équation (1.2) admet unique solution $x^{(\infty)} \in \Omega$, c'est-à-dire : $G\big(x^{(\infty)}\big) = 0_E$, vérifié:

$$\exists d > 0, \ \| \left(G'(x^{(\infty)})\right)^{-1} \| = d. \tag{1.3}$$

On va construire une condition sur $x^{(0)} \in \Omega$ afin que la suite de pente fixe suivante:

$$\begin{cases} x^{(0)} \in \Omega, \\ x^{(k+1)} = x^{(k)} - \left(G'(x^{(0)})\right)^{-1} G(x^{(k)}), \quad \text{pour tous} \quad k \ge 0, \end{cases} \tag{1.4}$$

converge vers $x^{(\infty)}$. Utilisant une ide similaire celle dvelopps par Ahus [1], on obtient le thorme de convergence suivant:

Theorem 1.1. *Soit $x^{(0)} \in \Omega$ tel que $\left\|x^{(0)} - x^{(\infty)}\right\|_E \le \delta\,(3dL)^{-1}$ avec $0 < \delta < 1$, alors:*

$$\forall k \ge 1, \ \left\|x^{(k)} - x^{(\infty)}\right\|_E \le (\delta)^{k+1} (3dL)^{-1}. \tag{1.5}$$

Numériquement, nous proposons une nouvelle suite d'itérative plus adaptée à la programmation numérique, obtenu en appliquant la méthode de Nyström avec règle Trapezoidale et la méthode de Kantorovich avec les functions chapeau de projection. On montre que cette nouvelle suite de rang fini converge parfaitement vers la solution exacte.

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Solution of Fredholm-type integral inclusion in b-metric spaces via new fixed point theorem

Hala Nadjet*
Mailing address: Department of mathematics, University Echahid Hamma Lakhdar, El Oued, Algeria
E-mail: halanadjat@gmail.com

Khaled Habita
Mailing address: Department of mathematics, University Echahid Hamma Lakhdar, El Oued, Algeria
E-mail: habitakhaled@gmail.com

Abstract
An existence theorem for Fredholm-type integral inclusion is establish in b-metric spaces. We first introduce fixed point theorems for new F-contraction in the setting of b-metric spaces, then we apply our fixed point theorem to prove the existence theorem for Fredholm-type integral inclusion.

1 Introduction

This work is concerned with the existence Solution of Fredholm-type integral inclusion in b-metric spaces via new fixed point theorem. For this purpose, let $X := C([a,b],\mathbb{R})$ be the space of all continuous real valued functions on $[a,b]$. Note that X is complete b-metric space by considering $d(x,y) = \sup_{t\in[a,b]} |x(t)-y(t)|^2$ with s = 2.

$$x(t) \in h(t) + \int_a^b P(t,s,x(s))ds, \qquad t \in J = [a,b]. \tag{1.1}$$

where $h \in X$ and $P : J \times J \times \mathbb{R} \to CB(\mathbb{R})$.

2 Main results

Theorem 2.1. *Let (X,d,s) be a complete b-metric space, $\alpha : X \times X \to [0,\infty)$ a function. Let $S,T : X \to CB(X)$ be multi-valued mappings such that for all $x,y \in X$*

$$\tau + F(s^3 H(Sx,Ty)) \leq F(\psi(N_s(x,y))), \tag{2.1}$$

with $\alpha(x,y) \geq s^2$, where $F \in \mathcal{F}_s$, $\psi \in \Phi$ and

$$M_s(x,y) = \max\left\{d(x,y), D(x,Tx), D(y,Ty), \frac{D(x,Ty)+D(y,Tx)}{2s}\right\}. \tag{2.2}$$

Suppose that the following conditions are satisfied.

Key Words and Phrases: *b-metrics spaces, multivalued F-contraction, α-admissible, integral inclusion*

(i) T is α_s admissible;

(ii) There exist $x_0 \in X$, and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq s^2$;

(iii) For every sequence $\{x_n\}$ in X such that x_n converges to x in X and $\alpha(x_n, x_{n+1}) \geq s^2$, for all $n \in N$, then $\alpha(x_n, x) \geq s^2$, for all $n \in \mathbb{N}$.

Then T has a fixed point.

Our hypotheses are on the following data :

(A) : for each $x \in X$, the multivalued operator $P_x(t,s) := P(t,s,x(s))$, $t,s \in J \times J$, is lower semi-continuous.

(B) There exists a continuous function $\eta : J \times J \to [0, +\infty)$ such that

$$|q_{x_1}(t,s) - q_{x_2}(t,s)|^2 \leq \eta(t,s)|x_1(s) - x_2(s)|^2.$$

For all $x_1, x_2 \in X$ with $(x_1, x_2) \in E(G)$ and $x_1 \neq x_2$, all $q_{x_1} \in P_{x_1}, q_{x_2} \in P_{x_2}$ and for each $(t,s) \in J \times J$;

(C) : there exists $\tau > 0$ such that

$$\sup_{t \in J} \int_a^b |\eta(t,s)| ds \leq \frac{e^{-\tau}}{8}.$$

(D) : There exist $x_0 \in X$ and $x_1 \in Tx_0$ such that $(x_0, x_1) \in E(G)$.

(E) : For each $x \in X$ and $y \in Tx$ with $(x,y) \in E(G)$, we have $(y,z) \in E(G)$ for all $z \in Ty$.

(F) : For every sequence $\{x_n\}$ in X such that $x_n \to x \in X$ and $(x_n, x_{n+1}) \in E(G)$ for all $n \in \mathbb{N}$, we have $(x_n, x) \in E(G)$ for all $n \in \mathbb{N}$.

Theorem 2.2. *Under assumptions $(A) - (F)$ the integral inclusion* (1.1) *has a solution in X.*

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Scaled Laguerre functions collocation method to solve high-order ordinary differential equations on the half-line

Boutarcha Sofiane*
**Department of mathematics , University of Bordj Bou Arreridj, El Anasser, Algeria*
E-mail: sofiane.boutarcha@univ-bba.dz

Rahmoune Azedine
Department of mathematics , University of Bordj Bou Arreridj, El Anasser, Algeria
E-mail: a.rahmoune@univ-bba.dz

Abstract
The aim of this work is to approximate the solution of a high-order ordinary differential equation on the half-line. The collocation method is based on the scaled Laguerre functions. At first, we describe some properties of the approximation of this function, and then, operational matrices of differentiation are given to reduce the problem to a linear algebraic equations system. Finally, we illustrate some numerical results to examine the accuracy and efficiency of the proposed method compared with other approaches.
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1 Introduction

The present work focuses on the numerical solution of the linear high-order ordinary differential equation of the form

$$u^{(m)}(x) + \sum_{i=0}^{m-1} \boldsymbol{a}_i(x) u^{(i)}(x) = f(x), m \in \mathbb{N}, x \geq 0 \tag{1.1}$$

With the initial boundary conditions

$$u^{(r)}(0) = \alpha_i, \lim_{x \to \infty} u(x) = 0, i = 0, 1, ..., m-1 \tag{1.2}$$

Where the functions $\boldsymbol{a}_0(x), \boldsymbol{a}_1(x), ..., \boldsymbol{a}_{m-1}(x)$ are continuous on the half line, $\alpha_0, \alpha_1, ..., \alpha_{m-1}$ are constants and u is unknown function to be determined. This problem has important applications, and it appears in various issues such as physics and chemical reaction. The solvability of Eq.(1) has been discussed by many authors, for instance, the authors of [2] used the exponential Jacobi pseudospectral method to approximate the solution of high-order ordinary differential equation (1). Our approach is based on solving problems on unbounded domains, where the solution is decaying at infinity (decays algebraically or exponentially), the authors of [3, 1] used it to solve integral and differential equations defined on the half-line.

Key Words and Phrases: *high-order ordinary differential equation, Scaled Laguerre functions, The half-line.*

2 Main results

Example 2.1. Consider the second-order differential equation

$$u^{(2)}(x) + \frac{6}{x}u^{(1)}(x) + 14u(x) = 4x^2e^{-x^2}, \quad x \in [0, \infty) \tag{2.1}$$

subject to

$$u(0) = 1, \quad u^{(1)}(0) = 0, \quad u(\infty) = 0. \tag{2.2}$$

The analytical solution is $u(x) = e^{-x^2}$. If we apply scaled Laguerre function collocation method for different degrees n with different scaling factors β, we have the following results

n	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$
4	5.44e-01	3.35e-01	8.86e-02	1.74e-01	6.93e-02	4.03e-02
8	3.08e-01	7.24e-02	4.17e-03	3.36e-03	1.18e-03	1.06e-03
16	5.04e-02	1.24e-03	2.45e-04	6.12e-05	1.62e-05	3.70e-06
32	1.27e-03	2.55e-05	3.99e-06	1.71e-07	5.05e-08	7.27e-09
64	3.66e-05	5.23e-08	6.22e-09	5.94e-11	5.97e-13	3.19e-14
128	8.39e-08	5.06e-11	1.93e-14	1.57e-15	1.25e-15	7.91e-16

Table 1: Example 1: A comparison of the discrete L^2-error for different factor β .

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PERIODICITY AND POSITIVITY IN NEUTRAL NONLINEAR LEVIN-NOHEL INTEGRO-DIFFERENTIAL EQUATIONS

Bessioud Karima*

Mailing address: Department of Mathematics and Computer Science, Mohamed-Cherif Messaadia University, Souk Ahras, Algeria
E-mail: karimabessioud1@gmail.com

Abstract
We consider the following neutral nonlinear Levin-Nohel integro-differential with variable delay

$$\frac{d}{dt}x(t)+\int_{t-\tau(t)}^{t}a(t,s)\,x(s)\,ds+\frac{d}{dt}g(t,x(t-\tau(t)))=0, \tag{0.1}$$

By using Krasnoselskii's fixed point theorem we obtain the existence of periodic and positive periodic solutions and by contraction mapping principle we obtain the existence of a unique periodic solution.

For $T>0$ let P_T be the set of all continuous scalar functions $x(t)$, periodic in t of period T.
we assume that

$$a(t+T,s+T)=a(t,s)\,,\ \tau(t+T)=\tau(t)\geq\tau^*>0,\forall t,\ s\in\mathbb{R}. \tag{0.2}$$

The function $g(t,x)$ is periodic in t of period T, it is also globally Lipschitz continuous in x. That is $g(t+T,x)=g(t,x)$, and there is positive constant E such that $|g(t,x)-g(t,y)|\leq E\|x-y\|$.
The next lemma is crucial to our results.

Lemma 0.1. *$x\in P_T$ is a solution of equation (0.1) if and only if $x\in P_T$ satisfies*

$$x(t)=-g(t,x(t-\tau(t)))-\left(1-e^{-\int_{t-T}^{t}A(z)dz}\right)^{-1}\int_{t-T}^{t}\left[L_x(s)-A(s)\,g(s,x(s-\tau(s)))\right]e^{-\int_{s}^{t}A(z)dz}ds, \tag{0.3}$$

where

$$L_x(t)=\int_{t-\tau(t)}^{t}a(t,s)\left(\int_{s}^{t}\left(\int_{u-\tau(u)}^{u}a(u,\nu)\,x(\nu)\,d\nu\right)du\right)+g(t,x(t-\tau(t)))-g(s,x(s-\tau(s)))\,ds, \tag{0.4}$$

and $A(t)=\int_{t-\tau(t)}^{t}a(t,s)\,ds$.

Key Words and Phrases: *Fixed points, positivity, periodicity, Levin-Nohel integro-differential equations.*

1 Main results

we state Krasnoselskii's fixed point theorem which enables us to prove the existence of a periodic solution. we express equation (0.3) as $(H\varphi)(t) = (B\varphi)(t) + (C\varphi)(t)$ where $C,\ B : P_T \to P_T$ are given by $(B\varphi)(t) = -g(t, \varphi(t-\tau(t)))$, and $(C\varphi)(t) = -\left(1 - e^{-\int_{t-T}^{t} A(z)dz}\right)^{-1} \int_{t-T}^{t} [L_\varphi(s) - A(s)\, g(s, \varphi(s-\tau(s)))]\, e^{-\int_s^t A(z)dz} ds.$
To simplify notations, we introduce the following constants.

$$\begin{aligned}
\eta &= \left(1 - e^{-\int_{t-T}^{t} A(z)dz}\right)^{-1},\ \rho = \sup_{s\in[t-T,t]} \left(\int_{s-\tau(s)}^{s} |a(s,w)|\, dw\right),\\
\gamma &= \sup_{s\in[t-T,t]} e^{-\int_s^t A(z)dz},\ \delta = \sup_{s\in[t-T,t]} \left(\sup_{w\in[t-T,t]} \int_w^s \left(\int_{u-\tau(u)}^{u} |a(u,\nu)|\, d\nu\right) du\right).
\end{aligned}$$

Lemma 1.1. *$C : P_T \to P_T$ is completely continuous and the image of C contained a compact set.*

Lemma 1.2. *If $E < 1$, then B is a contraction.*

Theorem 1.1. *Let $\alpha = \sup_{t\in[0,T]} |g(t,0)|$ and J be a positive constant satisfying the inequality $EJ + \alpha + \eta\gamma T(\rho(\delta J + 3(EJ+\alpha))) \le J$. Then equation (0.1) has a solution in M.*

Theorem 1.2. *Suppose (0.2) hold. If $(E + \eta\gamma T\rho(\delta + 3E)) < 1$, then equation (0.1) has a unique T -periodic solution.*

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PART F

Modeling and optimization

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Mathematical Modeling of Hepatitis C Infection

Nadjat HAMIDAT*

Mailing address: Department of Mathematics and Computer Science, Abdelhamid Ibn Badis University, Mostaganem, Algeria
E-mail: nadjat.hamidat.etu@univ-mosta.dz

Houria ALI MERINA

Mailing address: Department of Mathematics and Computer Science, Abdelhamid Ibn Badis University, Mostaganem, Algeria
E-mail: alimerina.houria@univ-mosta.dz

Abstract
The realisation of many acts, whether medical or personal, requires the use of reusable devices or objects, which must then undergo a procedure of sterilization or disinfection. A defect in these procedures may be responsible for infections such as : Hepatitis C, HIV and many other serious diseases, for this reason, the control of disinfection activities, sterilization and the establishment of a quality system guarantee the control of processes and limit exposure to these diseases. An epidemiological model of hepatitis C transmission has been developed based on ordinary differential equations. This model has integrated the characteristics of the disease (contagiousness and various other stages), its transmission modes (indirect : the material here plays the role of the vector of transmission of the disease). Our goal in this work is to show the impact of sterilization of material on the evolution of hepatitis C disease.

1 Introduction

Models and simulations of all kinds are tools for dealing with reality. Humans have always used mental models to better understand the world around them:to make plans, to consider different possibilities, to share ideas with others, to test changes, and to determine whether or not the development of an idea is feasible.
Viral hepatitis is an international public health problem, comparable to other major communicable diseases such as HIV, tuberculosis or malaria, or more recently Covid-19. In this work we are interested in viral hepatitis C.
In 2016, the World Health Organization issued global elimination targets for hepatitis C virus (HCV), including an 80% reduction in HCV incidence by 2030. Overall, epidemic modeling has and continues to play a critical role in informing HCV elimination strategies worldwide.

Key Words and Phrases: *Hepatitis C, Dynamic, Materials, sterilization, Epidemiological model*

2 Main results

- Develop a model using ordinary whole-order differential equations.
- Integrate different disease prevention strategies into the developed model.
- Complete mathematical analysis of the proposed model.
- Digitally simulate the results obtained.

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Optimal solution for minimization problem subject to differential system

Dib Karima*

LAOTI, FSEI, Université Mohammed Seddik Benyahia-Jijel, Algérie
E-mail: dibkarima2019@gmail.com

Azzam-Laouir Dalila

LAOTI, FSEI, Université Mohammed Seddik Benyahia-Jijel, Algérie
E-mail: laouir.dalila@gmail.com, dalilalaouir@univ-jijel.dz

Abstract
The objective of this work is to present the existence of optimal solutions of an optimal control problem subject to a couple of differential inclusions both of them governed by time and state-dependent maximal monotone operators of the form

$$(\mathcal{P})\begin{cases}-\dot{u}(t) \in \mathcal{A}(t,\xi(t))u(t) + f(t,u(t),v(t)) \quad a.e.\ t \in [0,T]\\ u(t) \in D(\mathcal{A}(t,\xi(t))) \quad \forall t \in [0,T]\\ -\dot{v}(t) \in \mathcal{B}(t,\xi(t))v(t) + g(t,u(t),v(t)) \quad a.e.\ t \in [0,T]\\ v(t) \in D(\mathcal{B}(t,\xi(t))) \quad \forall t \in [0,T]\\ u(0) = u_0 \in D(\mathcal{A}(0,\xi(0))), \quad v(0) = v_0 \in D(\mathcal{B}(0,\xi(0))).\end{cases}$$

1 Introduction

Let H be a real separable Hilbert space and $I = [0,T]$ $(T > 0)$. In the present work, we are mainly interested in the application of the existence and uniqueness of solutions of the above evolution system $(\mathcal{P})$, governed by time and state-dependent maximal monotone operators $\mathcal{A}(t,x)$ and $\mathcal{B}(t,x)$ of H to the problem of minimizing the cost function

$$\int_0^T L(t,u(t),v(t),\dot{u}(t),\dot{v}(t))\,dt$$

For this target, we study the existence and uniqueness of absolutely continuous solutions of $(\mathcal{P})$ under the following hypotheses

Hypotheses

$(H_1(\mathcal{A}))$ There exist a nonnegative real constant λ and a nonnegative and nondecreasing real function $\beta \in W^{1,1}(I,\mathbb{R})$ such that

$$dis(\mathcal{A}(t,x),\mathcal{A}(s,y)) \leq |\beta(t) - \beta(s)| + \lambda\|x - y\| \quad \forall t,s \in I, \forall x,y \in H.$$

Key Words and Phrases: *Absolutely continuous variation, maximal monotone operator, single valued perturbations, optimization problem, pseudo-distance.*

$(H_2(\mathcal{A}))$ There exists a nonnegative real constant $d_{\mathcal{A}}$, such that

$$\|\mathcal{A}^0(t,x)(y)\| \leq d_{\mathcal{A}}(1+\|x\|+\|y\|) \quad \forall (t,x,y) \in I \times H \times D(A(t,x)).$$

$(H_3(\mathcal{A}))$ For any bounded subset $K \subset H$, the set $D(\mathcal{A}(I \times K))$ is relatively ball-compact, that is, its intersection with any closed ball of H is relatively compact.
$(H_1(\mathcal{B}))$ There exist a nonnegative real constant α satisfying $\alpha\lambda < 1$, and a nondecreasing real function $\eta \in W^{1,1}(I,\mathbb{R})$ such that

$$dis(\mathcal{B}(t,x),\mathcal{B}(s,y)) \leq |\eta(t)-\eta(s)| + \alpha\|x-y\| \quad \forall t,s \in I, \forall x,y \in H.$$

$(H_2(\mathcal{B}))$ There exists a nonnegative real constant $d_{\mathcal{B}}$, such that

$$\|\mathcal{B}^0(t,x)(y)\| \leq d_{\mathcal{B}}(1+\|x\|+\|y\|) \quad \forall (t,x,y) \in I \times H \times D(\mathcal{B}(t,x)).$$

$(H_3(\mathcal{B}))$ For any bounded subset $K \subset H$, the set $D(\mathcal{B}(I \times K))$ is relatively ball-compact. (H_f^1) There exist a nonnegative real constant L_f such that

$$\|f(t,x,y)\| \leq L_f(1+\|x\|+\|y\|) \quad \forall (t,x,y) \in I \times H \times H.$$

(H_f^2) There exist a nonnegative real function $\omega_f \in L^1(I,\mathbb{R})$ such that

$$\|f(t,x,y)-f(t,x',y')\| \leq \omega_f(t)(\|x-x'\|+\|y-y'\|) \quad \forall (t,x,y),(t,x',y') \in I \times H \times H.$$

(H_f^1) There exist a nonnegative real constant L_g such that

$$\|g(t,x,y)\| \leq L_g(1+\|x\|+\|y\|) \quad \forall (t,x,y) \in I \times H \times H.$$

(H_g^2) There exist a nonnegative real function $\omega_g \in L^1(I,\mathbb{R})$ such that

$$\|g(t,x,y)-g(t,x',y')\| \leq \omega_g(t)(\|x-x'\|+\|y-y'\|) \quad \forall (t,x,y),(t,x',y') \in I \times H \times H.$$

2 Main results

Theorem 2.1. *Assume that for every $t \in I$, $\mathcal{A}(t) : D(\mathcal{A}(t)) \rightrightarrows H$ (resp. $\mathcal{B}(t) : D(\mathcal{B}(t)) \rightrightarrows H$) is a maximal monotone operator satisfying $(H_{\mathcal{A}}^1)$, $(H_{\mathcal{A}}^2)$ and $(H_{\mathcal{A}}^3)$ (resp. $(H_{\mathcal{B}}^1)$, $(H_{\mathcal{B}}^2)$ and $(H_{\mathcal{B}}^3)$). Assume also that $t \longmapsto J_1^{\mathcal{A}(t)}(x)$ (resp. $t \longmapsto J_1^{\mathcal{B}(t)}(x)$) is measurable for any fixed $x \in H$. Let $f,g : I \times H \times H \longrightarrow H$ satisfying respectively, (H_f) and (H_g).*
Let $L : I \times H \times H \times H \times H \longrightarrow [0,\infty[$ be a lower semicontinuous mapping such that $L(t,x,y,\cdot,\cdot)$ is convex on $H \times H$, for any $(t,x,y) \in I \times H \times H$. Then for $(u_0,v_0) \in D(\mathcal{A}(0)) \times D(\mathcal{B}(0))$, the problem of minimizing the cost function

$$\int_0^T L(t,u(t),v(t),\dot{u}(t),\dot{v}(t))\,dt$$

subject to problem $(\mathcal{P})$, has an optimal solution.

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Optimizing over the efficient set of the binary bi-objective knapsack problem

Lachemi Nadia*
Faculty of Mathematics, Department of Operations Research, Laboratory AMCD-RO, USTHB, Bab-Ezzouar, Algiers, Algeria
E-mail: Lacheminadia@hotmail.com

Chaabane Djamal
Faculty of Mathematics, Department of Operations Research, Laboratory AMCD-RO, USTHB, Bab-Ezzouar, Algiers, Algeria
E-mail: chaabanedj@yahoo.fr

Abstract
In this work, we focus on the problem of optimizing a linear function over the efficient set of a 0/1 bi-objective knapsack problem. Such a function represents the main criterion of the problem posed. The resolution process is based essentially on dynamic programming. The proposed method provides a subset of efficient solutions including one which optimizes the main criterion without having to enumerate all the efficient solutions of the problem. A numerical experiment is reported, different instances with large sizes of the associated efficient sets are considered to show the efficiency of our algorithm compared with a previous algorithm.

1 Introduction

A binary bi-objective knapsack problem denoted BOKP, can be stated mathematically as follow:

$$(BOKP)\begin{cases} \max\ Z_i(x) &= \displaystyle\sum_{j=1}^{n} p_j^i x_j,\ i=1,2 \\ & \displaystyle\sum_{j=1}^{n} w_j x_j \leq W, \\ & x_j \in \{0,1\}\ \forall j \in \{1,\dots,n\} \end{cases} \tag{1.1}$$

All coefficients p_j^i, w_j and W are supposed to be positive integers, $j=\overline{1,n},\quad i=1,2$ to avoid trivial solutions, we suppose that:

$w_j \leq W,\quad \forall j=1,...,n,$ and $\displaystyle\sum_{j=1}^{n} w_j > W.$

The set of efficient solutions is denoted E. The problem of optimization over the efficient set of the $BOKP$ is given by:

$$(P_E)\begin{cases} max\,\phi(x) &= \displaystyle\sum_{j=1}^{n} d_j x_j \\ & x=(x_1\ \dots\ x_n)' \in E \end{cases} \tag{1.2}$$

Where $d_j, j \in \{1,\dots,n\}$ is supposed to be a positive integer.

Key Words and Phrases: *Multiple objective programming, Bi-objective knapsack problem, dynamic programming, efficient set, optimal solution.*

2 Main results

In this work an algorithm is developed to optimize a linear function over the efficient set of a binary bi-objective knapsack problem without having to explicitly enumerate all the efficient solutions. Our proposal [2] is based on dynamic programming instead of linear programming or cut techniques which are the most used in the literature see [3]. The method consists of m stages of dynamic programming ($m \leq n$). However, at each stage we use some dominance relations proposed in [1] in order to eliminate some partial solutions that cannot provide efficient solutions. Another relation that we propose is used to discard some partial solutions that can not improve the main criterion, this relation allows to eliminate early more solutions either efficient or not. In order to obtain an optimal solution, we should keep the solutions in decision space in memory while the algorithm is running; this should require more memory space and a longer computation time with dynamic programming. Our algorithm employs an efficiency test program to ensure the efficiency of a solution and returns the corresponding efficient solution in decision space without storing the decision variables in memory.
To asses its performance, our algorithm is compared to a generic algorithm proposed in [4]; on a well-known instances of BOKP in terms of the percentage of nondominated solutions computed before arriving at an optimal solution, and the amount of CPU time used by each algorithm. The experimental study shows that the suggested algorithm performs better in the majority of the instances that were taken into consideration.

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A large-update of interior-point methods for convex quadratic programming based on a parametrized kernel function

Youssra Bouhenache *

Mailing address: Department of Mathematics, University of Mohammed Seddik Benyahia, Jijel, Algeria
E-mail: youssrabouhenache1997@gmail.com

Wided Chikouche

Mailing address: Department of Mathematics, University of Mohammed Seddik Benyahia, Jijel, Algeria
E-mail: wided.chikouche@gmail.com

Abstract
Interior-point methods provide a powerful approach for solving convex quadratic programming problem. Recently, S. Guerdouh, et al. [1] investigated a new parametrized kernel function. The new kernel function has a hyperbolic barrier term. In this paper, we generalize the analysis presented in the above paper for convex quadratic programming problems. For large-update interior point methods the iteration bound is the best currently known bound for primal-dual interior-point methods.

1 Introduction

Convex quadratic programming (CQP) is an optimization problem of minimizing a convex quadratic objective function subject to linear constraints.
We present the standard CQP problem (P) with its dual problem (D) as follows

$$(P) \qquad \min\left\{c^T x + \frac{1}{2}x^T Qx \ : \ Ax = b, \ x \geq 0\right\},$$

where $Q \in S^n_+$, $A \in \mathbb{R}^{m\times n}$, rank $(A) = m$, $b \in \mathbb{R}^m$, and $x \in \mathbb{R}^n$ with $m \leq n$,

$$(D) \qquad \max\left\{b^T y - \frac{1}{2}x^T Qx \ : \ A^T y - Qx + s = c, \ s \geq 0\right\},$$

where $y \in \mathbb{R}^m$ and $s, c \in \mathbb{R}^n$.
The concept of interior point method (IPM) for solving linear programming (LP) problems was first introduced by Karmarkar 1984 [2]. In 1994, the so-called primal-dual interior point methods (IPMs) used for solving CQP was later suggested by Nesterov and Nemirovski [3], they extended IPMs from LP to more general convex optimization problems. The purpose of this paper is to propose a new IPM for CQP based on the parametrized kernel function recently introduced in [1].

Key Words and Phrases: *Convex quadratic programming, Hyperbolic kernel function, Large-update methods.*

2 Main results

In this paper we extended the results obtained for a new parametric kernel-function-based IPMs introduced by Guerdouh et al. in [1] for LO to CQP problems. The kernel function has a hyperbolic barrier Term. We adopt the basic analysis used in [1] and revise them to be suited for the CQP case. we show that the large-update interior-point methods based on this kernel function has $\boldsymbol{O}\left(pn^{\frac{p+2}{2(p+1)}}\log\frac{n}{\epsilon}\right)$ iterations complexity for large-update methods which enjoyed the currently best known iteration bound. The iteration bounds are as good as the bounds for the LO case.

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Existence and Uniqueness of the Weak Solution for a Contact Problem

Fadlia Besma*

MMS Laboratory,Department of Mathematics, Faculty of Exact Sciences, Mentouri Brothers Constantine University, Constantine, Algeria
E-mail: besmafadlia@gmail.com

Dalah Mohamed

MMS Laboratory,Department of Mathematics, Faculty of Exact Sciences, Mentouri Brothers Constantine University, Constantine, Algeria
E-mail: mdalah.ufmc@gmail.com

Faizi Rima

LMA Laboratory, Department of Mathematics, University of Badji Mokhtar Annaba, Annaba, Algeria
E-mail: rima24math@gmail.com

Abstract
In this paper, We study the antiplane frictional contact models for electro-viscoelastic materials, both in quasistatic case. The material is assumed to be viscoelastic with short term memory and the friction is modelled with Tresca's law. we derive a variational formulation for each model that is in the form of an elliptic variational inequality for the displacement field, then we prove existence and uniqueness results for the weak solution.

1 Introduction

The piezoelectric effect results from the coupling between the electrical and the mechanical properties in which the body has the ability to produce an electrical field when a mechanical stress is present and, conversely, under the action of an electric field the body undergoes a mechanical stress.
The piezoelectricity was discovered by the brothers Curie in 1880 (Jacques and Pierre Curie). The piezoelectric materials generally are physically strong and chemically inert, and they are relatively inexpensive to manufacture.
The composition, shape and dimension of piezoelectric ceramic elements can be tailored to meet the requirements of a specific purpose.

Key Words and Phrases: *electro-viscoelastic, Tresca's law, variational formulation, weak solution*

2 Main results

we prove the existence of a unique weak solution to the model.

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Unbounded perturbation for an evolution problem

Nouha Boudjerida*

Mailing address: Department of Mathematics, University Mohamed Sadik Ben Yahia, jijel, Algeria
E-mail: nouhaboudjerida@gmail.com

Doria Affane

Mailing address: Department of Mathematics, University Mohamed Sadik Ben Yahia, jijel, Algeria
E-mail: affanedoria@yahoo.fr

Mustapha Fateh Yarou

Mailing address: Department of Mathematics, University Mohamed Sadik Ben Yahia, jijel, Algeria
E-mail: mfyarou@yahoo.com

Abstract
In this work, we present an existence result for a nonconvex perturbed sweeping process where the perturbation satisfy a weaker notion of Lipschitzianity with respect to the ρ-Hausdorff distance.

1 Introduction

The general class of differential inclusions known as the sweeping process has been introduced and thoroughly studied in the period of $70's$ by J. J. Moreau (see [3]) plays an important role in elastoplasticity and dynamics. The main purpose of this work is to consider, in the setting of finite dimensional space, a perturbed sweeping process problem for a closed prox-regular moving set depending on both the time and the state and has an absolutely continuous variation with respect to the Hausdorff distance, that differential inclusion can be expressed in the form

$$(\mathcal{SPP}_F)\begin{cases} -\dot{u}(t) \in N_{C_b(t,u(t))}(u(t)) + F(t,u(t)), \quad \text{a.e. } t \in [0,T], \\ u(t) \in C_b(t,u(t)) \quad \forall t \in [0,T], \\ u(0) = u_0 \in C_b(0,u_0), \end{cases}$$

where $F : [0,T] \times b\overline{B} \rightrightarrows \mathbb{R}^n$ represent the perturbation with nonempty closed values.

Key Words and Phrases: *Normal cone, prox regular sets, perturbation, set-valued map.*

2 Main results

Let $\psi : [0,T] \to \mathbb{R}^d$ be an absolutely continuous mapping with bounded measurable derivatives such that $\psi(0) = 0$ and $Q : \mathbb{R}^d \rightrightarrows [0,T] \times \mathbb{R}^d$ be a set-valued mapping define by

$$Q(\psi) = \left\{(t,y) \in [0,T] \times \mathbb{R}^d; \|\psi(t) - y\| \leq b,\ \forall b > 0\right\}.$$

We consider the following assumption :
$(\mathcal{H}_1^C)$ there is some constant $r \geq 0$ such that, for each $(t,u) \in Q(\psi)$, the sets $C(t,u)$ are nonempty closed uniformly r-prox -regulars values;
$(\mathcal{H}_2^C)$ there exists a constant $L \in [0,1[$ satisfies :

$$haus(C_b(t,u), C_b(s,v)) \leq |\chi(t) - \chi(s)| + L\|u - v\|,\ \forall\ (t,u),(s,v) \in Q(\psi),$$

where $\chi : [0,T] \to \mathbb{R}^+$ is an absolutely continuous mapping.
Let $F : [0,T] \times b\overline{B} \rightrightarrows \mathbb{R}^d$ with $b > 0$, be a set-valued mapping with nonempty closed values such that :
$(\mathcal{H}_1^F)$ $F(\cdot, w(\cdot))$ is measurable for every function $w \in \mathcal{C}_{\mathbb{R}^d}([0,T])$ with $\|w(t)\| \leq b$ for all $t \in [0,T]$;
$(\mathcal{H}_2^F)$ there exists $\alpha(\cdot),\ \beta(\cdot) \in L^2_{\mathbb{R}^+}([0,T])$, such that, for all $(t,y) \in [0,T] \times b\overline{B}$,

$$d(0, F(t,y)) \leq \alpha(t) + \beta(t)\|y\|;$$

$(\mathcal{H}_3^F)$ for all $\rho \geq 0$ and $y_1 \neq y_2$ with (t,y_1), (t,y_2) in $[0,T] \times b\overline{B}$,

$$haus_\rho(F(t,y_1), F(t,y_2)) < \beta(t)\|y_1 - y_2\|.$$

Theorem 2.1. *Let $\mathbb{R}^d$ the d-dimensional Euclidean space, $r > 0$ and Assume that $(\mathcal{H}_1^C)$, $(\mathcal{H}_2^C)$, $(\mathcal{H}_1^F)$, $(\mathcal{H}_2^F)$ and $(\mathcal{H}_3^F)$ hold. Then for every $u_0 \in C_b(0,u_0)$, the problem $(\mathcal{SPP}_F)$ has an absolutely continuous solution $u : [0,T] \to \mathbb{R}^d$ satisfying*

$$\|\dot{u}(t)\| \leq n_1(t),\ a.e.\ t \in [0,T],$$

with $n_1(t) = \frac{1}{1-L}\big(\dot{\chi}(t) + (2-L)(\alpha(t) + \beta(t)n)\big)$ *and* $n = \frac{1}{L-1}(\|u_0\| + 2 - L) - \frac{1}{(1-L)^2}\int_0^T \dot{\chi}(s)ds.$

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Non-convex Sweeping Process Problem with Drift

Arroud Chemseddine*

Department of Mathematics , University of Jijel, Algeria
E-mail: arroud.math@gmail.com

Giovanni Colombo

Department of Mathematics, University of Padova, Italy
E-mail: colombo@math.unipd.it

Abstract
In this work we studied a problem of free endpoint Bolza problem for the controlled Moreau process ;
Minimize $h(x(T))$ subject to

$$\begin{cases} \dot{x}(t) \in -N_{C(t)}(x(t)) + f(x(t), u(t))), \\ x(0) = x_0 \in C(0) , \quad u(t) \in U \end{cases} \tag{0.1}$$

Where $C(t)$ is a closed non-convex moving set, with normal cone $N_{C(t)}(x)$ at $x \in C(t)$, U being the control set and f being smooth.
We try to prove necessary optimality conditions in the form of Pontryagin maximum principle.

1 Introduction

Jean Jacques Moreau in his paper [2] studied a special type of differential inclusion called sweeping process, its wording is the differential inclusion

$$\dot{x}(t) \in -N_{C(t)}(x(t)), \quad x(0) = x_0 \in C(0), a.e \quad t \in [0, T], \tag{1.1}$$

Where $C(t)$ is a nonempty closed set in a Hilbert space H and $N_{C(t)}(x)$ is the normal cone to $C(t)$ at $x \in H$, with $N_{C(t)}(x) = \emptyset$ whenever $x \in H \backslash C(t)$. The essential idea of this process is to depict the movement of a point contained inside a set, and as the set moves, the point is swept by it. The sweeping process covers several mechanical problems, as in elastoplasticity quasistatics and dissipative systems in electrical circuits. The extended sweeping process when external forces are present takes the form

$$\dot{x}(t) \in -N_{C(t)}(x(t)) + f(t, x(t)), \quad x(0) = x_0 \in C(0), a.e \quad t \in [0, T], \tag{1.2}$$

where $f : I \times H \to H$ is a mapping that is measurable with respect to the time-variable and Lipschitz with respect to the state-variable. This differential inclusion (1.2) considered by Moreau to deal with mechanical contact problems.

Key Words and Phrases: *Moreaus sweeping process, adjoint equation, Optimal control, Maximality condition*

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Weather Forecasting enhacement by an ARIMA-LSTM hybrid model

GASMI Laid*

Mailing address:Department of Mathematics and Computer Science Adrar University, Algeria.
E-mail: gasmi1983@gmail.com

KABOU Salaheddine

Mailing address: Mailing address:Department of Mathematics and Computer Science Adrar University, Algeria
E-mail: kabou.salaheddine@yahoo.fr

Abstract
Time series forecasting is an important topic in finance, economics,and business. This work studies the applicability and potential of forecasting techniques such as ARIMA,as well as recurrent neural networks, in particular the (LSTM) model. The proposed approach uses the unique strengths of ARIMA and LSTM to improve forecast accuracy. The results clearly show that the proposed method (ARIMA-LSTM). Here, the goal is to create a hybrid technique that can handle both linear and nonlinear interactions and is most appropriate for most situations. The results clearly show that the proposed method (ARIMA-LSTM); achieves the best prediction accuracies for the series studied.

1 Introduction

Weather forecasting has become an important area of research in recent decades. It is always best to establish a linear relationship between the input weather data and the corresponding target data. But with the discovery of nonlinearity in the nature of meteorological data, attention has shifted to wards the nonlinear prediction of meteorological data. So it has two parts as mentioned below:
1. Rain prediction, i.e. whether it will rain or not.
2. Forecast of different meteorological factors that will help us predict rainfall. To achieve this second objective, a hybrid time series prediction model is built, which will try to learn the relationship between the weather factor and time.

Key Words and Phrases: *Time series, ARIMA, LSTM, Hybrid model.*

2 Main results

In this poster we presented the advantage of ARIMA-LSTM hybridization to improve the accuracy of time series forecasts. This work proposes a forecasting technique by separating a time series dataset into linear and nonlinear components. Then, autoregressive integrated moving average (ARIMA) and artificial neural network (ANN) models are used to separately recognize and predict the reconstructed detailed and approximate components, respectively. In this way, the proposed approach uses the unique strengths of ARIMA and LSTM to improve forecast accuracy. The results clearly show that the proposed method achieves the best prediction accuracies for the series studied.

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Etude comparative entre deux méthodes hybrides du gradient conjugué en utilisant différent types de recherche linéaire

Ouaoua Mohamed Lamine*

Laboratoire LMFN, Département de Mathématiques, Université Ferhat Abbas Sétif1, Algérie
E-mail: mohamedlamine.ouaoua@univ-setif.dz

Khelladi Samia and Benterki Djamel

Laboratoire LMFN, Département de Mathématiques, Université Ferhat Abbas Sétif1, Algérie
E-mail: samia.boukaroura@univ-setif.dz, djbenterki@univ-setif.dz

Abstract
La méthode du gradient conjugué est l'une des méthodes les plus efficaces pour résoudre les problèmes d'optimisation non linéaire sans contraintes.
Dans ce travail, on a fait une étude comparative nmérique entre deux méthodes hybrides du gradient conjugué, parmi les plus récentes, la méthode **MMDL** et **MLSCD,** en utilisant différentes règles de rcherche linéaire inexacte. Les tests numériques ont été effectués sur plusieurs fonctions tests et pour différentes dimensions (n).

1 Introduction

Soit le problème d'optimisation sans contraintes:

$$(P\ )\begin{cases} \min f(x) \\ x \in \mathbb{R}^n \end{cases}, \quad \text{où } f : \mathbb{R}^n \longrightarrow \mathbb{R}$$

Parmi les méthodes les plus utilisées pour résoudre ce type de problèmes, on a la méthode du gradient conjugué.Elle génère une suite $\{x_k\}_{k\in\mathbb{N}^*}$ de la façon suivante:

$$\begin{cases} x_1 & \text{point initial} \\ x_{k+1} = x_k + \alpha_k d_k & k \geq 1 \end{cases}.$$

Le pas $\alpha_k \in \mathbb{R}$ est déterminé par une recherche linéaire exacte ou inexacte.
Les directions d_k sont calculées de façon récurrente par les formules suivantes:

$$d_k = \begin{cases} -g_k & k = 1 \\ -g_k + \beta_k d_{k-1} & k \geq 2 \end{cases},$$

où $g_k = \nabla f(x_k)$ et β_k est un scalaire.

Key Words and Phrases: *Optimisation sans contraintes, Méthode du gradient conjugué, Recherche linéaire inexacte, Convergence globale, Méthode hybride*

Les différentes valeurs attribuées à β_k définissent les différentes variantes de la méthode du gradient conjugué. Parmi les β_k les plus connus, on a:

$$\begin{aligned}
\beta_k^{CD} &= -\frac{\| g_k \|^2}{g_{k-1}^T d_{k-1}}, \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{\| g_{k-1} \|^2}, \quad \beta_k^{LS} = -\frac{g_k^T y_{k-1}}{g_{k-1}^T d_{k-1}}, \\
\beta_k^{DHSDL} &= \frac{\| g_k \|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} \mid g_k^T g_{k-1} \mid}{\mu \mid g_k^T d_{k-1} \mid + d_{k-1}^T y_{k-1}} - t\frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, \quad \mu > 1, \ t > 0, \\
\beta_k^{DLSDL} &= \frac{\| g_k \|^2 - \frac{\|g_k\|}{\|g_{k-1}\|} \mid g_k^T g_{k-1} \mid}{\mu \mid g_k^T d_{k-1} \mid - d_{k-1}^T g_{k-1}} - t\frac{g_k^T s_{k-1}}{d_{k-1}^T y_{k-1}}, \quad \mu > 1, \ t > 0,
\end{aligned}$$

où $y_{k-1} = g_k - g_{k-1}$, $s_{k-1} = x_k - x_{k-1}$ et $\| \, . \, \|$ désigne la norme euclidienne.

2 Etude comparative et résulats

On considère la méthode **MLSCD**, donnée avec la direction de recherche

$$d_k = \begin{cases} -g_1 & k = 1 \\ -\left(1 + \beta_k^{LSCD} \frac{g_k^T d_{k-1}}{\|g_k\|^2}\right) g_k + \beta_k^{LSCD} d_{k-1} & k \geq 2, \end{cases} \text{, tel que}$$

$$\beta_k^{LSCD} = \max\left\{0, \min\left\{\beta_k^{LS}, \beta_k^{CD}\right\}\right\},$$

Pour la seconde méthode hybride **MMDL,** la diretion de recherche est donnée par :

$$d_k = \begin{cases} -g_1 & k = 1 \\ -\left(1 + \beta_k^{MMDL} \frac{g_k^T d_{k-1}}{\|g_k\|^2}\right) g_k + \beta_k^{MMDL} d_{k-1} & k \geq 2 \end{cases} \text{, tel que}$$

$$\beta_k^{MMDL} = \max\left\{0, \min\left\{\beta_k^{DHSDL}, \beta_k^{DLSDL}\right\}\right\},$$

Dans ce travail, on a fait une étude comparative numérique entre ces deux méthodes hybrides du gradient conjugué (**MMDL** et **MLSCD),** en utilisant différentes règles de recherche linéaire inexacte, à savoir celle d'Armijo, de Wolfe et de Backtracking, pour calculer le pas de déplacement α_k. Les tests numériques ont été effectués sur plusieurs fonctions tests et pour différentes dimensions (n).

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An Effectual Logarithmic Barrier Method without Line Search for linear Optimization

Maiza Asma*

Mailing address: Department of Mathematics, Ferhat Abbas University of Setif-1, Algeria
E-mail: asma.maiza@univ-setif.dz

Leulmi Assma

Mailing address: Department of Mathematics, Ferhat Abbas University of Setif-1, Algeria
E-mail: as_smaleulmi@yahoo.fr

Abstract
This paper presents a logarithmic barrier method without line search for solving linear programming problem. The descent direction is the classical Newton's one. However, the displacement step is determined by a simple and efficient technique based on the notion of the majorant function approximating the barrier function.

1 Introduction

Linear optimization or linear programming problems are an important class of optimization problems that helps to find the feasible region and optimize the solution in order to have the highest or lowest value of the function. These, it can be viewed as part of a great revolutionary development which has given mankind the ability to state general goals and to lay out a path of detailed decisions to be taken in order to achieve the "best" for its goals when faced with practical situations of great complexity. It is now well established as an important and very active branch of applied mathematics. The wide applicability of linear programming models and being a rich mathematical theory which underlying these models and methods developed to solve them have been the driving forces behind the rapid and continuing evolution of the subject. In this paper, we propose a logarithmic barrier interior-point method for solving linear programming problems. In fact, the main difficulty to be anticipated in establishing an iteration in such a method will come from the determination and computation of the step-size.
We consider the following linear programming problem

$$(D)\begin{cases} \min\limits_{x} b^T x \\ A^T x \geq c, \ x \in \mathbb{R}^m. \end{cases}$$

Where $A \in \mathbb{R}^{m\times n}$, such that $rangA = m < n$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$.

Key Words and Phrases: *Linear Programming, Logarithmic Barrier methods. Majorant function*

2 Main results

The problem (D) is approximated by the following perturbed problem (D_η)

$$(D_\eta)\begin{cases} \min \ f_\eta(x) \\ x \in \mathbb{R}^m, \end{cases} \tag{2.1}$$

with the penalty parameter $\eta > 0$, and f_η is the barrier function defined by

$$f_\eta(x) = \begin{cases} b^T x + n\eta \ln \eta - \eta \sum_{i=1}^{n} \ln \langle e_i, \ A^T x - c \rangle & \text{if} \quad A^T x - c > 0 \\ +\infty & \text{if not.} \end{cases}$$

where $(e_1, e_2, ..., e_n)$ is the canonical base in $\mathbb{R}^n$. We are interested then in solving the problem (D_η). We know that if the matrix $H = \bigtriangledown^2 f_\eta(x)$ is positive definite, then the problem (D_η) is strictly convex, if it has a solution it is unique. In 2008, B. Merikhi and J.P. Crouzeix proposed an interior point method to solve (D_η) of logarithmic barrier type. The main advantage of this method is the calculation of an optimal solution. $(x_{k+1} = x_k + \alpha_k d_k)$.
- d_k the descent direction takes the Newton direction defined by: $[\nabla^2 f_\eta(x) d_k = -\nabla f_\eta(x)$.
- α_k we are interested in the theoretical performances of this method, we study more closely the effective calculation of the step of displacement by a new technique concerning the upper bound functions. we approximate the function:$\varphi(\alpha) = \frac{1}{\eta}(f_\eta(x + \alpha d) - f_\eta(x))$. by simple upper bound functions. We can show that:

$$\varphi(\alpha) = n\left(\sum_{i=1}^{n} z_i\right)\alpha - \|z\|^2\alpha - \sum_{i=1}^{n} \ln(1 + z_i\alpha), \ \alpha \in [0, \widehat{\alpha}].$$

such that, $\varphi(\alpha)$ verifies the following properties : $\|z\|^2 = n(\bar{z}^2 + \sigma_z^2) = \varphi''(0) = -\varphi'(0)$, $\varphi(0) = 0$. In our case, we take $x_i = 1 + tz_i$, so we have $\bar{x} = 1 + t\bar{z}$ and $\sigma_x = \alpha\sigma_z$. Which gives me the lower bound function, then:

$$\widetilde{\varphi}_0(\alpha) = \frac{\|z\|^2}{\beta_0}\alpha - k\ln(1 + \frac{\|z\|^2}{\beta_0}\alpha).$$

Such as : $\beta_0 = \bar{z} + \frac{\sigma_z}{\sqrt{n-1}}$. The logarithms are well defined as soon as $\alpha \leq \widehat{\alpha}_0$. So we deduce the following reduction for all $\alpha \in [0, \widehat{\alpha}_0[$: $\widetilde{\varphi}_0(\alpha) \geq \varphi(\alpha)$. With $\widetilde{\varphi}_0$ is convex and satisfies the conditions.

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Logarithmic barrier method via a new minorant function for Convex Quadratic Programming

Choubeila Souli*

Department of Mathematics, Ferhat Abbas University of Setif-1, Algeria
E-mail: choubeila.souli@univ-setif.dz

Assma Leulmi

Department of Mathematics, Ferhat Abbas University of Setif-1, Algeria
E-mail: as_smaleulmi@yahoo.fr

Abstract
This paper presents a logarithmic barrier method without line search for solving Convex Quadratic Programming problem. The descent direction is the classical Newton's one. However, the displacement step is determined by a simple and efficient technique based on the notion of the majorant function approximating the barrier function.

1 Introduction

Linearly constrained convex quadratic programming problems arise in several areas of applications, such as economic, social, public planning and manufacturing. Several approaches and numerous algorithms have been proposed to solve this class of problems. Quadratic programming problems share many of the combinatorial properties of linear programming problems. These properties allow devising algorithms extending the simplex method to solve quadratic problems. In this paper, we propose a logarithmic barrier interior-point method for solving linear programming problems. In fact, the main difficulty to be anticipated in establishing an iteration in such a method will come from the determination and computation of the step-size.
We consider the following linear programming problem

$$(PQ)\begin{cases}\min \;\; q(x) & = \frac{1}{2}x^tQx \\ A^tx \geq b & x \in \mathbb{R}^m\end{cases}$$

Where where Q is a $\mathbb{R}^{n\times n}$ symmetric semidefinite matrix, $A \in \mathbb{R}^{m\times n}$, such that $rangA = m < n$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$.

Key Words and Phrases: *Convex Quadratic Programming, Logarithmic Barrier methods. Mainorant function.*

2 Main results

The problem (PQ) is approximated by the following perturbed problem (D_η)

$$(PQ_\eta)\begin{cases} \min\ q_\eta(x) \\ x \in \mathbb{R}^m, \end{cases} \tag{2.1}$$

with the penalty parameter $\eta > 0$, and f_η is the barrier function defined by

$$q_\eta(x) = \begin{cases} q(x) - \eta \sum_{i=1}^{m} \ln \langle e_i,\ Ax - b\rangle & if \quad A^T x - b > 0 \\ +\infty & otherwise. \end{cases}$$

where $(e_1, e_2, ..., e_n)$ is the canonical base in $\mathbb{R}^n$. We are interested then in solving the problem (PQ_η). We know that if the matrix $H = \nabla^2 q_\eta(x)$ is positive definite, then the problem (PQ_η) is strictly convex, if it has a solution it is unique. In 2008, B. Merikhi and J.P. Crouzeix proposed an interior point method to solve (PQ_η) of logarithmic barrier type. The main advantage of this method is the calculation of an optimal solution. $(x_{k+1} = x_k + \alpha_k d_k)$.
- d_k the descent direction takes the Newton direction defined by: $\nabla^2 q_\eta(x) d_k = -\nabla q_\eta(x)$.
- α_k we are interested in the theoretical performances of this method, we study more closely the effective calculation of the step of displacement by a new technique concerning the minorant function. We approximate the function: $\varphi(\alpha) = \frac{1}{\eta}\left(q_\eta(x + \alpha d) - q_\eta(x)\right)$. by simple minorant function. We can show that:

$$\varphi(\alpha) = n\left(\sum_{i=1}^{n} z_i\right)\alpha - \|z\|^2\alpha - \sum_{i=1}^{n} \ln(1 + z_i\alpha) + \frac{1}{2\eta}\alpha^2 d^t Q d - \frac{1}{\eta}\alpha d^t Q d,\ \alpha \in [0, \widehat{\alpha}].$$

Such that $z_i = \frac{\langle e_i, Ad\rangle}{\langle e_i, Ax-b\rangle}$, $\varphi(\alpha)$ verifies the following properties :

$$\|z\|^2 = n(\bar{z}^2 + \sigma_z^2) = \varphi''(0) = -\varphi'(0),\ \varphi(0) = 0.$$

In our case, we take $x_i = 1 + tz_i$, so we have $\bar{x} = 1 + t\bar{z}$ and $\sigma_x = \alpha\sigma_z$. Which gives me the lower bound function, then: $\widetilde{\varphi}_0(\alpha) = \frac{\|z\|^2}{\beta_0}\alpha - k\ln(1 + \frac{\|z\|^2}{\beta_0}\alpha) - \frac{1}{r}\widehat{\alpha} d^t Q d$. Such as : $\beta_0 = \bar{z} + \sigma_z\sqrt{n-1}$ The logarithms are well defined as soon as $\alpha \leq \widehat{\alpha}\widehat{\alpha}_0 : \widetilde{\varphi}_0(\alpha) \geq \varphi(\alpha)$. With $\widetilde{\varphi}_0$ is convex and satisfies the conditions.

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Les nombres de domination dans les réseaux

BEKHTI Yamina[1]
(1, 2) *Département de Recherche opérationnelle, Université des Sciences et de la Technologie Houari Boumedienne, Alger, Algérie*
E-mail: ybekhti@usthb.dz

Méziane AÏder[2], **Mohammed Lalou** [3], **Hamamache Kheddouci**[4]
(3, 4) *Laboratoire LIRIS, Universite Claude Bernard Lyon 1, France*
E-mail: meziane.aider@usthb.edu.dz, mlalou@liris.cnrs.fr,
hamamache.kheddouci@univlyon1.fr

Abstract

La domination positive est une notion combinatoire et caractéristique qui permet de modéliser la propagation de l'influence positive entre des individus et des groupes d'individus en relation (via les réseaux sociaux) pour lutter contre les fléaux sociaux. Le paramètre associé à cette notion est le nombre de domination positive. Il représente le cardinal minimum d'un ensemble d'individus pouvant influer positivement sur tous les individus d'un groupe constitué. Le problème de détermination de ce paramètre dans le graphe modélisant le réseau est NP-dur dans le cas général. Ce qui exclut l'utilisation de méthodes exactes. Ceci conduit soit à recourir à des méthodes approchées, soit à identifier des classes particulières de graphes pour étudier ce paramètre.

Nombres de domination

Domination traditionnelle

Soit $G = (V, E)$ un graphe simple. Un sous-ensemble D de sommets de $V(G)$ est un ensemble dominant si tout sommet de $V - D$ a au moins un voisin dans D. Le cardinal minimum d'un ensemble dominant de G, noté $\gamma(G)$, est appelé nombre de domination.

Domination positive

Un ensemble dominant positif est un sous-ensemble P de sommets dans G, si chaque sommet u dans $V - P$ est dominé par au moins $\rho d(u)$ sommets dans P, où:
ρ: appelé facteur d'influence et est $0 < \rho < 1$, (souvent $\rho = 0.5$).
Autrement dit,

$$\forall u \in V \backslash P : |N(u) \cap P| \geq \rho d(u).$$

Le problème s'écrit comme suit :

Dom Pos : Recherche d'un ensemble dominant positif

Données : Un graphe non orienté G.;
Résultat : Un ensemble P dominant positif dans G, de cardinal minimum.

Key Words and Phrases: *Ensemble dominant positif, Réseaux sociaux en ligne, Fléaux sociaux.*

Domination positive totale

Un ensemble dominant positif total est un sous-ensemble T de sommets dans G, si chaque sommet u dans V est dominé par au moins $\rho d(u)$ sommets dans T.
Autrement dit,

$$\forall u \in V : |N(u) \cap V| \geq \rho d(u).$$

Le problème s'écrit comme suit :

Dom Pos : Recherche d'un ensemble dominant positif total

Données : Un graphe non orienté G.;
Résultat : Un ensemble T dominant positif total dans G, de cardinal minimum.

Complexité

Wang et al. [?] ont montré que les problèmes de recherche d'ensemble dominant positif de cardinal minimum dans un graphe donné sont des problèmes NP-complets. Wang et al. [?] ont par ailleurs démontré par des L-réductions à partir des problèmes de couvertures de graphes cubiques, que les problèmes de domination positive sont APX-durs.

Conclusion

Les nombres de domination sont utilisés dans divers situations concrètes et leurs développements théoriques ont contribués dans la résolution de problèmes pratiques tels que les problèmes reliés aux réseaux de communications et informatiques, systèmes de surveillances par Radars, réseaux électriques, ... Dans le cadre de notre recherche, on estime exploiter ces essentielles notions de domination et les caractéristiques combinatoires des graphes réseaux pour éliminer certains fléaux sociaux du monde physique via les réseaux sociaux en ligne comme Facebook, Instagram, Tweeter, ...

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Optimization Approach for Modelling Dielectric Behavior of Composite Materials in Wide Frequency Band

Abdelhalim Brahimi*
Scientific instrumentation laboratory (LIS), Department of Electronics, Faculty of technology, Ferhat Abbas Sétif 1 University, Setif 19000, Algeria
E-mail: abdelhalim.brahimi@univ-setif.dz

Nacerdine Bourouba
Scientific instrumentation laboratory (LIS), Department of Electronics, Faculty of technology, Ferhat Abbas Sétif 1 University, Setif 19000, Algeria
E-mail: bourouband@yahoo.fr

Labiba Chioukh
Scientific instrumentation laboratory (LIS), Department of Electronics, Faculty of technology, Ferhat Abbas Sétif 1 University, Setif 19000, Algeria
E-mail: labiba.chioukh@univ-setif.dz

Nacerdine Bouzit
Scientific instrumentation laboratory (LIS), Department of Electronics, Faculty of technology, Ferhat Abbas Sétif 1 University, Setif 19000, Algeria
E-mail: bouzitnacerdine@yahoo.fr

Rabah Delfouf
Scientific instrumentation laboratory (LIS), Department of Electronics, Faculty of technology, Ferhat Abbas Sétif 1 University, Setif 19000, Algeria
E-mail: rabah.delfouf@univ-setif.dz

Abstract
Composite materials are materials with high mechanical performance, which can be shaped at will by the designer and therefore endowed with unlimited potential. These materials are being developed today in practically all fields and are the source of formidable challenges in various high-tech projects. They have become essential for construction in several fields such as aerospace, shipbuilding, automotive.... As their name suggests, they are composed of several constituents which mainly come in two phases: the matrix and the reinforcement. In the same matrix of epoxy resin (RE), ZnO powder was evenly mixed with one of two titanate powders, CaTiO3 (CT) or MgTiO3 (MT). The dielectric properties of these samples were obtained using the time domain spectroscopy (TDS) measurement technique in the range [DC-10 GHz]. During this work, low-frequency analyses (500 MHz) were carried out, mainly focusing on the static permittivity and conductivity variation as a function of zinc oxide volume fractions. The results allowed us to confirm the validity and effectiveness of the modified Lichtenecker Law (MLL). Good agreement can be seen between the MLL permittivity values and experimental ones for ternary composites. This study interest lies on these materials application in components used in microelectronics and telecommunication such as antennas, filters, resonators, wave absorbers, capacitors, charge storage devices, substrates and cavities.

Key Words and Phrases: *Composite material, modelling, MLL, permittivity, conductivity, time domain spectroscopy*

1 Introduction

The protocol used to characterize the different samples is the Time Domain Spectroscopy (TDS) in the range [DC-10 GHz] by performing also a particular study at low frequency (500 MHz) [1]. Additionally, a comparative study is established on two types of titanates added to zinc oxide following different proportions. This has been carried out to illustrate the effect of the two titanates (CT and MT), in the presence of ZnO, on the permittivity and conductivity of these ternary composites. According to previous studies carried out on the modified Lichtenecker law [2] for the multi-phase mixtures complex permittivity evaluation, the predicted values achieved with this modeling law were in good agreement with the results found experimentally. The proposed model must be validated by comparing it with the experimental data [3].

2 Main results

A noticeable effect has been recorded at the low frequency, the addition of CT made it possible to increase the permittivity more than MT while the latter contributed to obtain the lowest conductivity. This investigation aims to highlight the effect of titanate and zinc oxide on the dielectric behavior of the composite as well as on the shape factor provided by the modified Lichtenecker law. It follows an optimization approach of the predictive model of this law based on a better smoothing of this shape factor in order to allow a connection between the theoretical and experimental results. The results allowed us to confirm the validity and effectiveness of the modified Lichtenecker Laws (MLL). Significant agreement accord can be shown between the MLL predicted values and experimentally ones for ternary combinations. The performance of the proposed law is confirmed by compared it with Lichtenecker law values. The permittivity values of the ternary composites determined using this model show good agreement with the experimental results. This concordance was supported by the small relative errors quantified from these findings. Additionally, the polynomial approximation method applied to the MLL model ternary model is responsible for the model optimization merit.

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Sentinels for the identification of pollution term

Merabti Nesrine Lamya*

Mailing address: Department of Mathematics and Informatics, University of Larbi Ben M'hidi, Oum el Bouaghi, Algeria
E-mail: merabti.nesrine@univ-oeb.dz

Rezzoug Imad

Mailing address: Department of Mathematics and Informatics, University of Larbi Ben M'hidi, Oum el Bouaghi, Algeria
E-mail: rezzoug.imad@univ-oeb.dz

Abstract
In this work, we are interseted to study the method of sentinel to identify the pollution terms system with incomplete data. The main tool is the adjoint state. We use the method of HUM "Hilbert uniqueness method". The Taylor development of the first order of sentinel is gives us the identification of the pollution terms.

1 Introduction

The modelling of environmental problems leads to mathematical system with missing data. In 1795 Gauss and Legendre elaborate the method of least square, it is the most popular parameter identification thechnique. This thechnique consists in minimizing the distance between observed values y_{obs} and calculated values $y(v)$. After that J.L.Lions introduced another method in 1992 which called sentinel method. This method provides informations on a parameter or an approximation of the latter or which called the pollution term $(\lambda\widehat{\epsilon})$. To hope to be able to obtain some information therefore it's necessary to observe y. We observe the state on an observatory O.

2 Main results

In this work, we define the parabolic system on an open of R^n, $\partial\Omega = \Gamma$,

$$\begin{cases} \dfrac{\partial y}{\partial t} + Ay + f(y) = \epsilon + \lambda\hat{\epsilon} & \text{in} \quad \Omega \times (0,T) \\ y(0) = y^0 + \tau\hat{y}^0 & \\ y = 0 & \text{on} \quad \Sigma = \Gamma \times (0,T) \end{cases} \tag{2.1}$$

- A : is an elleptic differential operator second operator such that

$$Ay = -\frac{\partial}{\partial x_i}(a_{ij}\frac{\partial y}{\partial x_j})$$

Key Words and Phrases: *Sentinel method, pollution term, Controllability, incomplete data*

- f : a nonlinear operator.
- $\lambda\hat{\epsilon}$: pollution term.
- y^0 : initial condition is in a Hilbert or Banach space and

$$\begin{cases} \|y^0\| \leq 1 \\ \lambda, \tau \text{ are small.} \end{cases}$$

The objective of our work is to try to estimate $(\lambda\widehat{\epsilon})$ without trying to known or estimate the missing term $\tau\widehat{y}^0$. It suffices to show that the problem is equivalent to an exact zero controllability whith the HUM "Hilbert Uniqueness Method". The sentinel function, once constructed of the pollution term is deduced by the Taylor developement of the first order of S such that

$$S(\lambda, \tau) = \int_0^T \int_O (h+u)y(x,t,\lambda,\tau)dxdt$$

To gain the control u, we use a classical method called penalization method with :

$$J_\epsilon(u,z) = \frac{1}{2}\|u\|_{L^2(]0,T[\times O)} + \frac{1}{2\epsilon}\left\|-\frac{\partial z}{\partial t} + A^*z + f'(y_0)z - (h+u)\chi_O\right\|^2_{L^2(Q)}$$

and

$$(P_\epsilon)\left\{ \min_{(u,z)\in A^\epsilon} J_\epsilon(u,z) \right.$$

with

$$A^\epsilon = \left\{ (u,z)\text{tel que} \;\middle|\; \begin{array}{c} -\dfrac{\partial z}{\partial t} + A^*z + f'(y_0)z - (h+u)\chi_O \in L^2(Q) \\ z(T) = z(0) \text{ in } \Omega \\ z = 0 \text{ on } \Sigma \end{array} \right\}$$

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Scalarization method for finding the efficient frontier in multiobjective problems

Wafa Bouguern*

Mailing address: Department of mathematics, Bordj Bou Arreridj university, Setif, Algeria
E-mail: wafa.bouguern@univ-bba.dz

Smail Addoune

Mailing address: Department of mathematics, Bordj Bou Arreridj university, Bordj Bou Arreridj, Algeria
E-mail: addoune@gmail.com

Abstract
This paper is dedicated to study some scalarization methods commonly used in multiobjective optimization, the properties of these methods are examined with respect to basic features such as ordering cone, convexity and boundedness. We made numerical tests and a comparison between these different methods.

1 Introduction

Several computational methods have been proposed for characterizing pareto optimal solutions depending on the different scalarizations of the multiobjective optimization problem. Among the many possible ways of scalarizing the multiobjective linear programming problems, the Weighting method, ε-constraint method, Benson's method, Elastic Constraint method.
For determining solutions of the multiobjective optimization problem (MOP)

$$(MOP) \qquad \begin{cases} \min_x f(x) := (f_1(x), \dots, f_p(x)) \\ \text{subject to} \\ x \in X. \end{cases}$$

a widespread approach is the transformation of this problem to a scalar-valued parameter dependent optimization problem. This is done for instance in the Weighted Sum method [2], where we solve

$$(WSP(\lambda)) \qquad \min_x \sum_{k=1}^{p} \lambda_k f_k(x)$$

The Weighted Sum problem $(WSP(\lambda))$ uses the vector of weights $\lambda \in \mathbb{R}^p_{\geq}$ as a parameter.

Key Words and Phrases: *Weighted Sum method, ε-constraint method, Benson's method, Elastic Constraint method, Multiobjective optimization, Pareto front.*

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Another scalarization is based on the minimization of only one of the p objectives while all the other objectives are transformed into constraints by introducing upper bounds. This scalarization is called ε-constraint method and is given by

$$(EC_j(\varepsilon)) \qquad \begin{cases} min\ f_j(x) \\ x \in X \\ s.c \\ f_k(x) \leq \varepsilon_k \quad k = \{1, ...p\} \,/\, \{j\}\,, \quad k \neq j \end{cases}$$

Here the parameters are the upper bounds $\varepsilon_k, k = \{1, ...p\} \,/\, \{j\}$ for a $k \in \{1, ...p\}$ Surveys about different scalarization approaches can be found in [2] [1].
For the ε-constraint method we have no results on properly efficient solutions. In addition, the scalarized problem $(EC_j(\varepsilon))$ may be hard to solve in practice due to the added constraints $f_k(x) \leq \varepsilon_k$. In order to address this problem we can relax these constraints by allowing them to be violated and penalizing any violation in the objective function. Ehrgott and Ryan (2002) used this idea to develop the Elastic Constraint scalarization [2], the last method that we will present and discuss in this paper is the Benson's method. Finally we made numerical tests and a comparison between these different methods.

2 Main results

Weighted sum method finds very few solutions, and moreover, it does not make it possible to find solutions enclosed in concavities, contrary to ε-constraints method.
When μ(parameter of elastic constraint method) tends to ∞ the problem of elastic constraints approaches to the problem of ε-constraints.
If the initial solution is chosen appropriately, Benson's method correctly computes efficient solutions.
We have shown that scalarization methods for multicriteria optimization problems are not able to find all effective solutions.

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Salp Swarm Optimizer Combined with Chaotic Maps

Nacer Meriem*
Mailing address: Mathematics, Constantine 1, Setif, Algeria
E-mail: Souhilanacer24@gmail.com

Hamaizia Tayeb
Mailing address: Mathematics, Constantine 1, Setif, Algeria
E-mail: h2tayeb@gmail.com

Abstract
Salp Swarm Algorithm (SSA) is one of the most recently proposed meta-heuristic optimization algorithms driven by the simulation behavior of salps, it has manifested its capability in solving various optimization problems and many real-life applications. However, similar to most of the meta-heuristic algorithms, it suffered from stagnation in local optima and low convergence rate. So, there is a need to enhance SSA to speed its convergence and effectiveness to solve complex problems. Recently, chaos is one of the most common mathematical approaches employed to boost the performance of this genre of algorithms. Thus, to mitigate these problems, chaotic maps are used to control the balance between the exploration and exploitation in this algorithm (SSA). The proposed Algorithm is evaluated using unimodel and multimodel benchmark test functions. The comparative results shows that the chaotic maps significantly enhances the performance of the SSA algorithm.

1 Introduction

Recently, meta-heuristic algorithms have become surprisingly very popular. This is due to proving their superiority in solving many optimization problems. Among them, swarm intelligence (SI) algorithms form a very sturdy category, they are based on the swarming behaviour of biological creatures in searching for food, escaping predators or survival. The Salp Swarm Algorithm (SSA) is a relatively new swarm intelligence algorithm to simulate the foraging behavior of the sea swarm slap. The essence of the SSA is a random search optimization algorithm. It has the shortcomings of low accuracy in the later stage of iteration and is easy to get stuck at local optima. As a meta-heuristic algorithm, the searching behavior of SSA is divided into two main phases: exploration and exploitation phases. In exploration phase, it can efficiently discover the search space mostly by randomization, but it may face abrupt changes. In exploitation phase, it converges toward the most promising region. But, SSA often traps into local optima due to its stochastic nature and lack of balancing between exploration and exploitation. Thus, from this point, many studies have been presented to improve the performance of SSA and to overcome these defects. in this work, we used a chaotic mapping sequence to take place the random parameter, which significantly improved the convergence rate and resulting precision of SSA.

Key Words and Phrases: *Optimization, Meta-heuristic optimization, Salp swarm algorithm, Chaos*

2 Main results

1 Chaotic salp swarm algorithm (CSSA)

1.1 Chaotic parameters

The original SSA has mainly three main parameters which affect its performance. These parameters are C_1, C_2 and C_3. C_2 and C_1 are the two main parameters influencing the updating position of a salp. Thus, they significantly impact on balancing between exploration and exploitation. In this study, new chaotic map is employed to adjust a C_2 parameter of SSA. Equation 2.1 shows the updating of C_2 parameter according to the chaotic map. Equation 2.2 shows the updated position of a salp according to the chaotic map.

$$C_2 = t^{l+1}. \tag{2.1}$$

Where l represent the iteration.

$$x_j^1 = \begin{cases} F_j + C_1((ub_j - lb_j)t^{l+1} + lb_j), & C_3 \geq 0; \\ F_j - C_1((ub_j - lb_j)t^{l+1} + lb_j), & C_3 \leq 0. \end{cases} \tag{2.2}$$

1 Experiments and analysis

The performance of newly developed variant of CSSA tested over a set of two benchmark problems with different characteristics. we apply CSSA for each function. Results of all algorithm are recoded for a population of 30 and maximum iteration 500. Linear transformation to compare different test function results by the help of min-max normalization in range $[0,1]$. The two functions are:

$$f_1(x) = -\cos(x_1)cos(x_2)e^{(-(x_1-\pi)^2-(x_2-\pi)^2)},\ f_2(x) = -\sum_{i=1}^{n} \sin(x_i)\left[\sin\left(\frac{ix_i^2}{\pi}\right)\right]^{2m}.$$

2 Numerical results

The numerical results are displayed in Tab.1.

		SSA	CSSA
f_1	best value	-1	-1
	worst value	-1	-1
	(x,y)	$(3.1416, 3.1416)$	$(3.1416, 3.1416)$
	number of iteration	500	200
	T/s	1.839424	1.7782230
f_2	best value	-1.8013	-1.8013
	worst value	-1.8013	-1.8013
	(x,y)	$(2.2029, 1.5708)$	$(2.2029, 1.5708)$
	number of iteration	500	150
	T/s	1.939353	1.895803

Table 1: optimization results over 30 runs for f_1 and f_2

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Some results about EP operators

Safa Menkad
LTM, Department of Mathematics, University of Batna 2, Algeria
E-mail: s.menkad@univ-batna2.dz

Sohir Zid
LTM, Department of Mathematics, University of Batna 2, Algeria
E-mail: s.zid@univ-batna2.dz

Abstract
A bounded linear operator A on a Hilbert space H is said to be an EP operator if ranges of A and A^* are equal and A has a closed range. In this work we present some relationships between an EP operator T, its λ-Aluthge transform $\Delta_\lambda(T)$ and the Moore-Penrose inverse T^+.

1 Introduction

let $\mathcal{H}$ be a complex Hilbert space and let $\mathcal{B}(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. For an arbitrary operator $T \in \mathcal{B}(\mathcal{H})$, we denote by $\mathcal{R}(T)$, $\mathcal{N}(T)$ and T^* for the range, the null subspace and the adjoint operator of T, respectively. It is well known that for every operator $T \in \mathcal{B}(\mathcal{H})$, there is a unique factorization $T = U|T|$, where $\mathcal{N}(U) = \mathcal{N}(T) = \mathcal{N}(|T|)$, U is a partial isometry, i.e. $UU^*U = U$ and $|T| = (T^*T)^{\frac{1}{2}}$ is the modulus of T. This factorization is called the polar decomposition of T. From the polar decomposition, the λ-Aluthge transform of T is defined for any $\lambda \in [0,1]$, by $\Delta_\lambda(T) = |T|^\lambda U|T|^{1-\lambda}$. In particular, for $\lambda = \frac{1}{2}$, $\Delta(T) = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$ is called the Aluthge transform of T. Now, we recall the notion of the Moore-Penrose inverse that will be used in this paper. For $T \in \mathcal{B}(\mathcal{H})$, the Moore-Penrose inverse of T is the unique operator $T^+ \in \mathcal{B}(\mathcal{H})$ which satisfies:

$$TT^+T = T, \;\; T^+TT^+ = T^+, \;\; (TT^+)^* = TT^+, \;\; (T^+T)^* = T^+T.$$

It is well known that the Moore-Penrose inverse of T exists if and only if $\mathcal{R}(T)$ is closed. It is easy to see that $\mathcal{R}(T^+) = \mathcal{R}(T^*)$, TT^+ is the orthogonal projection of $\mathcal{H}$ onto $\mathcal{R}(T)$ and that T^+T is the orthogonal projection of $\mathcal{H}$ onto $\mathcal{R}(T^*)$. The operator T is said to be EP operator, if $\mathcal{R}(T)$ is closed and $TT^+ = T^+T$. Clearly

$$T \text{ EP} \Longleftrightarrow \mathcal{R}(T) = \mathcal{R}(T^*) \Longleftrightarrow \mathcal{N}(T) = \mathcal{N}(T^*).$$

The notion of EP matrix was introduced in 1950 by Schwerdtfeger [5]. A few years later, Campbell and Meyer [2] extended the notion of EP matrix to bounded operator with a closed range defined on a Hilbert space. In this work, firstly, we shall show a necessary and sufficient condition for the range of $\Delta_\lambda(T)$ to be closed. Secondly, we investigate when an operator and its λ-Aluthge transform both are EP.

Key Words and Phrases: *Hilbert space, EP operator , Moore-Penrose inverse, λ-Aluthge transform*

2 Main results

The λ-Aluthge transform preserves many properties of the original operator. However, an operator $T \in \mathcal{B}(\mathcal{H})$ may have a closed range without $\Delta_\lambda(T)$ having a closed range as shown by the following example.

Example 2.1. Let $T = \begin{pmatrix} A & 0 \\ (I - A^*A)^{\frac{1}{2}} & 0 \end{pmatrix} \in \mathcal{B}(\mathcal{H} \oplus \mathcal{H})$, where A is a contraction and $\mathcal{R}(A)$ is not closed. Then $T^*T = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, is an orthogonal projection. Hence T is a partial isometry. This implies that $\mathcal{R}(T)$ is closed and $T = T|T| = TT^*T$ is the polar decomposition of T. Therefore, for $\lambda \in]0, 1]$, we have $\Delta_\lambda(T) = (T^*T)^\lambda T(T^*T)^{1-\lambda} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$. So $\mathcal{R}(\Delta_\lambda(T))$ is not closed.

In the next result, we provide a necessary and sufficient condition for the range of $\Delta_\lambda(T)$ to be closed .

Proposition 2.1. *Let $\lambda \in]0, 1]$ and $T \in \mathcal{B}(\mathcal{H})$ with closed range. Let P be an idempotent with range $\mathcal{R}(T)$ and Q be an idempotent with kernel $\mathcal{N}(T)$. Then*

$$\mathcal{R}(\Delta_\lambda(T)) \textit{ is closed if and only if } \mathcal{R}(QP) \textit{ is closed.}$$

The following result, which is one of the main results of this section, generalizes Theorems 3.3 and 3.15 obtained for complex matrices in [4] to the closed range operators on an arbitrary Hilbert space.

Theorem 2.2. *For $T \in \mathcal{B}(\mathcal{H})$ with closed range and $\lambda \in]0, 1]$, we have*

$$T \textit{ is an EP operator} \iff \Delta_\lambda(T) \textit{ is EP and } \mathcal{R}(T) = \mathcal{R}(\Delta_\lambda(T)).$$

Remark 2.1. Without the condition $\mathcal{R}(T) = \mathcal{R}(\Delta_\lambda(T))$, the reverse implication does not hold. To see this let $T = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} \in \mathcal{B}(\mathcal{H} \oplus \mathcal{H})$. Then $\mathcal{R}(T)$ is closed and $T^+ = \begin{pmatrix} 0 & 0 \\ I & 0 \end{pmatrix}$. Furthermore, $T^2 = 0$. Hence $\Delta_\lambda(T) = 0$ is EP, while T is not EP because $TT^+ = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} = T^+T$.

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On the numerical range of the Aluthge transform of T

Sohir Zid*

Mailing address: Departement of mathematics, University of Batna 2, Batna, Algeria
E-mail: s.zid@univ-batna2.dz

Safa Menkad

Mailing address: Departement of mathematics, University of Batna 2, Batna, Algeria
E-mail: s.menkad@univ-batna2.dz

Abstract
Let $T \in \mathcal{B}(\mathcal{H})$ be a bounded linear operator on a Hilbert space $\mathcal{H}$, and $T = U|T|$ be its polar decomposition. The Aluthge transform of T is defined by $\Delta(T) = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$. In this paper, We discuss some results on numerical range via the Moore-Penrose inverse and Aluthge transformation.

1 Introduction

For $T \in \mathcal{B}(\mathcal{H})$, there is a unique factorization $T = U|T|$, where $\mathcal{N}(U) = \mathcal{N}(T) = \mathcal{N}(|T|)$. This factorization is called the polar decomposition of T. From the polar decomposition, the Aluthge transform of T is defined by

$$\Delta(T) = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}, \quad T \in \mathcal{B}(\mathcal{H}).$$

This transform was introduced in [1] by Aluthge. One of the interests of the Aluthge transform lies in the fact that it respects many properties of the original operator. For example T has a nontrivial invariant subspace if an only if $\Delta(T)$ does . Another important property is that T and $\Delta_\lambda(T)$ have the same spectrum . So T is invertible if and only if $\Delta_\lambda(T)$ is invertible. Also the *-Aluthge transform $\Delta^{(*)}(T)$ of T is $\Delta^{(*)}(T) = |T^*|^{\frac{1}{2}}U|T^*|^{\frac{1}{2}}$[2].
We denote by $\delta(\mathcal{H})$ the class of operator $T \in \mathcal{B}(\mathcal{H})$ which satisfies $U^2|T| = |T|U^2$. The operator T is called binormal if TT^* and T^*T commute. In this paper, We discuss some results on numerical range via the Moore-Penrose inverse and Aluthge transformation.

Key Words and Phrases: *numerical range , Aluthge transformation, Moore-Penrose inverse , binormal operators*

2 Main results

Recall that the numerical range $W(T)$ of T is defined by $W(T) = \{< Tx, x >, \|X\| = 1\}$.

Theorem 2.1. *Let $T \in \delta(\mathcal{H})$ is binormal. Then the following statements Hold.*

(1) If T is invertible, then $W(\Delta(T^{-1})) = W(\Delta(T))^{-1}) \subseteq W(U^*)W(|T|^{-1})$

(2) If T is invertible, then $W(\Delta(T^{-1})) = W(\Delta(T))^{-1}) \subseteq W(U^*)W(|T^{-1}|)$.

(3) If T is an EP operator, then $W(\Delta(T^{+})) = W(\Delta(T))^{+}) \subseteq W(U^*)W(|T|^{-1})$

(4) If T is an EP operator, then $W(\Delta(T^{+})) = W(\Delta(T))^{+}) \subseteq W(U^*)W(|T^{-1}|)$.

(5) If T is an EP operator, then $W(\Delta(T) = W(\Delta^{(*)}(T)) \subseteq W(U)W(|T|)$

(6) If T is an EP operator, then $W(\Delta(T) = W(\Delta^{(*)}(T)) \subseteq W(U)W(|T^*|)$

Proof. (1) Let $x \in \mathcal{H}$, such that $||X|| = 1$. Then

$$\begin{aligned}\left\langle \Delta(T^{-1})x, x\right\rangle = \left\langle (\Delta(T))^{-1}x, x\right\rangle &= \left\langle (|T|^{\frac{-1}{2}}U^*|T|^{\frac{-1}{2}}x, x\right\rangle \\ &= \left\langle U^*\frac{|T|^{\frac{-1}{2}}x}{|||T|^{\frac{-1}{2}}x||}, \frac{|T|^{\frac{-1}{2}}x}{|||T|^{\frac{-1}{2}}x||}\right\rangle \left\langle |T|^{-1}x, x\right\rangle\end{aligned}$$

Thus $W(\Delta(T^{-1})) = W(\Delta(T))^{-1}) \subseteq W(U^*)W(|T|^{-1})$.
(2)Direct replacement shows that

$$\begin{aligned}\left\langle (\Delta(T))^{-1}x, x\right\rangle &= \left\langle \Delta(T^{-1})x, x\right\rangle \\ &= \left\langle (|T^{-1}|^{\frac{1}{2}}U^*|T^{-1}|^{\frac{1}{2}}x, x\right\rangle \\ &= \left\langle U^*|T^{-1}|^{\frac{1}{2}}x, |T^{-1}|^{\frac{1}{2}}x\right\rangle \\ &= \left\langle U^*\frac{|T^{-1}|^{\frac{1}{2}}x}{|||T^{-1}|^{\frac{1}{2}}x||}, \frac{|T^{-1}|^{\frac{1}{2}}x}{|||T^{-1}|^{\frac{1}{2}}x||}\right\rangle \left\langle |T^{-1}|x, x\right\rangle\end{aligned}$$

Hence $W(\Delta(T^{-1})) = W(\Delta(T))^{-1}) \subseteq W(U^*)W(|T^{-1}|)$.
with similar arguments, one proves (3),(4) ,(5) and (6). □

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Multilabel Image classification approach Using Conditional Random Field

BENBRAIKA Souad*
Ecole Normale Supérieure de Bousaada, Algeria
E-mail: benbraika.souad@gmail.com

Abstract
In this work, we propose a multi-label classification approach based Conditional Random Field (CRF) for unamend aerial vehicle (UAV) images. Where the main idea of the proposed model is that, exploit the both spatial contextual information and cross-correlation between labels to give a reasonable prediction.

1 Introduction

The exploitation of the spatial contextual information is a common technique with a view to improve the results of classification. Markov Random Fields [2] Conditional Random Fields [3] are known to be one of most promising tools for encapsulating spatially neighbouring information in the classification model. Recently, it gained highly attention as application in the remotely sensed field that one of the contribution of the extended works is Multilabel Conditional Random Field (ML_CRF)[1], in which our proposed work treate the main idea of their contribution. We start by the background of CRF concept. Let X be the observed data from an input image where $X = \{x_i \in X/i = 1, \dots, n\}$ andx_i is the observation data from the i^th site. Let Y be their corresponding labels, defined as $Y = \{y_i \in Y/i = 1, \dots, n\}$. The posterior over the labels Y given the observed data X is defined as : $P(Y|X) = \frac{1}{Z} exp\{-E(Y, X)\}$ where Z is a normalizing constant. The energy function $E(Y, X)$ is expressed as the sum of unary and pairwise terms:

$$E(Y,X) = E_{data}(Y,X) + E_{spatial}(Y,X) = \sum_{i\in\mathcal{S}} V_1(y_i, x) + \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{N}_i} V_2(y_i, y_j, x)$$

with $\mathcal{S}$ being the set of total n sites and where $\mathcal{N}_i$ is the neighborhood of site i. Knowing that the order of the neighbourhood system plays an important role in the spatial term. Generally, the first order (4-neighborhood) and the second order (8-neighborhood) of neighborhood system are the most neighboring system useful in this framework of research.

2 Proposed method and Main results

In ordinary multilabel classification, the both observed data and their label are defined through several classification maps X_k with $k \in C = \{1, 2, \dots, c\}$, such that$X_k = \{x_{ki} \in X_k/i = 1, \dots, n\}$. The basic concept of ML_CRF is to change the interaction potential termV_2 from pairwise term to a ternary one :

$$\begin{aligned} P(Y_k|X_k) &= \frac{1}{Z} exp\Big(\sum_{i=1}^{n} V_1(y_{ki}, X_k) + \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^k}\sum_{l\in\mathcal{N}_k^i} V_2(y_{ki}, y_{kj}, y_{li}, X_k)\Big) \\ &= \frac{1}{Z} exp\Big(\sum_{i=1}^{n} V_1(y_{ki}, X_k) + \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i^k} I_1(y_{ki}, y_{kj}, X_k) + \sum_{i=1}^{n}\sum_{l\in\mathcal{N}_k^i} I_2(y_{ki}, y_{li} X_k)\Big) \end{aligned} \tag{2.1}$$

Key Words and Phrases: *Conditional Random Fields (CRFs), spatial contextual information, Multilabel classification,.*

Accordingly, V_1 is the unary potential. I_1 is the pairwise interaction potential that is explaining the spatially neighbouring information between adjacent tiles of tile i within the same class $k.I_2$ is the cross-correlation potential term that explains the information between different labels within same tile i. Note that the cross-correlation term was written only by using the two labels variable and has no relation on the observed data (i.e. the classification maps). The local probabilities of each tile are conditioned by its multilabel vector descriptor in addition to the corresponding neighboring tiles over the same label map level. Starting from an initial multilabel combination generated from the output of the MLP classifier, at each iteration, the ICM maximizes the conditional MAP estimation:

$$\widehat{y_{ki}} \leftarrow \underset{y_{ki}}{argmax}\ P(y_{ki}|y_{k\mathcal{N}_i^k}, y_{k\mathcal{N}_k^i}, X_k)$$

. In order to evaluate the performance of the proposed classification method, we exploited two real datasets of UAV images acquired over two different locations. The effectiveness of the proposed framework is evaluated by the two well kwon accuracy measures, namely, the sensitivity (SEN : good detection), the specificity (SPE :) and their average (AVG). For the sake of comparison, we confront the proposed multilabel scheme with unary scores strategy obtained by means of RGB BOWs features coupled with an AE network and an MLP classifier (termed as ML-unary), along with the traditional monolabel CRF reference method that was run on each binary map (class) independently of the others, not taking into account the multilabel context (cross-correlation between maps). In the following, this reference method is called ML-CRF. The Full-ML-CRF stands for the proposed method, which exploits also the interclass correlation as described earlier. The results are summarized in the following Table .

	Dataset 1			Dataset 2		
	Accuracy (%)			Accuracy (%)		
METHOD	**SPE**	**SEN**	**AVG**	**SPE**	**SEN**	**AVG**
ML-Unary	97.1	47.9	72.5	98.2	62.6	80.4
ML-CRF	86.8	53.4	70.1	92.5	70.6	81.5
Full-ML-CRF	82.2	**70.3**	**76.2**	90.8	**75.9**	**83.4**

Our proposed strategy (Full-ML-CRF) outperforms both the ML-Unary and ML-CRF methods in terms of average accuracyscoring 76.2% and 83.4% for data sets 1 and 2, respectively. It records an increment of around 4% and 6% in data set 1,and 3% and 2% in data set 2 over the ML-unary and ML-CRF methods, respectively.

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Solving a multi-level multi-objective linear programming problem using a new method

Zohra Sabrina Delhoum *

delhoum.zahra@yahoo.fr: Oran Graduate School of Economics, Oran, Algeria
E-mail: delhoum.zahra@yahoo.fr

Abstract
In this paper, we present a new method for the resolution of a multi-level multi-objective linear programming problem with bounded variables based on the adaptive method for solving multi-level multi-objective linear programming with bounded variables. The peculiarity of this method is that it has the suboptimality criterion that allows us to find $\epsilon-$efficient solutions, and unlike to other methods, the method we are going to present has the advantage of solving exactly these kind of problem and manipulating the constraints as they are without any transformation. We will present the detailed algorithm of the method and give a numerical example to demonstrate its applicability.

1 Introduction

In this paper, we will propose a new method for the resolution of a multi-level multi-objective linear programming problem with bounded variables based on the adaptive method for solving a multi-objective programming problem [2]. During the resolution process, the solution is obtained hierarchically and by level. The strategy Stackelberg [5] was used as a concept to find efficient solutions.

The adaptive method [3] is considered as an intermediate method between the interior methods and the simplex one. Indeed, using the adaptive method, the first efficient solution can be an extreme point, an interior point or any point on the edge. The advantage of this method lies in its speed. Convergence results of this method are immediately deduced from those of the direct support method. Furthermore, the method integrates a suboptimal criterion which permits to stop the algorithm with a desired accuracy to find solutions $\epsilon-$efficient. This criterion could be useful in practical applications.

This paper is devoted to present this method

Key Words and Phrases: *Multi-level programming, Multi-objective, Linear programming, Bounded variables, Adaptive method, Pareto optimal solution, Slater optimal solution, $\epsilon-$efficiency*

2 Main results

We consider the following multi-level multi-objective linear programming problem (MMLP) with bounded variables:

$$\begin{cases} \max\limits_{\overline{x}_1} Z_1(x) = C_1 x, \quad (L_1) \\ \vdots \\ \max\limits_{\overline{x}_h} Z_h(x) = C_h x, \quad (L_h) \\ x \in S. \end{cases} \tag{2.1}$$

The method presented in this paper allows us to solve this program. The algorithm of this method is effective, fast, simple, and permits a time reduction in the whole optimization process. It avoids any preliminary transformation of the decision variables. Indeed, it handles the bounds such as they are without any modification. Another particularity of our method is that it uses a suboptimal criterion which can stop the algorithm with a desired precision. An illustrative numerical example was given to demonstrate the applicability of this algorithm.

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Extended method for equilibrium problems using Bregman distance

ZEGHAD BOCHRA*

Laboratory of Fundamental and Numerical Mathematics LMFN, Department of Mathematics, Setif-1 Ferhat Abbas University, 19000, Algeria.
E-mail: bochra.zeghad@univ-setif.dz

Abstract
In this paper, we introduce a new version method for solving equilibrium problems by using Bregman distance . We also study convergence results of this algorithm under the assumption that the bifunctions is pseudomonotone and Lipschitz-type continuous.

1 Introduction

The mirror descent method proposed first by Nemirovski and Yudin to solve convex optimization problems, became widely popular to solve huge-scale optimization problems ([2]). The algorithm replaces the Euclidean distance with a so-called Bregman distance as the regularizer for projection. This idea provides a significant convergence speed-up for high-dimensional optimization problems [1]. The purpose of this paper is to introduce an iterative extension and improvement of the mirror descent method, which is suitable for solving equilibrium problem $EP(f;\ C)$. This is stated as follows:

$$\text{find } x \in C \quad \text{such that} \quad f(x,y) \geq 0, \forall y \in C, \tag{1.1}$$

where C is a nonempty, closed, and convex subset of a real linear space E, $f : E \times E \to \mathbb{R}$ be a bifunction.

2 Main results

Throughout this paper, we assume that the following assumptions hold on f
Assumption 2.1: Let $f : E \times E \to \mathbb{R}$ satisfies the following conditions:

(A1) $f(x,x) = 0$, for all $x \in C$ and f is pseudomonotone on C, i.e.,

$$f(x,y) \geq 0 \Rightarrow f(y,x) \leq 0, \qquad \forall x,y \in C$$

(A2) f is Lipschitz-type condition on E, i.e., there exist two positive constants c_1 and c_2, such that

$$f(x,y) + f(y,z) \geq f(x,z) - c_1 \|x-y\| - c_2 \|y-z\|, \quad \forall x,y,z \in C.$$

Key Words and Phrases: *equilibrium problems, mirror descent method, pseudomonotonicity, Bregman distance.*

(A3) $f(x,.)$ is convex and subdifferentiable on E for each fixed $x \in C$.

(A4) The solution set $sol(f;C)$ of $EP(f;C)$ is nonempty.

It has been proved that under the conditions **(A1)-(A3)**, the solution set $sol(f;C)$ of $EP(f;C)$ is closed and convex ([3]).
We propose the following algorithm for solving $EP(f;C)$:
Algorithm 1
Initialization: Choose $x_1 \in C_0, y_1 \in C$, $\lambda > 0$,
Step 1: Compute

$$x_{n+1} = \arg\min_{y\in C} \{\lambda f(y_n, y) + D_h(y, x_n)\},$$

Step 2: Compute

$$y_{n+1} = \arg\min_{y\in C} \{\lambda f(y_n, y) + D_h(y, x_{n+1})\},$$

Step 3: Set $n := n+1$ and go to Step 1.

Lemma 2.1. *Let $\{x_n\}$ and $\{y_n\}$ be the two sequences generated by Algorithm 1.*

i $< \nabla h(x_n) - \nabla h(x_{n+1}), y - x_{n+1} > \leq \lambda \left(f(y_n, y) - f(y_n, x_{n+1})\right), \forall y \in C.$

ii *If $x_{n+1} = x_n = y_n$, then $y_n \in sol(f;C)$.*

iii *For all $p^* \in sol(f;C)$, the following inequality holds:*

$$\begin{aligned} D_h(p^*, x_{n+1}) &\leq D_h(p^*, x_n) - (1 - \tfrac{2\lambda L_2}{\sigma}) D_h(x_{n+1}, y_n) \\ &-(1 - \tfrac{2(2+\sqrt{2})\lambda L_1}{\sigma}) D_h(y_n, x_n) + \tfrac{2\sqrt{2}\lambda L_1}{\sigma} D_h(y_{n-1}, x_n). \end{aligned} \tag{2.1}$$

Lemma 2.2. *The sequences $\{x_n\}$ and $\{y_n\}$ generated by Algorithm 1 are bounded.*

Theorem 2.1. *Let f be a bifunction : $E \times E \to \mathbb{R}$ satisfied conditions* (**A1**)-(**A2**)-(**A4**), *and $\lambda \in (0, \frac{\sigma}{2(L_2+\sqrt{2}L_1)})$. Then the sequences $\{x_n\}$ and $\{y_n\}$ generated by Algorithm 1 converge to some point $p^* \in sol(f;C)$.*

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Operators where T^n and T^+ commute for some positive integer n $\in$ N

Elgues Anissa*

Mailing address: Departement of mathematics, Faculty of sciences, University of Batna 2, Batna, Algeria
E-mail: a.elgues@univ-batna2.dz

Abstract
A new class of operators, larger than Ep operators , named n Ep operators is introduced. Basic properties are given, An operator T $\in$ B(H)with closed range is named n-Ep operator if $T^nT^+ = T^+T^n$ for some positive integer n $\in$ N .

1 Introduction

Let T be a bounded linear operator on a Hilbert space H. In Our work we introduce a new class of operators which is a generalization of the Ep operators , called n-Ep operators .We give some basic properties of this operators.Secondly, We prove that if T is Moore-Penrose invertible, then T is 2n- EP if and only if it's Moore-Penrose inverse is too 2n-EP. But, this need not be true in case of $(2n+1)$, As a consequence, we generalize Fugled -Putnam theorem for Ep operators to n- EP operators.

2 Main results

Main results

Definition 2.1. Let $T \in$ B(H) with closed range ,then T called n EP operator if $T^nT^+ = T^+T^n$ for any positive integer n in N

Proposition 2.1. *Let $T \in L(H)$ with closed range , If T is n EP operator then T^+ is not.*

Exemple Let $T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ is nilpotent with nilpotency index 3 so it's 3- EP operator .

$T^+ = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $(T^+)^3 = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$(T^+)^3T \neq T(T^+)^3$, Thus T^+ is not 3 EP.

Key Words and Phrases: *Ep operators, n Ep operators , closed range, Moore-Penrose inverse*

Proposition 2.2. *Let $T \in B(H)$ with closed range. Then T is 2n-Ep if and only if T^+ is 2n-Ep.*

Fuglede theorem was generalized for two normal operators by Putnam, which is well-known as Fuglede-Putnam theorem and is stated as follows.

Theorem 2.1. *Let S, T be bounded normal operators on H and $M \in B(H)$, If $MS = TM$, then $MS^* = T^*M$.*

Fuglede-Putnam theorem is not true in general for EP operators on Hilbert spaces.P. SAM JOHNSON , VINOTH A AND K. KAMARAJ proved that under some conditions the theorem holds good.result just replaces the adjoint operation by the Moore-Penrose inverse in the Fuglede theorem .

Theorem 2.2. *Let T, S be EP operators on H and $M \in B(H)$. If $MT = SM$ and $MT^*T = S^*SM$, then $MT^* = S^*M$.*

In the following Theorem, we generalize the last theorem obtained for Ep operators to n-EP operators on an arbitrary Hilbert space.

Theorem 2.3. *Let $T, S \in B(H)$ with closed ranges be n EP operators on H and $A \in B(H)$. If $AT = SA$ and $A(T^n)^*T^n = (S^n)^*S^nA$, then $A(T^n)^* = (S^n)^*A$.*

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PART G

Probability and statistics

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Estimation Bayésienne sous des données progressivement censurées et des fonctions de perte équilibrées

Khawla Boudjerda

Département de mathématiques, université Mohamed Seddik Ben Yahia, Jijel, Algerie
E-mail: khawla18000@gmail.com

Ali Lalouci

Département de mathématiques et informatiques, centre universitaire A. Boussouf, Mila, Algerie
E-mail: ali.lalouci@gmail.com

Abstract
Le but principale de ce travail est de développer plusieurs types de fonctions de perte pour estimer par la méthode Bayésienne les paramètres et les caractéristiques de la distribution de Lindey basée sur les données progressivement censurées de type II. On utilise les fonctions de perte équilibrée de trois différentes fonctions de perte usuelle qui sont : la fonction de perte quadratique, la fonction de perte Entropy et la fonction de perte Linex. Les estimateurs sont donnés sous forme d'intégrales, nous appliquons les méthodes de Monte-Carlo et en particulier l'algorithme de Metropolis-Hastings pour procéder à des simulations et à une analyse des données. Une comparaison à l'aide du critère IMSE est faite. Nous avons effectué une étude sur les données réelles pour illustrer les résultats obtenus.

1 Introduction

Les données censurées sont utilisées lorsqu'on peut pas observer l'information complète de taux de panne des unités. Les plans de données censurées les plus connus sont de type I et de type II, mais ces types de censures n'ont pas la flexibilité d'autoriser le retrait d'unités à des points à part le point terminal de l'expérience. Pour ces raison, on considère un plan de données censurées plus général qui sont les données progressivement censurées de type II décrit comme suit : On considère une expérience dont on a n unités indépendantes sont placées dans le test, dans la première panne, R_1 unités sont retirées au hasard de $(n-1)$ unités restantes. Dans la deuxième panne,R_2 unités du reste $n-2-R_1$ unités sont retirées. Le test continu jusqu'à la m-ième panne où $R_m = n - m - R_1 - R_2 - ... - R_{m-1}$ unités sont retirées. Plusieurs auteurs ont abordés des études sur les données progressivement censurées, G.Hofman et all (2005) ont montré que les schémas progressivement censurés de type II améliorent les schémas censurés de type II dans de nombreuses situations. Pour plus de détails sur les données progressivement censurées voir Balackrishnan et Aggarwala (2000), Balackrishnan et Hussain (2007), Prandhan et Kundu (2009), Basak et al (2009).
Dans ce papier, on s'intéresse à l'estimation Bayésienne des paramètres, de la fonction de fiabilité

Key Words and Phrases: *Loi de Lindley, critère IMSE, données progressivement censurées, fonctions de perte équilibrées, algorithme de Metropolis-Hastings.*

et de la fonction de taux de panne de la loi de Lindley sous des données progressivement censurée de type II et des fonctions de perte équilibrées de la forme :

$$L_{w,\delta_0}(\theta, \delta) = wL(\delta_0, \delta) + (1 - w)L(\theta, \delta),$$

avec $0 < w < 1$, δ_0 est l'estimateur de θ et $L(\theta, \delta)$ est une fonction de perte usuelle, (Pour plus de détails sur les fonction de perte équilibrées voir Zellner (1994), Ghosh et Strawderman (1999), Jafari Jozani et al (2006)).
Le reste de ce papier est organisé comme suit. La section suivante décrit le modèle considéré ainsi que ses fonctions caractéristique. Dans la section 3, nous effectuons une estimation Bayésienne sous différentes fonctions de perte (symétrique et asymétrique). La section 4 présente le calcul des estimateurs Bayésiens sous les fonctions de perte équilibrées. Une simulation des résultats par les méthodes MCMC (Monté Carlo par chaine de Markov) est réalisée dans la section 6. Finalement, la dernière section conclut le papier et présente un exemple d'étude de cas sur les données réelles.

2 Main results

Les résultats principales de ce travail sont les suivants :

Les risques a posteriori des estimateurs Bayesiens des paramètres et des fonctions caractéristiques de la loi de Lindley sous les fonctions de perte équilibrées sont plus petits par rapport aux estimateurs Bayésiens sous les fonctions de perte usuelle pour certaines valeurs de w.
Les résultats d'estimation sous un plan de données progressivement censurées sont mieux par rapport aux plans des données complètes et censurées.

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Nonparametric relative error estimation using a functional regressor by the k Nearest Neighbors smoothing

Bellatrach Nadjet*
Mailing address:Department of Mathematics, Kasdi Merbah University of Ouargla,Ouargla 30000, Algeria
E-mail: bellatrachnadjet@gmail.com

Bouabsa Wahiba
Mailing address: Department of mathematics, Djillali Liabès University of Sidi Bel Abbès, Sidi Bel Abbès, Algeria
E-mail: bouabsa_w@yahoo.fr

Abstract
The nonparametric estimator of the conditional hazard function is studied in this research utilizing the k nearest neighbors (k-NN) estimation method. which a random variable with values in a semi-metric space is used to determine a scalar response variable. We establish the asymptotic normalcy and provide this estimator's almost complete convergence. The success of this method is then demonstrated by a comparison with the kernel method estimation presented by Ferraty [2] , Laksaci, and Mechab [3] for both simulated and actual data.

1 Introduction

The literature has paid quite some attention to nonparametric hazard rate estimation when the data are functional. The first work which deals with this question is Ferraty et al. [2]. They established the almost complete convergence of the kernel estimate of the conditional hazard function in the independent case.
This work deals with the nonparametric estimation with k nearest neighbors method k-NN, more precisely we consider a kernel estimator of the hazard function constructed from a local window to take into account the exact k nearest neighbors with real response variable Y and functional curves X.

2 Main results

2.1 Asymptotic proporties by the k-NN method

Theorem 2.1. *In the continuity type model and under the hypotheses (H1), (H2), (H4), (H5) and (H6), suppose that $k = k_n$ is a sequence of positive real numbers such that $\frac{k_n}{n} \to 0$ and $\frac{\log n}{k_n} \to 0$, then we have:*

$$\lim_{n\to\infty} \widehat{h}^x(y) = h^x(y) \qquad a.co.$$

Key Words and Phrases: *Functional data; Asymptotic normality; The conditional hazard function; k-NN estimator; Rate of convergence; Random bandwidth*

Lemma 2.1. *Under the hypotheses of Theorem 2.1, we have:*

$$\lim_{n\to\infty} \widehat{f}^x(y) = f^x(y) \qquad a.co.$$

and

$$\lim_{n\to\infty} \widehat{F}^x(y) = F^x(y) \qquad a.co.$$

2.2 Similation study

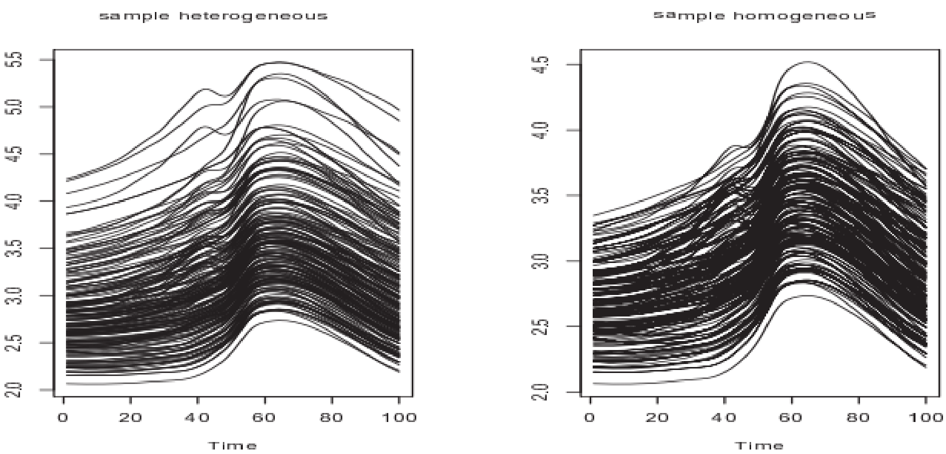

Figure 1: The learning curves.

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Kernel estimation of the conditional distribution function for heterogenous functional data

Leulmi Sarra*

Mailing address: Department of mathematique, University Constantine 1, Constantine, Algeria
E-mail: math17sara@yahoo.fr

Boudada Halima

Mailing address: Department of mathematique, University Constantine 1, Constantine, Algeria
E-mail: boudadah@yahoo.com

Abstract
Recently, many topics concerning the analysis of the functional independent and identically distributed data (see [2]). Whereas, the heterogeneous data are very usefel in practice, for example, patients are typically a very heterogeneous population as they differ with many factors including demographics, diagnostics test results and medical histories. In this work, we focus on a functional nonparametric kernel estimation of the conditional distribution function when the observations are independent and heteregenously distributed. Then, under standard condition, we establish the pointwise almost complete convergence, with rate, of the proposed estimator.

1 Introduction

Functional data analysis is a branch of statistics that has been the subject of many studies and developments in the recent years. It deals with the analysis and theory of data that are in the form of functions, images and shapes, or more general objects. Note that the modelization of functional data is becoming more and more popular since the publication of the monograph of [3] on functional data analysis, for the parametric models. However, the first results concerning the nonparametric models were obtained by [1] who established the almost complete pointwise consistency of kernel regression estimator. These results have been extended in [2] by treating the pointwise almost complete for divers kenel estimators when the observations are independent and identically distributed.
Moreover, the heterogeneity is one of major features of a big data and heterogenous data result in problems in data integration and big data analytics. For this, in this work, we proposed a nonparametric functional estimator of the conditional distribution function when the observations are independent and heterogeneousely distributed.
To our knowledge, the pontwite almost complete convergence of our proposed estimator has not been studied in statistical literature. So, in this work, we address this problem.

Key Words and Phrases: *Kernel Method, Heterogenous data, Functional data, Almost complete convergence*

2 Main results

let us draw n pairs $(X_i, Y_i)_{\{i=1,\dots,n\}}$ of random variables independent and heterogeneousely distributed taking valued in $\mathbf{F} \times \mathbb{R}$, where $\mathbf{F}$ is a infinite-dimensional space equipped with a semi-metric d.
Our goal is to estimate the conditional distribution function of Y given $X = x$ by $F_Y^x(y) = P(Y/X = x)$, which obtained as the solution for a of the following minimization problem

$$\min_{a\in\mathbb{R}} \sum_{i=1}^{n} \left[H\left(g^{-1}(y - Y_i)\right) - a\right]^2 K\left(h^{-1}d(X_i, x)\right),$$

where the functions K and H are kernels with $H(u) = \int_{-\infty}^{u} H_0(v)dv$ where H_0 is a kernel of type 0 and $h := h_n$ (resp. $g := g_n$) is a sequence of strictly positive real numbers which plays a smoothing parameter role.
We can easilly derive the following explicit estimator

$$\widehat{F}_n^x(y) = \frac{\sum_{i=1}^{n} K\left(h^{-1}d(X_i, x)\right) H\left(g^{-1}(y - Y_i)\right)}{\sum_{i=1}^{n} K\left(h^{-1}d(X_i, x)\right)}. \tag{2.1}$$

Then we establish its pointwise almost complete convergence (a.co.), with its rate.

Theorem 2.1. *Under standard conditions in our context, we have*

$$\sup_{y\in S_{\mathbb{R}}} |\widehat{F}_n^x(y) - F_Y^x(y)| \to O, \quad (a.co.)$$

where $S_{\mathbb{R}}$ is a fixed compact subset of $\mathbb{R}$.

Notice that this convergence is stronger tha the almost sure one. We remark also that the obtained rate of the convergence (in our context) is the same as that of [2] when the observations are independent and identically distributed.

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Pricing Options under α-Hypergeometric Uncertain Volatility Models and their Connection to 2BSDEs

Zaineb Mezdoud*

Mailing address: Department of mathematic, University Badji Mokhtar,Annaba, Algeria
E-mail: zaineb.mezdoud@gmail.com

Khaoula Bouguetof

Mailing address: Department of mathematic, University Larbi Tebessi, Tebessa, Algeria
E-mail: khaoula.bouguetof@gmail.com

Abstract
We propose a α-hypergeometric model with uncertain volatility (UV) where we derive a worst-case scenario for option pricing. The approach is based on the connexion between a certain class of nonlinear partial differential equations of HJB-type (G-HJB equations), that govern the nonlinear expectation of the UV model and that provide an alternative to the difficult model calibration problem of UV models, and second-order backward stochastic differential equations (2BSDEs). Using asymptotic analysis for the G-HJB equation and the equivalent 2BSDE representation, we derive a limit model that provides an accurate description of the worst-case price scenario in cases when the bounds of the UV model are slowly varying. The analytical results are tested by numerical simulations using a deep learning based approximation of the underlying 2BSDE..

1 Introduction

The classical option pricing problem based on the seminal work by Black and Scholes assumes that the volatility of the underlying asset is constant over time. While the Black-Scholes model is still considered an important paradigm for option pricing, there is plenty of empirical evidences that the assumption of constant volatility is not adequate. In order to come up with more realistic models, various strategies have been proposed to treat the volatility of asset prices as a stochastic process . One of the most famous representative of the large class of stochastic volatility models is the Heston model that has become the basis of many other models, such as jump diffusion models , α-hypergeometric models , or various forms of uncertain volatility models (UVM) such as, all of which can be considered as extensions of the Black-Scholes model and which share many features with the model considered in this article.
One of the common feature of all stochastic volatility models is that the volatility process can only be indirectly observed through the asset price, which poses specific challenges for the parameter estimation (or: *calibration*) of these models. Standard approaches are

Key Words and Phrases: *α-hypergeometric stochastic volatility model, uncertain volatility model, 2BSDE, deep learning based discretisation of 2BSDE*

based on maximum likelihood estimation using (filtered) time series data or fitting of the implied volatility surface. In the Heston model, the price hits zero in finite time unless the Feller condition is imposed. As a consequence, the underlying optimisation problems are typically endowed with constraints, which pose additional problems in model calibration. Here we use an alternative approach, in which we consider the unknown diffusion coefficient of the stochastic volatility model a bounded random variable. Specifically, we focus on the UVM developed by and consider an α-hypergeometric stochastic volatility model of the form

$$dX_t = rX_t dt + X_t q e^{V_t} dW_t^1 \tag{1.1a}$$

$$dV_t = (a - be^{\alpha V_t})dt + \sigma dW_t^2, \tag{1.1b}$$

where W_t^1 and W_t^2 are correlated Brownian motions, with $d(W_t^1, W_t^2) = \rho dt$ for some $|\rho| \leq 1$, and $b, \alpha, \sigma > 0$ and $a \in \mathbb{R}$ are constants; the parameter q is unknown; the only information available is that $q \in [\sigma_{\min}, \sigma_{\max}]$ for some $\sigma_{\max}, \sigma_{\min} \in \mathbb{R}_+^*$. This implies that the volatility β_t of the risky asset under the risk-neutral measure $\mathbb{Q}$,

$$dX_t = rX_t dt + X_t \beta_t dW_t^1 \,, \tag{1.2}$$

where $r \in \mathbb{R}$ is the risk-free interest rate, is stochastic with $\underline{\sigma}_t \leq \beta_t \leq \overline{\sigma}_t$, with

$$\underline{\sigma}_t = \sigma_{\min} F(V_t)\,, \quad \overline{\sigma}_t := \sigma_{\max} F(V_t) \quad \text{for } \; 0 \leq t \leq T\,.$$

Here $F > 0$ is a differentiable increasing function that we choose to be $F(v) = e^v$.
Our aim is to derive worst-case pricing scenarios for the seller in the spirit of the work [?], without needing to calibrate the model exactly. To this end, we rescale time in the volatility equation in (1.1) according to $t \mapsto \delta t$, which yields

$$dX_t = rX_t dt + X_t q e^{V_t} dW_t^1 \tag{1.3a}$$

$$dV_t = \delta(a - be^{\alpha V_t})dt + \sqrt{\delta}\sigma dW_t^2 \tag{1.3b}$$

and allows us to smoothly interpolate between an UVM and a fixed volatility model. The parameter $\delta > 0$ symbolizes the reciprocal of the time-scale of the process V, and thus the standard UVM can be formally obtained by sending $\delta \to 0$, in which case $V_t = v$ and

$$dX_t^0 = rX_t^0 dt + qX_t^0 e^v dW_t^1. \tag{1.4}$$

Varying δ sheds some light on the importance of the stochastic volatility equation for the worst-case scenario: when the variation of the volatility is slow, the market price of the asset is not very volatile, so this price remains stable; in the opposite case, it may becomes too volatile and therefore more risky.

2 Main results

2.1 Worst-case scenario price

Let $\Theta = [\sigma_{\min}, \sigma_{\max}]$. For any $\delta > 0$, the worst-case scenario price at time $t < T$ is defined as

$$P^\delta := P^\delta(t; x, v) = \exp(-r(T-t)) \sup_{q \in \Theta} E_{(t;x,v)}[h(X_T^\delta)]. \tag{2.1}$$

If $\delta = 0$, we define

$$P^0 := P^0(t; x, v) = \exp(-r(T-t)) \sup_{q \in \Theta} E_{(t;x,v)}[h(X_T^0)]. \tag{2.2}$$

Where $E_{(t;x,v)}[\cdot]$ is the conditional expectation given $\mathcal{F}_t$ with $X_t^\delta = x$ and $V_t = v$.

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Local Approach of The Smoothing Parameter Selection in Kernel Density Estimation of The Inventory Model (R, s, S)

Khineche Halima*

Mailing address: Department of Mathematics, University of Biskra, Biskra, Algeria
E-mail: halima.khineche@univ-biskra.dz

Cherfaoui Mouloud

Mailing address: Research Unit LaMOS (Modeling and Optimization of Systems), University of Bejaia, Bejaia, Algeria
E-mail: mouloudcherfaoui2013@gmail.com

Abstract
The objective of this work is the estimation of the transition matrix P associated with the discrete Markov chain describing the inventory model (R, s, S). Firstly, we proposed to use the nonparametric discrete kernel method to estimate the components of the matrix P. Then, we exposed a new local approach for the smoothing parameters selection which relies on the principle of the minimization of a specific error criterion for each component of the matrix P. Finally, we carried out a comparative numerical study to illustrate the impact of the choice of smoothing parameters on the performance of the estimator of P.

1 Introduction

The exploitation of the discrete kernel method at work [2, 3], where the authors proved the applicability of the method of the kernel in estimate discrete functions with discrete support and introduce some discrete kernel estimator properties. In this work, we propose to consider a discrete time discrete Markov chain where our objective is to analyze the problem of the choice of the smoothing parameter of a discrete associated kernel estimator of a transition matrix corresponding to the inventory model of type (R,s,S) has been proved (see [1]). In [5], the authors solved the problem of selecting the smoothing value in the kernel estimation context of a Markov chain modeling a queuing system. Recently, the authors [4] proposed to use the matrix norms which have an impact on the quality of the P estimator, they suggested choosing the optimal smoothing parameter. However, in all of the existing work on the estimation of the inventory model transition matrix (R, s, S) the authors have considered only the case of global choice. For this purpose, we propose to select the local smoothing parameter at each component of the matrix P. Thus, the transition matrix of the Markov chain X is given by :

Key Words and Phrases: *Inventory management, Discrete stochastic process, Discrete kernel estimation, Smoothing parameter, Errors.*

$$\hat{P} = \begin{array}{c|cccc|ccc}
 & 0 & 1 & & s & s+1 & & S \\
\hline
0 & \sum\limits_{S}^{+\infty} \hat{a}_k & \hat{a}_{S-1} & \cdots & \hat{a}_{S-s} & \hat{a}_{S-s-1} & \cdots & \hat{a}_0 \\
1 & \sum\limits_{S}^{+\infty} \hat{a}_k & \hat{a}_{S-1} & \cdots & \hat{a}_{S-s} & \hat{a}_{S-s-1} & \cdots & \hat{a}_0 \\
 & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
s & \sum\limits_{S}^{+\infty} \hat{a}_k & \hat{a}_{S-1} & \cdots & \hat{a}_{S-s} & \hat{a}_{S-s-1} & \cdots & \hat{a}_0 \\
\hline
s+1 & \sum\limits_{s+1}^{+\infty} \hat{a}_k & \hat{a}_s & \cdots & \hat{a}_1 & \hat{a}_0 & 0 & 0 \\
 & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & 0 \\
S & \sum\limits_{S}^{+\infty} \hat{a}_k & \hat{a}_{S-1} & \cdots & \hat{a}_{S-s} & \hat{a}_{S-s-1} & \cdots & \hat{a}_0
\end{array} \tag{1.1}$$

For estimating the matrix P, we propose to use the kernel method defined by :

$$\hat{a}_k = \hat{P}(X = k) = \hat{f}(k) = \frac{1}{n}\sum_{i=1}^{n} K_{k,h}(X_i), \quad k \in \mathbb{N} \tag{1.2}$$

2 Main results

We show the results obtained in the local selection of the smoothing parameter of the transition matrix P of the Markov chain associated with the stock model (R, s, S). Then, we present the results of a study of simulations using MATLAB, which evaluates the characteristics of the estimator for different sample sizes from different distributions. Following that, we will discuss the results by comparing two smoothing parameters, the local approach and the global approach, on the quality of the estimator.

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Statistical Inference of A New Extension of The Power Lindley distribution

Kahloul Bariza*
Mailing address: Department of Mathematics ,University of Mohamed Khider, Biskra, Algeria
E-mail: bariza.kahloul@univ-biskra.dz

Djabrane yahia
Mailing address: Department of Mathematics, University of Mohamed Khider, Biskra, Algeria
E-mail: yahia.djabrane@univ-biskra.dz

Abstract
In this paper, we developed a new distribution called Power Lindley Generalized Gamma distribution. The new distribution contains Lindley, Power Lindley, gamma Lindley introduced by Zeghdoudi et al[3], and XLindley introduced by Chouia et al[4] distributions as special cases. Some of its statistical properties are studied. The parameters of the new distribution are estimated by using the method of maximum likelihood. An application of the new distribution to the COVID-19 data set is presented, and we compared it with some other well-known distributions.

1 Introduction

Statistical distributions are commonly used to describe real-world phenomena and used in different areas of applied sciences. Due to the usefulness of statistical distributions, their theory is widely studied. However, known probability distributions remain unable to represent data for some real phenomena accurately. Therefore many new flexible distributions using different methods have been developed and applied to describe these real phenomena.
Lindley[2] proposed the one parameter Lindley distribution in the context of Fiducial and Bayesian Statistics. Recently, Most of the Lindley distribution extensions have been derived because of their great useful to analyze lifetime data. Ghitany et al[1] proposed an extension of the Lindley distribution with a two-parameter called power Lindley distribution, the probability density function is defined by

$$f_1(x,\alpha,\theta) = \frac{\alpha\theta^2}{1+\theta}(1+x^\alpha)x^{\alpha-1}\exp(-\theta x^\alpha),\ x>0,\ \alpha,\theta>0.$$

In this work, we introduced a new mixed distribution, which is based on mixtures of the Power Lindley and Generalized gamma distributions, which will be called The Power Lindley Generalized Gamma distribution(PLGG for short).

Key Words and Phrases: *Power Lindley distribution, Generalized Gamma distribution, maximum likelihood estimation, COVID-19 data.*

2 The Power Lindley Generalized Gamma distribution

Let f_1 be the PDF of the Power Lindley distribution and f_2 be the PDF of the Generalized Gamma distribution introduced by Stacy(1962). We define a new distribution as a mixture of f_1 and f_2 with mixing proportion $0 \leq p \leq 1$ and $1-p$, respectively, as follows:

$$f(x) = pf_1(x, \alpha, \theta) + (1-p)f_2(x, \alpha, \lambda, \theta).$$

by putting $p = \delta(\theta+1)/(\delta(\theta+1)+\eta)$. So the PDF of the new distribution is given by

$$f(x, \theta, \alpha, \lambda, \delta, \eta) = \frac{[\Gamma(\lambda)\delta\alpha\theta^2(1+x^\alpha)x^{\alpha-1} + \eta\alpha\theta^\lambda x^{\lambda\alpha-1}]}{\Gamma(\lambda)(\delta(\theta+1)+\eta)} \exp(-\theta x^\alpha), \ x > 0. \tag{2.1}$$

where $\theta, \alpha, \lambda > 0$ and $\delta, \eta \geq 0$. We denote the PLGG$(\theta, \alpha, \lambda, \delta, \eta)$. In this work, we present some properties of the PLGG distribution like as survival and hazard functions, the moments, Mean, and variance.

3 Estimation of parameters

In this section, we use the method of Maximum Likelihood Estimates (MLE) to estimate the parameters of the new distribution. The Log-likelihood function of the PLGG distribution is given by

$$\begin{aligned} Logl(x_i, \theta, \alpha, \lambda, \delta, \eta) &= \sum_{i=1}^{n} \log(\delta\theta^2(1+x_i^\alpha)x_i^{\alpha-1} + \frac{\eta\theta^\lambda x_i^{\lambda\alpha-1}}{\Gamma(\lambda)}) + n\log(\alpha) - n\log(\delta(\theta+1)+\eta) \\ &\quad -\theta\sum_{i=1}^{n} x_i^\alpha. \end{aligned}$$

The MLEs can be obtained by maximizing The log-likelihood function numerically. The maximization can be performed by using the `maxLik()`function in R software.
Finally, we fit the PLGG distribution to real data set of COVID-19 and compare it with another distributions.

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Non-parametric regression for incomplete, dependent and functional data

Boudada Halima*

Mailing address: Mathematics Department, Frères Mentouri University, Constantine 1, Constantine, Algeria.
E-mail: boudadah@yahoo.com

Leulmi Sarra

Mailing address: LAMSAD Laboratory, Mathematics Department, Frères Mentouri University, Constantine 1, Constantine, Algeria.
E-mail: math17sara@yahoo.fr

Abstract
The aim of this work is to study the asymptotic properties of the regression function when the variable of interest is real and subjected to a left truncation by another random variable while the covariate is of functional type, that is to say has values in a infinite dimensional space. In this study, a locally linear modelling was used for the functional estimation by considering an estimator adapted to a polynomial of one degree. Under standard assumptions, the pointwise almost sure convergence with rate has been established when the observations have obeyed a certain type of dependence (α-mixing)

1 Introduction

The statistical problems related to the study of functional random variables have been experiencing a growing interest in the literature for many years. The development of this theme of research is ongoing and motivated by the abundance of data measured on increasingly reliable grids, this is for example the case, in meteorology, medicine,$\cdots$ etc. Studying the relationship between two random variables in order to predict one (the explained variable Y) given the other (the explanatory variable X) is one of the most important topics in functional data analysis. There are several ways to approach this forecasting problem for example: regression which is based on conditional expectation, [1] have introduced kernel estimators of this function when the dependent random variable is real and the independent random variable is of functional type. This nonparametric functional method is based essentially on an extension of the Nadaraya-Watson kernel estimator. Although this estimator is known to have good characteristics, its asymptotic bias depends on the derivative of the marginal density function. This makes the locally linear method not only more general but rather has better advantages. For this reason, this method is always attractive to several statisticians. Mention is made, for example, of the work of [2].

Key Words and Phrases: *Functional analysis, left truncation, α-mixing, local linear estimation*

Another difficulty encountered in statistics is the lack of information during the various observations on various scales of studies and analyses. This lack information introduces an obstacle during observation and obtaining the result. In statistical terminology, most often, these deficiencies are called censures and truncations which cause some disturbances at the level of the observed data.
Other wise, the fact of assuming that the studied data are always independent is unrealistic in practice, which is why statisticians and probabilists have focused their studies on another type of data which are dependent data as [3].
The aim of this work is to study, under standard assumptions, the pointwise almost sure convergence of the local linear estimator of the regression function under a dependent and functional truncation.

2 Main results

Let be $(Y_i, T_i, X_i)\, n$ independent and identically distributed copies of (Y, T, X) such as (Y, X) is a random vector that takes values in $\mathbb{R} \times \mathcal{F}$ with $\mathcal{F}$ an infinite dimensional space devoted with a semi metric d and T is a real random variable with unknown distribution function G. Beside, X and Y are supposed $\alpha-$mixed.
Under left truncation model, the local linear estimtor of the regression function $m(x) = E(Y/X = x)$ is expressed by

$$m_n(x) = \frac{\sum_{i=1}^{n} W_{ij}(x)Y_j}{\sum_{i=1}^{n} W_{ij}(x)}, \qquad \left(\frac{0}{0} := 0\right),$$

with $W_{ij}(x)$ are a calculated weights

Theorem 2.1. *Under a standard conditions in our context, we have*

$$m_n(x) - m(x) \longrightarrow 0 \qquad a.s$$

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Estimation non paramétrique pour des données fonctionnelles, censurées et dépendantes

Leulmi Farid*

Mailing address: Department of mathematique, University Constantine 1, Constantine, Algeria
E-mail: farid_leulmi@yahoo.fr

Leulmi Sarra

Mailing address: Department of mathematique, University Constantine 1, Constantine, Algeria
E-mail: math17sara@yahoo.fr

Résumé
Dans ce travail, nous nous intéressons à traiter une méthode d'estimation non paramétrique plus robuste et plus efficace que la méthode de noyau. Le modèle considéré ici est le modèle censuré aléatoirement à droite qui est le plus utilisé dans différents domaines pratiques. Dans un premier temps, nous proposons un estimateur de la régression en utilisant la méthode linéaire locale. Nous étudions sa convergence ponctuelle presque complète avec vitesse.

1 Introduction

Les données fonctionnelles peuvent apparaître dans différents domaines des sciences appliquées comme la chimie, environnement, biométrie, économétrie, médecine, …, etc. L'étude du lien entre une variable expliquée réelle et une variable explicative a des valeurs dans un espace de dimension infini (variable aléatoire fonctionnelle) est une des études les plus importantes en statistique, cette étude est réalisée en estimant la fonction de régression. Nous citons [1] pour l'estimation non paramétrique de la régression dans le cas complet et indépendant.
L'hypothèse fondamentale dans la construction de la plupart des modèles et tests statistiques est en générale l'indépendance des observations. Or, plusieurs exemples pratiques prouvent que cette hypothèse est souvent irréalisable, et notamment lorsqu'il s'agit de l'étude des séries chronologiques. La motivation constante qui a dicté le choix de la problématique de la thèse consiste clairement à une contribution à la gestion de la dépendance. Nous référons [3] pour l'estimation non paramétrique de la régression dans le cas complet et dépendant.
Une autre difficulté rencontrée en statistique est le manque d'information durant les différentes observations sur diverses échelles d'études et d'analyses. Ce manque

Key Words and Phrases: *Convergence presque complète, Dépendance, Données fonctionnelles, Données censurées*

d'information introduit un empêchement lors de l'observation et l'obtention du résultat. Dans la terminologie statistique, le plus souvent, on nomme ces manques par les censures et les troncatures qui provoquent quelques troubles au niveau des données observées. La censure à droite est le phénomène le plus couramment rencontré lors du recueil de données de survie, c'est la forme de censure qui reçoit le plus d'attention dans la littérature. Nous citons [2] pour l'estimation non paramétrique de la régression dans le cas censuré et indépendant.
C'est pour ces raisons, la problématique abordée dans ce travail est l'estimation non paramétrique de modèle conditionnel à variable explicative fonctionnelle lorsque la variable de réponse est supposée réelle et soumise à une censure à droite sous la condition de dépendance.

2 Résultats Principaux

Nous considérons le modèle de régression non paramétrique

$$T = m(X) + \epsilon,$$

dans lequel le couple aléatoire (X, T) est tel que T correspond à une durée de survie qui peut être censurée à droite par une autre v.a. C indépendante de T. L'erreur ϵ représente la variation de le variable d'intérêt (réponse) $T \in \mathbb{R}$ autour de le variable explicative (co-variable) $X \in \mathcal{F}$, avec $\mathcal{F}$ est un espace fonctionnelle.
Notre but est d'estimer l'espérance conditionnelle de T sachant X dans une optique non paramétrique. De manière analogue aux données complètes (voir [1]), l'estimateur local linéaire de la fonction de régression m, est donnée par

$$\widetilde{m}(x) = \frac{\sum_{i,j=1}^{n} W_{ij}(x) \frac{\delta_j Y_j}{S_n(Y_j)}}{\sum_{i,j=1}^{n} W_{ij}(x)}, \tag{2.1}$$

où $Y = \min(T, C)$ et $\delta = 1_{\{T \leq C\}}$. Ensuite nous traitons la convergence presque complète (p.co.) de $\widetilde{m}$, avec leurs vitesse.

Theorem 2.1. *Sous des conditions standard dans notre contexte, nous avons*

$$\widetilde{m}(x) - m(x) \to O, \quad (p.co.)$$

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Simulating the behavior of the heteroscedastic regression model for left truncated data

Saloua ALIOUCHE*

Mailing address: Faculty of Mathematics, University of Science and Technology Houari Boumediene, BP 32, El-Alia 16111, Algeria
E-mail: saliouche@usthb.dz

Ourida SADKI

Mailing address: Faculty of Mathematics, University of Science and Technology Houari Boumediene, BP 32, El-Alia 16111, Algeria
E-mail: osadki@usthb.dz

Abstract
The focus of this paper is on estimating heteroscedastic regression function for left truncated data. We study the asymptotic properties of the proposed estimator and give its convergence rate. Some simulations are given to confirm the performance of the theoretical results for these model.

1 Introduction

Let (X_i, Y_i), $1 \leq i \leq N$, be a vector of independent and identically distributed (iid) random variables (rv) distributed as $(X, Y) \in \mathbb{R}^d \times \mathbb{R}$, where Y is a variable of interest, and X is a covariate, admit a distribution function (df) F and V respectively.
Often these rv are considered as lifetimes that can not be fully observed. Let $(T_i)_{1\leq i \leq N}$ be a variable of left truncation with df G. We suppose that (X, Y) is independent of T.
In the random left-truncation model (RLT), we observe (X_i, Y_i, T_i) only if $Y_i \geq T_i$, whereas neither is observed if $Y_i < T_i$ $(i = 1, ..., N)$ with N is the sample size. Without possible confusion, we still denote (X_i, Y_i, T_i) $(1 \leq i \leq n)$, $n \leq N$ the actually observed rv.
Let $\alpha = \mathbb{P}(T \leq Y)$, be the probability of observing at least one pair of (X, Y, T).
In order to study the relation between X and Y, we consider the heteroscedastic regression model $Y = m(X) + \sigma(X)\varepsilon$ (1), where we assume that ϵ and X are independent, and the functions $m(.)$ and $\sigma(.)$ take the particular forms

$$m(x) = \int_0^1 F^{-1}(s|x)J(s)ds \ \ and \ \ \sigma^2(x) = \int_0^1 F^{-1}(s|x)^2 J(s)ds - m^2(x),$$

where $F^{-1}(s|x) = \inf\{y : F(y|x) \geq s\}$ is the quantile function of Y given x, $J(.)$ is the score function satisfied $\int_0^1 J(s)ds = 1$. The heteroscedastic regression model (1) has been extensively studied in the right censored case (see Sujika and Van Keilegom (2018) [?]).

Key Words and Phrases: *Asymptotic properties, Conditional quantile function, Heteroscedastic regression, Truncated data.*

The method proposed here consists to first estimate the conditional distribution $F(y|x)$ and then estimate $m(.)$ and $\sigma(.)$ in model (1).
The conditional distribution $F(y|x)$, can be written as $F(y|x) = \frac{F_1(x,y)}{v(x)}$, where $F_1(.,.)$ is the first derivative with respect to x of the joint df of (X,Y), and v is the density of the covariate. Lemdani et al. (2009) [2] proposed a kernel estimator of $F(.|.)$ defined by

$$\tilde{F}(y \mid x) \quad = \quad \frac{F_{1,n}(x,y)}{v_n(x)} = \frac{\frac{\alpha_n}{nh_{1,n}^d}\sum_{i=1}^{n} G_n^{-1}(Y_i)k_d\left(\frac{x-X_i}{h_{1,n}}\right)K_0\left(\frac{y-Y_i}{h_{2,n}}\right)}{\frac{\alpha_n}{nh_{1,n}^d}\sum_{i=1}^{n} G_n^{-1}(Y_i)k_d\left(\frac{x-X_i}{h_{1,n}}\right)},$$

where k_d is a density function on $\mathbb{R}^d$ (multivariate kernel), K_0 is a df, $h_{1,n}$, $h_{2,n}$ are two bandwidths towards 0 when $n \to \infty$, G_n is the Lynden-Bell (1971) [3] estimator of G and α_n is the He and Yang (1998) [1] estimator of α.
This motivates the introduction of the following estimators of $m(.)$ and $\sigma(.)$

$$\hat{m}(x) = \int_0^1 \tilde{F}^{-1}(s|x)J(s)ds \quad and \quad \hat{\sigma}^2(x) = \int_0^1 \tilde{F}^{-1}(s|x)^2 J(s)ds - \hat{m}^2(x).$$

2 Main results

Under some regular assumptions on the bandwidths, the kernel, the joint and marginal densities, the score function, $m(.)$ and $\sigma(.)$, we establish some asymptotic properties of our estimator. Our result is illustrate in the following theorem, we give in particular the almost sure convergence rate.

Theorem 2.1.

$$\sup_x |\hat{m}(x) - m(x)| = O\left(max\left\{\sqrt{\frac{\log n}{nh_{1,n}^d}}, h_{1,n}^2 + h_{2,n}^2\right\}\right), P - a.s \quad n \ \to \infty.$$

To support Theorem 2.1, some simulations have been realized to study the performance of the estimator for different combinations of sample sizes n and percentage of RLT α in the two cases: homoscedastic and heteroscedastic regression models and on $\mathbb{R}$ and $\mathbb{R}^d$.

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Spectral estimation for unevenly spaced time series

Charef Eddine Mansouri*

Mailing address:Laboratory of MSTD, Department of Probability and Statistics, University of Science and Technology Houari Boumediene, Algiers, Algeria
E-mail: cmansouri@usthb.dz

Tarek Medkour

Mailing address: Department of Probability and Statistics, University of Science and Technology Houari Boumediene, Algiers, Algeria
E-mail: tmedkour@usthb.dz

Abstract
In this paper we consider the problem of spectral analysis for unevenly spaced time series. We extend the result of quantile periodogram developed in [2] for unevenly sampling. The ultimate goal of this study is to investigate the asymptotic property of the quantile periodogram for unevenly spaced time series. We demonstrate the usefulness of the proposed periodogram in detecting the hidden frequency from the time series with big gaps and outliers. We evaluate the performance of our periodogram function using simulations.

1 Introduction

The analysis of time-series data is becoming increasingly common in astrophysics research, and will continue to grow in various fields of study. An unevenly (or unequally or irregularly) spaced time series is a sequence of observation time and value pairs (t_n, Y_n) in which the spacing of observation times is not constant.The analysis of unevenly spaced time series are limited by issues such as unevenly sampled data, measurement uncertainties, and the statistical methods we employ. A several methods were originally developed in statistics and signal processing have been applied to study the periodic pattern in uneven spaced time series. Wherefore, astronomers and statisticians used the periodogram for detecting a hidden frequency in the unevenly spaced time series. Therefore, most of the popular periodogram methods are based on least square regression fitting of the observing data with a linear combination of trial functions ($\cos(\omega t), \sin(\omega t)$). The least squares periodogram , also called the Lomb-Scargle (LS) periodogram proposed a modification on the ordinary periodogram to deal with unevenly samples.The Lomb-Scargle periodogram is efficient for Gaussian white noise, while it suffers from considerable degradation of performance when the noise has a heavy-tailed distribution. To overcome this probleme we propose a new periodogram that are constructed from trigonometrique quantile regression. The main contribution of the proposed periodogram is to evaluate the significance of the periodic patterns for unevenly spaced time series.

Key Words and Phrases: *Quantile regression, Periodicity, Unevenly spaced, time series analysis*

2 Main results

2.1 Lomb-Scargle periodogram

The Lomb-Scargle periodogram is a common and useful tool in the frequency analysis of unevenly spaced time series.

2.1.1 The Model

Suppose a set of discret-time noisy samples $y(t_k)$, $k = 1, 2, .., n$, is acquired by sampling on unequally time instants t_k from a continuous- time signal $y(t)$

$$\begin{aligned} y(t_k) &= \beta_1(\omega)\cos(\omega(t_k-\varphi)) + \beta_2(\omega)\sin(\omega(t_k-\varphi)) + \epsilon(t_k), \\ &= x^T(t_k,\omega)\beta(\omega) + \epsilon(t_k), \end{aligned} \tag{2.1}$$

where $\beta(\omega) = [\beta_1(\omega), \beta_2(\omega)]^T$ is the regression coefficients, ω could be any real number, the errors $\epsilon(t_k)$ are independent with zero mean and variance σ_ϵ^2 and φ is defined by :

$$\varphi = \frac{1}{2\omega}\arctan\left(\frac{\sum_{k=1}^{n}\sin(2\omega t_k)}{\sum_{k=1}^{n}\cos(2\omega t_k)}\right) \tag{2.2}$$

The coefficient vector $\beta(\omega)$ can be obtained by minimizing the mean squared error of fitting as:

$$\begin{aligned} \hat{\beta}(\omega) &= \arg\min_{\beta\in\mathbb{R}^2}\sum_{k=1}^{n}\left(y(t_k) - \beta_1\cos(\omega(t_k-\varphi)) - \beta_2\sin(\omega(t_k-\varphi))\right)^2, \\ &= \arg\min_{\beta\in\mathbb{R}^2}\|y(t_k) - x^T(t_k,\omega)\beta(\omega)\|_2^2, \end{aligned} \tag{2.3}$$

The Lomb-Scargle periodogram [3, 4] is given by

$$\begin{aligned} P_{LS}(\omega) &= \frac{1}{2\hat{\sigma}^2}\frac{\left[\sum_{k=1}^{n}((y(t_k)-\bar{y})\cos(\omega(t_k-\varphi))\right]^2}{\sum_{k=1}^{n}\cos^2(\omega(t_k-\varphi))} + \frac{\left[\sum_{k=1}^{n}((y(t_k)-\bar{y})\sin(\omega(t_k-\varphi)))\right]^2}{\sum_{k=1}^{n}\sin^2(\omega(t_k-\varphi))}, \\ &= \frac{1}{2\hat{\sigma}^2}(\hat{\beta}_1^2(\omega) + \hat{\beta}_2^2(\omega)), \end{aligned} \tag{2.4}$$

2.2 Quantile Periodogam for spectral analysis of unevenly spaced time series

A quantile periodogram is proposed. It is constructed from quantile regression estimates in a trigonometric linear model.

$$\tilde{\beta}_\tau(\omega) = \arg \min_{(\beta_{\tau,1},\beta_{\tau,2})\in\mathbb{R}^2} \sum_{k=1}^{n} \rho_\tau(y(t_k) - \beta_{\tau,1}\cos(\omega(t_k-\varphi)) - \beta_{\tau,2}\sin(\omega(t_k-\varphi))), \quad (2.5)$$

where $0 < \tau < 1$ and and $\rho_\tau(u) = u(\tau - I(u) < 0)$ is the check function. Therefore, instead of fitting the models mentioned in 2.1 we fit the model below:

$$y(t_k) = x^T(t_k,\omega)\beta(\omega) + \epsilon(t_k), \quad (2.6)$$

The quantile periodogram is defined by:

$$Q_\tau(\omega) = \frac{n}{4}\|\tilde{\beta}_\tau(\omega)\|_2^2 \quad (2.7)$$

3 Results

Theorem 3.1. *First, we start by giving some technical assumption which are required.*

(C1) $f_{t_k}(u) := F'_{t_k}(u)$ *exists for all* u *and* $F_{t_k}(u+\lambda) - F_{t_k}(\lambda) = f_{t_k}(\lambda)u + O(u^{d+1})$ *uniformly for* $|u| \le u_0$.

(C2) $F_{t_k}(\lambda) = \tau$ *and* $f_{t_k}(\lambda) = \kappa$ *for all* k.

(C3) $n^{-1} \lim_{n\to+\infty} \sum_{k=1}^{n} x^T(t_k,\omega)x(t_k,\omega) = \Omega$ *a positive definite matrix.*

Under the assumptions (C_1), (C_2), *and* (C_3) *we have :*

$$\sqrt{n}(\tilde{\beta}_\tau(\omega)) - \beta(\omega)) \overset{\mathcal{A}}{\sim} N(0, \eta^2\Omega^{-1}) \quad (3.1)$$

This asymptotic distribution for the regression quantile can be used to investigate the asymptotic distribution of the quantile periodogram defined in 2.7

Theorem 3.2. *Under the assumptions* (C_1), (C_2), *and* (C_3) *we have :*

$$\tilde{Q}_\tau(\omega) \overset{\mathcal{A}}{\sim} \Gamma(1, \eta^2) \quad (3.2)$$

4 Conclusion

In this paper, we investigated the application of different regression techniques (least squares, quantile regression) for periodicity detection in irregularly sampled time series. We show that the distribution of the quantile periodogram is a scaled gamma distribution. Simulations are reported on the performance of the proposed periodogram

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Theoretical study of mobilisation stochastic Keller-Segel model

Ali Slimani*

Department of Mathematics, University 20 Aout 1955, Algeria,
Laboratory applied mathematics and history and Didactics of mathematics (LAMAHIS)
E-mail: alislimani21math@gmail.com, ali.slimani@univ-skikda.dz

Abstract
In this paper, we investigate a stochastic chemotaxis model perturbed with additive noise and we study a result of existence and uniqueness of the local solution to the convection diffusion equation, we then prove that this solution is global if we added condition, the required results are obtained by stochastic analysis techniques, semigroup.

1 Introduction

Chemotaxis is a crucial model of cell contact. Chemical substances in the atmosphere have an effect on the movement of mobile organisms. Positive chemotaxis refers to movement in a direction where the chemical material concentration is higher, whereas negative chemotaxis refers to movement in a direction where there is a lower concentration of the chemical substance. The (Patlak) Keller-Segel system is a model that attempts to explain the chemotaxis phenomenon, which is a chemical attraction between organisms. The Keller-Segel model can be simplified to a nonlinear PDE. Patlak was the first to introduce the classical chemotaxis model, also known as the Keller-Segel model [20] [1953] E, Keller and L-Segel [20] [1970]. Many experts have studied the stochastic Keller-Segel equations.

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Key Words and Phrases: *Stochastic Keller-Segel model; Space-time white noise; chemotaxis; Gaussian process*

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Strong Solutions of Stochastic Differential Equations Driven by Spatial Parameters Semimartingale of McKean-Vlasov Type

Hafida BENBRAHIM*

Mailing address: Department of Mathematics, Mohamed Khider-Biskra University, Biskra, Algeria
E-mail: hafida.benbrahim@univ-biskra.dz

Abstract
The purpose of this work, is to study a new type of stochastic McKean-Vlasov equations in general form where the system is driven by continuous semimartingale which depend, nonlinearly, on both the state process as well as of its law. Our main objective is to prove a theorem of the existence and uniqueness of strong solutions to this type of equations. The proof is made up of two parts, firstly, under the linear growth conditions and using the Picard's iteration we prove the existence of its strong solution. Then we observe the pathwisse uniqueness of this solution for which the local characteristic of the semimartingale satisfies non-Lipischitz conditions.

1 Introduction

Kunita in ([**3**] Kunita, Stochastic Flows and Stochastic Differential Equations, Cambridge University Press, UK, 1990) studied stochastic differential equations (SDEs) driven by semimartingales with spatial parameters, as an extension of the classical Itô's SDE. Motivated by this results, We consider the following equation

$$\begin{cases} dX_t = F\left(dt, X_t, \mathbb{P}_{X_t}\right) \\ X_0 = \xi \end{cases} \tag{1.1}$$

where $F(t,x,\mu) = \left\{F^1(t,x,\mu), ..., F^d(t,x,\mu)\right\}$, $(x,\mu) \in \mathbb{R}^d \times P_2\left(\mathbb{R}^d\right)$, is a $C\left(\mathbb{R}^d \times P_2\left(\mathbb{R}^d\right); \mathbb{R}^d\right)$-valued, continuous semimartingale with local characteristic (q,b), and $\mathbb{P}_{X_t}$ is the law of X_t. Firstly, following [3], we introduce assumptions for a continuous semimartingale F so that the Itô integral is well defined. Next, under a linear growth conditions, we use the Picard's iteration, Kolmogorov's tightness criteria, and a suitable version of the Skorokhod theorem [2] to prove the existence of strong solution of this equation. Finally, the pathwise uniqueness is obtained under two non-Lipschitz conditions.

2 Main results

Definition 2.1. Let $\mathcal{P}_2\left(\mathbb{R}^d\right)$ be the space of probability measures with finite second moment equipped with the Wasserstein metric W_2. Assuming that $F(t,x,\mu) = \left\{F^1(t,x,\mu), ..., F^d(t,x,\mu)\right\}$ is a famlly of continuous $C\left(\mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right); \mathbb{R}^d\right)$-semimartingales with parameter $(x,\mu) \in \mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right)$ defined on a complete filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P}\right)$ with parameters $(x,\mu) \in \mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right)$. Let $F^i(t,x,\mu) = B^i(t,x,\mu) +$

Key Words and Phrases: *Stochastic McKean-Vlasov equations, non-Lipschitzian, local characteristic, strong solution.*

$M^i(t,x,\mu)$ be its the decomposition such that: $M^i(t,x,\mu)$ is a continuous d-dimensional local $(\mathcal{F}_t)_{t\geq 0}$-martingale, its quadratic variation is given by $\left\langle M^i(t,x,\mu), M^j(t,y,\nu)\right\rangle = \int_0^t q^{ij}(s,x,\mu,y,\nu)\,ds$, and $B^i(t,x,\mu) = \int_0^t b^i(s,x,\mu)\,ds$ is a continuous process of finite variation. the pair (q,b) is called the local characteristic of $F(t,x,\mu)$, where $q(s,x,\mu,y,\nu) = \left(q^{ij}(s,x,\mu,y,\nu)\right)_{d\times d}$ is symmetric and non-negative definite $d\times d$-matrix valued function and $b(s,x,\mu) = \left(b^i(s,x,\mu)\right)_{1\times d}$. if X_t is a predictable process such that

$$\int_0^T \left|q^{ij}(t,X_t,\mathbb{P}_{X_t},X_t,\mathbb{P}_{X_t})\right| dt < \infty \text{ a.s and } \int_0^T \left|b^i(t,X_t,\mathbb{P}_{X_t})\right| dt < \infty \text{ a.s.}$$

Then the generalized Itô integral $\int_0^T F(dt,X_t,\mathbb{P}_{X_t})$ is well defined.

Theorem 2.1. *Assuming that $X_0 = \xi \in L^2\left(\Omega,\mathcal{F}_0,\mathbb{P};\mathbb{R}^d\right)$ and the local characteristic (q,b) of $F(t,x,\mu)$ satisfies the following conditions:*
(C.1) *The functions $b(t,x,\mu)$ and $q(t,x,\mu,y,\nu)$ are Borel measurable and continuous in (t,x,μ) and (t,x,μ,y,ν), respectively.*
(C.2) *There exist a non-negative non-random function $K(t)$ with $\int_0^t [K(s)]^n ds < \infty$, $\forall n > 0$, such that, for every $(x,\mu) \in \mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right)$ and $t\in[0,T]$, q and b satisfy*

$$|b(t,x,\mu)| \leq K(t)\left(1+|x|+W_2(\mu,\delta_0)\right),$$
$$\|q(t,x,\mu,x,\mu)\| \leq 3K(t)\left(1+|x|^2+W_2^2(\mu,\delta_0)\right),$$

where δ_0 is the Dirac measure at 0.
(C.3) *Let $\sigma_1(x)$ and $\sigma_2(x)$ be a positive continuous functions, bounded on $[1,\infty[$, and satisfying $\lim_{x\downarrow 0} \frac{c(\sigma_i(x)+1)}{\log(x^{-2})} = 0$, $i=1,2$, where c is a positive constante. Assume that*

$$|b(t,x,\mu) - b(t,y,\nu)| \leq k\left(|x-y|\,\sigma_1(|x-y|) + W_2(\mu,\upsilon)\right),$$
$$\|q(t,x,\mu,x,\mu) - 2q(t,x,\mu,y,\nu) + q(t,y,\nu,y,\nu)\| \leq 2k\left(|x-y|^2\sigma_2(|x-y|) + W_2^2(\mu,\upsilon)\right)$$

where $k>0$ is a constant. Then the MV-SDE 1.1 admits a unique strong solution.

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On a Stochastic Differential System Driven by G-Brownian Motion and its Applications

El-Hacène Chalabi

Department of Pharmacy, Faculty of Medicine, Salah Boubnider University, Constantine, Algeria
E-mail: elhacene25@gmail.com

Abstract
In this paper we prove the convergence of Caratheodory's approximate solution to the unique solution for a stochastic system of differential equations driven by $G-$Brownian motion in the case where the coefficients of the equations are non-Lipschitzian and nonlinear growth. The system studied appears in the modeling of many diffusion phenomena in various sciences, particularly in biology and medicine.

1 Introduction

N. Caratheodory [2] is the first to introduce the Caratheodory approximation scheme in the study of ordinary differential equations. This approximation has been used by many mathematicians to prove the existence and uniqueness of solutions for differential equations with different conditions. Let's take for example, K.Liu in [4] has used this approximation for a class of infinite-dimensional stochastic equations with time delays. Furthermore, H.Young [5] has discussed the approximate solutions to stochastic differential delay equation.
Also, [3] F. Faiz Ullah proved that the Caratheodory approximation solution for stochastic differential equation driven by a $G-$Brownian motion. In short, we write $G-$SDEs converges to the unique solution of the $G-$SDEs under the Lipschitz and the linear growth conditions.
S. Peng established the theory of non linear expectation and defined the related stochastic calculus, especially stochastic integrals of Itô's type due to $G-$Brownian motion and derived the related Itô's formula. Also, the notion of G-normal distribution plays the same important role in the theory of non linear expectation as that of normal distribution with the classical probability.
The existence and the uniqueness theorems of the solution for the $G-$SDEs under different conditions was proved in [1]. In this paper we give the proof of the convergence of Caratheodory's approximate solution to the unique solution for the following stochastic

Key Words and Phrases: *G-expectation, G-brownian motion, G-stochastic differential equations, Caratheodory approximation scheme.*

systemes (S$G-$SDEs):

$$\begin{cases} X(t) = X_0 + \int\limits_{t_0}^{t} \varphi_1(s, X(s), Y(s))\, ds \\ \qquad + \int\limits_{t_0}^{t} \varphi_2(s, X(s), Y(s))\, d\langle B, B\rangle(s) + \int\limits_{t_0}^{t} \varphi_3(s, X(s), Y(s))\, dB(s) \\ Y(t) = Y_0 + \int\limits_{t_0}^{t} \psi_1(s, X(s), Y(s))\, ds + \\ \qquad + \int\limits_{t_0}^{t} \psi_2(s, X(s), Y(s))\, d\langle B, B\rangle(s) + \int\limits_{t_0}^{t} \psi_3(s, X(s), Y(s))\, dB(s) \end{cases} \tag{1.1}$$

Where (X_0, Y_0) is a given initial condition, $(\langle B(t), B(t)\rangle)_{t\geq 0}$ is the quadratic variation process of the $G-$Brownian motion $(B(t))_{t\geq 0}$ and all the coefficients $\varphi_i(t, x, y)$, $\psi_i(t, x, y)$, for $i = 1, 2, 3$ are non uniform Lipschitzian and non linear growt. These results are obtained by using the Caratheodory approximation scheme.
This paper is divided into three sections. The second section gives the necessary notations and results that we will use in this work. The third section proves the convergence of Caratheodory's approximate solution to the unique solution for S$G-$SDEs (1.1).

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Approche par la théorie des opérateurs pour les systèmes d'itération de fonctions aléatoires

Fetima LADJIMI *

Mailing address : Département de Mathématiques, Université Mouloud Mammeri de Tizi-Ouzou
Laboratoire de Mathématiques Pures et Appliquées, Tizi-Ouzou, Algérie.
E-mail: fetima.ladjimi@ummto.dz

Abstract Nous nous intéréssons à l'étude de comportement asymptotique des chaînes de Markov itératives i.e obtenues par itération de fonctions aléatoires. En utilisant des techniques d'opérateurs linéaires quasi-compacts, nous montrons l'existence et l'unicité d'une mesure invariante (stationnaire) pour ces chaînes, dans le cas où les fonctions aléatoires itérées sont Lipschitziennes. Nous appliquons cette approche à l'étude de comportement asymptotique de la chaîne de Diaconis-Freedman sur $[0,1]$.

1 Introduction

Les processus et chaînes de Markov sous la forme d'itérées de fonctions aléatoires ou de systèmes dynamiques sont intensément étudiés ces derniéres années. Rappelons que cela fait suite aux travaux de Doeblin et Fortet (1937), Norman (1972), Fuhrstenberg (1963),... Kifer (1986) assurant en particulier que toute chaîne de Markov peut être mise sous la forme itérative. Sous cette forme et suite à ces développements, les processus et chaînes de Markov ont offert un cadre théorique adapté aux théories de l'apprentissage et ont ouvert la voie à de nombreux champs d'application : fractals, traitement d'images, dynamique des populations, météorologie, actuariat,... Cela a permis, en particulier, l'émergence des techniques de simulation exacte (Algorithme de Propp et Wilson(1996)) réalisant ainsi une avancée remarquable dans les méthodes de simulation.
Dans la suite nous faisons une étude de ce type de chaînes en se basant sur l'approche par la théorie spéctrale des opérateurs linéaires quasi-compact.

2 Système d'itération de fonctions aléatoires lipschitziennes avec dépendance spatiale des probabilités de transition

Soit (E,d) un espace métrique compact et $\mathbb{L}\mathrm{ip}(E,E)$ l'espace de Banach des fonctions lipschitziennes de E dans E. On note par $\mathscr{H}_\alpha(E), 0<\alpha\leq 1$ l'espace des fonctions α-Hölder continues de E dans $\mathbb{C}$, défini par :

$$\mathscr{H}_\alpha(E) := \{f \in C(E) \mid \|f\|_\alpha := |f|_\infty + m_\alpha(f) < +\infty\}$$

Key Words and Phrases: *Loi stationnaire, Opérateur quasi-compact , Itération de fonctions aléatoires, Chaîne de Markov.*

où $m_\alpha(\varphi) := \sup_{\substack{x,y\in E \\ x\neq y}} \frac{|f(x)-f(y)|}{d(x,y)^\alpha}$ et $|\cdot|_\infty$ est la norme de la convergence uniforme .

Soit $(\mu_x)_{x\in E}$ une collection de mesures de probabilité sur $\mathbb{L}\mathrm{ip}(E,E)$ et l'on considère la chaîne de Markov $(X_n)_{n\geq 0}$ sur E dont le noyau de transition P est donné pour toute fonction borélienne bornée $\varphi : E \to \mathbb{C}$ et tout $x \in E$, par

$$P\varphi(x) = \int_{\mathbb{L}\mathrm{ip}(E,E)} \varphi(T(x))\mu_x(\mathrm{d}T).$$

Nous citons le résultat suivant donné dans (Ladjimi et Peigné [3], Proposition 2.2).

Theorem 2.1. *[Ladjimi and Peigné [3]] Supposons qu'il existe $\alpha \in (0,1]$ such that*

H1. $r := \sup_{\substack{x,y\in E \\ x\neq y}} \int_{\mathbb{L}\mathrm{ip}(E,E)} \left(\frac{d(T(x),T(y))}{d(x,y)}\right)^\alpha \mu_x(\mathrm{d}T) < 1,$

H2. $R_\alpha := \sup_{\substack{x,y\in E \\ x\neq y}} \frac{|\mu_x - \mu_y|}{d(x,y)^\alpha} < +\infty,$

H3. *Il existe $\delta > 0$ et une mesure de probabilité μ sur E telle que*

$$\forall x \in E \qquad \mu_x \geq \delta\mu$$

H4. *Il existe $x_0 \in E$ et une suite $(\xi_n)_{n\geq 0}$ des fonctions continues de semi-groupe T_μ engendré par le support de μ tels que*

$$\forall x \in E \quad \lim_{n\to+\infty} \xi_n(x) = x_0.$$

Il existe alors sur E une unique mesure de probabilité P-invariante ν. De plus, il existe des constantes $\kappa > 0$ et $\rho \in]0,1[$ telles que, pour toute fonction $\varphi \in \mathscr{H}_\alpha(E)$, on ait

$$\forall x \in E \quad |P^n\varphi(x) - \nu(\varphi)| \leq \kappa\rho^n. \tag{2.1}$$

3 Application à la chaîne de Diaconis-Freedman sur $[0,1]$

Nous appliquons les résultats précédents pour étudier le comportement asymptotique d'un exemple de chaînes de Markov itératives, Il s'agit de la chaîne de Diaconis- Freedman sur $[0,1]$ décrivant les mouvements d'une particule entre deux points attractifs.

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Estimation of functional regression with missing data at random

BENCHIHA Abbassia*

Laboratory of Statistics and Stochastic Processes, University of Djillali Liabes,BP 89,Sidi Bel-Abbès,22000,Algeria
E-mail: benchiha.abbassia@yahoo.fr

Abstract
In this paper, we consider the problem of the co-variability analysis between a functional variable X and a scalar response variable Y which is not totally observed. We use the local linear approach to model this relationship by constructing a local linear estimator of the regression operator when missing data occur in the response variable. Asymptotic results, in term of the pointwise almost complete consistencies, is established for the constructed estimator.
Motivated by its interaction with other applied fields, the functional data analysis (FDA) became a major topic of research in recent years. A key references on this topic are the monographs of Ramsay and Silverman (2005), Bosq (2000), Ferraty and Vieu (2006), Cuevas (2014), Zhang (2014) and Hsing and Eubank (2015). In this paper, we are interested by studying the local linear estimate of the regression operator when the regressor is of functional kind and the response variable presents some missing observations.

Notice that the local polynomial approach has various advantages over the kernel method, namely in the bias term (see, Fan and Gijbels 1996, for an extensive discussion on the comparison between both these methods). The local linear modeling has been recently introduced in the FDA by Bàillo and Grané (2009). They proved the L^2-consistency of the local linear estimate of the regression function when the explanatory variable takes values in a Hilbert space. Barrientos et al. (2010) have considered an alternative and simplified version which can be used for more general functional covariates. Chouaf and Laksaci (2012) as far as they are concerned have studied a spatial version of the estimate proposed by Barrientos et al. (2010). They established the almost complete consistency of this estimate when the observations are spatially dependent. We return to Demongeot et al. (2014) for the functional local linear estimate of the conditional cumulative distribution function. All these works are only concerned with the complete data case. Our main purpose, in this paper, is to study the case when the response variable has missing data at random (MAR).

1 Introduction

Consider n independent pairs of random variables (X_i, Y_i) for $i = 1, \dots, n$ that we assume drawn from the pair(X, Y). The latter is valued in $\mathcal{F} \times \mathbb{R}$, where $\mathcal{F}$ is a semi-metric space and d denotes a semi-metric.

Key Words and Phrases: *Almost complete convergence, Local linear method, Missing data, Uniform consistency, Regression operator*

Usually, the local polynomial estimate is constructed by approximating, locally, the non-parametric model by a polynomial function. In functional statistics, when the data are complete, Barrientos et al. (2010) have proposed to approximate the regression operator $R(x) = \mathbb{E}[Y|X = x]$ in a neighborhood of x by

$$R(Z) = a + b\beta(z, x), \text{ where } z \text{ is in a neighborhood of } x.$$

The scalars a and b are estimated by

$$\min_{(a,b)\in\mathbb{R}^2} \sum_{i=1}^{n} (Y_i - a - b\beta(X_i, x))^2 K(h^{-1}\varrho(x, X_i)) \tag{1.1}$$

where K is a kernel and $h = h_{K,n}$ is a sequence of positive real numbers, $\beta(.,.)$ is a known function from $\mathcal{F}^2$ into $\mathbb{R}$ such that, $\forall\xi \in \mathcal{F}$, $\beta(\xi, \xi) = 0$ and $\varrho(.,.)$ is a function of $\mathcal{F}\times\mathcal{F}$ such that $d(.,.) = |\varrho(.,.)|$. In what follows, we consider the case where the response variables are MAR data. In this case, we introduce a Bernoulli random variable δ such that $\delta = 1$ if Y is observed and $\delta = 0$ otherwise. The MAR consideration implies that the two variables Y and δ are independent conditionally to a given X, in the sense that

$$\mathbb{P}(\delta = 1|X, Y) = \mathbb{P}(\delta = 1|X) = P(X)$$

where $P(\cdot)$ is an unknown functional operator which is called the conditional probability of observing the response variable Y given the explanatory variable X.

2 Main results

Theorem 2.1. *Under hypotheses, we have* $|\widehat{R}(x) - R(x)| = O(h^{\kappa}) + O\left(\sqrt{\frac{\log n}{n\ \ \phi_x(h)}}\right)$, *a.co.*

Lemma 2.1. *Under the hypotheses of Theorem 2.1, we have that:*
$\left|\widehat{R}_D(x) - \mathbb{E}[\widehat{R}_D(x)]\right| = O_{a.co.}\left(\sqrt{\frac{\log n}{n\phi_x(h)}}\right)$ *and* $|\widehat{R}_N(x) - \mathbb{E}[\widehat{R}_N(x)]| = O_{a.co.}\left(\sqrt{\frac{\log n}{n\phi_x(h)}}\right)$.

Corollary 2.1. *Under the hypotheses of Lemma 2.1, there exists a positive real C such that* $\sum_{n=1}^{\infty} \mathbb{P}\left(\widehat{R}_D(x) < C\right) < \infty$.

Lemma 2.2. *Under hypotheses, we have:* $\left|\widehat{B}(x)\right| = O(h^{\kappa})$.

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Theoretical study of mobilisation stochastic Keller-Segel model

Ali Slimani*

Department of Mathematics, University 20 Aout 1955, Algeria,
Laboratory applied mathematics and history and Didactics of mathematics (LAMAHIS)
E-mail: alislimani21math@gmail.com, ali.slimani@univ-skikda.dz

Abstract
In this paper, we investigate a stochastic chemotaxis model perturbed with additive noise and we study a result of existence and uniqueness of the local solution to the convection diffusion equation, we then prove that this solution is global if we added condition, the required results are obtained by stochastic analysis techniques, semigroup.

1 Introduction

Chemotaxis is a crucial model of cell contact. Chemical substances in the atmosphere have an effect on the movement of mobile organisms. Positive chemotaxis refers to movement in a direction where the chemical material concentration is higher, whereas negative chemotaxis refers to movement in a direction where there is a lower concentration of the chemical substance. The (Patlak) Keller-Segel system is a model that attempts to explain the chemotaxis phenomenon, which is a chemical attraction between organisms. The Keller-Segel model can be simplified to a nonlinear PDE. Patlak was the first to introduce the classical chemotaxis model, also known as the Keller-Segel model [20] [1953] E, Keller and L-Segel [20] [1970]. Many experts have studied the stochastic Keller-Segel equations.

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Key Words and Phrases: *Stochastic Keller-Segel model; Space-time white noise; chemotaxis; Gaussian process*

Strong Solutions of Stochastic Differential Equations Driven by Spatial Parameters Semimartingale of McKean-Vlasov Type

Hafida BENBRAHIM*

Mailing address: Department of Mathematics, Mohamed Khider-Biskra University, Biskra, Algeria
E-mail: hafida.benbrahim@univ-biskra.dz

Abstract
The purpose of this work, is to study a new type of stochastic McKean-Vlasov equations in general form where the system is driven by continuous semimartingale which depend, nonlinearly, on both the state process as well as of its law. Our main objective is to prove a theorem of the existence and uniqueness of strong solutions to this type of equations. The proof is made up of two parts, firstly, under the linear growth conditions and using the Picard's iteration we prove the existence of its strong solution. Then we observe the pathwisse uniqueness of this solution for which the local characteristic of the semimartingale satisfies non-Lipischitz conditions.

1 Introduction

Kunita in (**[3]** Kunita, Stochastic Flows and Stochastic Differential Equations, Cambridge University Press, UK, 1990) studied stochastic differential equations (SDEs) driven by semimartingales with spatial parameters, as an extension of the classical Itô's SDE. Motivated by this results, We consider the following equation

$$\begin{cases} dX_t = F\left(dt, X_t, \mathbb{P}_{X_t}\right) \\ X_0 = \xi \end{cases} \tag{1.1}$$

where $F(t,x,\mu) = \left\{F^1(t,x,\mu), ..., F^d(t,x,\mu)\right\}$, $(x,\mu) \in \mathbb{R}^d \times P_2\left(\mathbb{R}^d\right)$, is a $C\left(\mathbb{R}^d \times P_2\left(\mathbb{R}^d\right); \mathbb{R}^d\right)$-valued, continuous semimartingale with local characteristic (q,b), and $\mathbb{P}_{X_t}$ is the law of X_t. Firstly, following [3], we introduce assumptions for a continuous semimartingale F so that the Itô integral is well defined. Next, under a linear growth conditions, we use the Picard's iteration, Kolmogorov's tightness criteria, and a suitable version of the Skorokhod theorem [2] to prove the existence of strong solution of this equation. Finally, the pathwise uniqueness is obtained under two non-Lipschitz conditions.

2 Main results

Definition 2.1. Let $\mathcal{P}_2\left(\mathbb{R}^d\right)$ be the space of probability measures with finite second moment equipped with the Wasserstein metric W_2. Assuming that $F(t,x,\mu) = \left\{F^1(t,x,\mu), ..., F^d(t,x,\mu)\right\}$ is a familly of continuous $C\left(\mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right); \mathbb{R}^d\right)$-semimartingales with parameter $(x,\mu) \in \mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right)$ defined on a complete filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P}\right)$ with parameters $(x,\mu) \in \mathbb{R}^d \times \mathcal{P}_2\left(\mathbb{R}^d\right)$. Let $F^i(t,x,\mu) = B^i(t,x,\mu) +$

Key Words and Phrases: *Stochastic McKean-Vlasov equations, non-Lipschitzian, local characteristic, strong solution.*

$M^i(t,x,\mu)$ be its the decomposition such that: $M^i(t,x,\mu)$ is a continuous d-dimensional local $(\mathcal{F}_t)_{t\geq 0}$-martingale, its quadratic variation is given by $\langle M^i(t,x,\mu), M^j(t,y,\nu)\rangle = \int_0^t q^{ij}(s,x,\mu,y,\nu)\,ds$, and $B^i(t,x,\mu) = \int_0^t b^i(s,x,\mu)\,ds$ is a continuous process of finite variation. the pair (q,b) is called the local characteristic of $F(t,x,\mu)$, where $q(s,x,\mu,y,\nu) = \left(q^{ij}(s,x,\mu,y,\nu)\right)_{d\times d}$ is symmetric and non-negative definite $d\times d$-matrix valued function and $b(s,x,\mu) = \left(b^i(s,x,\mu)\right)_{1\times d}$. if X_t is a predictable process such that

$$\int_0^T \left|q^{ij}\left(t,X_t,\mathbb{P}_{X_t},X_t,\mathbb{P}_{X_t}\right)\right| dt < \infty \text{ a.s and } \int_0^T \left|b^i\left(t,X_t,\mathbb{P}_{X_t}\right)\right| dt < \infty \text{ a.s.}$$

Then the generalized Itô integral $\int_0^T F\left(dt,X_t,\mathbb{P}_{X_t}\right)$ is well defined.

Theorem 2.1. *Assuming that $X_0 = \xi \in L^2\left(\Omega,\mathcal{F}_0,\mathbb{P};\mathbb{R}^d\right)$ and the local characteristic (q,b) of $F(t,x,\mu)$ satisfies the following conditions:*
(C.1) *The functions $b(t,x,\mu)$ and $q(t,x,\mu,y,\nu)$ are Borel measurable and continuous in (t,x,μ) and (t,x,μ,y,ν), respectively.*
(C.2) *There exist a non-negative non-random function $K(t)$ with $\int_0^t [K(s)]^n\,ds < \infty$, $\forall n > 0$, such that, for every $(x,\mu)\in\mathbb{R}^d\times\mathcal{P}_2\left(\mathbb{R}^d\right)$ and $t\in[0,T]$, q and b satisfy*

$$|b(t,x,\mu)| \leq K(t)\left(1+|x|+W_2(\mu,\delta_0)\right),$$
$$\|q(t,x,\mu,x,\mu)\| \leq 3K(t)\left(1+|x|^2+W_2^2(\mu,\delta_0)\right),$$

where δ_0 is the Dirac measure at 0.
(C.3) *Let $\sigma_1(x)$ and $\sigma_2(x)$ be a positive continuous functions, bounded on $[1,\infty[$, and satisfying $\lim_{x\downarrow 0}\frac{c(\sigma_i(x)+1)}{\log(x^{-2})} = 0$, $i=1,2$, where c is a positive constante. Assume that*

$$|b(t,x,\mu)-b(t,y,\nu)| \leq k\left(|x-y|\,\sigma_1(|x-y|)+W_2(\mu,\upsilon)\right),$$
$$\|q(t,x,\mu,x,\mu)-2q(t,x,\mu,y,\nu)+q(t,y,\nu,y,\nu)\| \leq 2k\left(|x-y|^2\sigma_2(|x-y|)+W_2^2(\mu,\upsilon)\right)$$

where $k>0$ is a constant. Then the MV-SDE 1.1 admits a unique strong solution.

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On a Stochastic Differential System Driven by G-Brownian Motion and its Applications

El-Hacène Chalabi

Department of Pharmacy, Faculty of Medicine, Salah Boubnider University, Constantine, Algeria
E-mail: elhacene25@gmail.com

Abstract
In this paper we prove the convergence of Caratheodory's approximate solution to the unique solution for a stochastic system of differential equations driven by $G-$Brownian motion in the case where the coefficients of the equations are non-Lipschitzian and nonlinear growth. The system studied appears in the modeling of many diffusion phenomena in various sciences, particularly in biology and medicine.

1 Introduction

N. Caratheodory [2] is the first to introduce the Caratheodory approximation scheme in the study of ordinary differential equations. This approximation has been used by many mathematicians to prove the existence and uniqueness of solutions for differential equations with different conditions. Let's take for example, K.Liu in [4] has used this approximation for a class of infinite-dimensional stochastic equations with time delays. Furthermore, H.Young [5] has discussed the approximate solutions to stochastic differential delay equation.
Also, [3] F. Faiz Ullah proved that the Caratheodory approximation solution for stochastic differential equation driven by a $G-$Brownian motion. In short, we write $G-$SDEs converges to the unique solution of the $G-$SDEs under the Lipschitz and the linear growth conditions.
S. Peng established the theory of non linear expectation and defined the related stochastic calculus, especially stochastic integrals of Itô's type due to $G-$Brownian motion and derived the related Itô's formula. Also, the notion of G-normal distribution plays the same important role in the theory of non linear expectation as that of normal distribution with the classical probability.
The existence and the uniqueness theorems of the solution for the $G-$SDEs under different conditions was proved in [1]. In this paper we give the proof of the convergence of Caratheodory's approximate solution to the unique solution for the following stochastic

Key Words and Phrases: *G-expectation, G-brownian motion, G-stochastic differential equations, Caratheodory approximation scheme.*

systemes (S$G-$SDEs):

$$\begin{cases} X(t) = X_0 + \int\limits_{t_0}^{t} \varphi_1(s, X(s), Y(s))\, ds \\ \qquad + \int\limits_{t_0}^{t} \varphi_2(s, X(s), Y(s))\, d\langle B, B\rangle(s) + \int\limits_{t_0}^{t} \varphi_3(s, X(s), Y(s))\, dB(s) \\ Y(t) = Y_0 + \int\limits_{t_0}^{t} \psi_1(s, X(s), Y(s))\, ds + \\ \qquad + \int\limits_{t_0}^{t} \psi_2(s, X(s), Y(s))\, d\langle B, B\rangle(s) + \int\limits_{t_0}^{t} \psi_3(s, X(s), Y(s))\, dB(s) \end{cases} \tag{1.1}$$

Where (X_0, Y_0) is a given initial condition, $(\langle B(t), B(t)\rangle)_{t\geq 0}$ is the quadratic variation process of the $G-$Brownian motion $(B(t))_{t\geq 0}$ and all the coefficients $\varphi_i(t, x, y)$, $\psi_i(t, x, y)$, for $i = 1, 2, 3$ are non uniform Lipschitzian and non linear growt. These results are obtained by using the Caratheodory approximation scheme.
This paper is divided into three sections. The second section gives the necessary notations and results that we will use in this work. The third section proves the convergence of Caratheodory's approximate solution to the unique solution for S$G-$SDEs (1.1).

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Approche par la théorie des opérateurs pour les systèmes d'itération de fonctions aléatoires

Fetima LADJIMI *

Mailing address : Département de Mathématiques, Université Mouloud Mammeri de Tizi-Ouzou
Laboratoire de Mathématiques Pures et Appliquées, Tizi-Ouzou, Algérie.
E-mail: fetima.ladjimi@ummto.dz

Abstract Nous nous intéréssons à l'étude de comportement asymptotique des chaînes de Markov itératives i.e obtenues par itération de fonctions aléatoires. En utilisant des techniques d'opérateurs linéaires quasi-compacts, nous montrons l'existence et l'unicité d'une mesure invariante (stationnaire) pour ces chaînes, dans le cas où les fonctions aléatoires itérées sont Lipschitziennes. Nous appliquons cette approche à l'étude de comportement asymptotique de la chaîne de Diaconis-Freedman sur $[0,1]$.

1 Introduction

Les processus et chaînes de Markov sous la forme d'itérées de fonctions aléatoires ou de systèmes dynamiques sont intensément étudiés ces dernières années. Rappelons que cela fait suite aux travaux de Doeblin et Fortet (1937), Norman (1972), Fuhrstenberg (1963),... Kifer (1986) assurant en particulier que toute chaîne de Markov peut être mise sous la forme itérative. Sous cette forme et suite à ces développements, les processus et chaînes de Markov ont offert un cadre théorique adapté aux théories de l'apprentissage et ont ouvert la voie à de nombreux champs d'application : fractals, traitement d'images, dynamique des populations, météorologie, actuariat,... Cela a permis, en particulier, l'émergence des techniques de simulation exacte (Algorithme de Propp et Wilson(1996)) réalisant ainsi une avancée remarquable dans les méthodes de simulation.
Dans la suite nous faisons une étude de ce type de chaînes en se basant sur l'approche par la théorie spéctrale des opérateurs linéaires quasi-compact.

2 Système d'itération de fonctions aléatoires lipschitziennes avec dépendance spatiale des probabilités de transition

Soit (E,d) un espace métrique compact et $\mathbb{L}\mathrm{ip}(E,E)$ l'espace de Banach des fonctions lipschitziennes de E dans E. On note par $\mathscr{H}_\alpha(E), 0<\alpha\leq 1$ l'espace des fonctions α-Hölder continues de E dans $\mathbb{C}$, défini par :

$$\mathscr{H}_\alpha(E) := \{f \in C(E) \mid \|f\|_\alpha := |f|_\infty + m_\alpha(f) < +\infty\}$$

Key Words and Phrases: *Loi stationnaire, Opérateur quasi-compact , Itération de fonctions aléatoires, Chaîne de Markov.*

où $m_\alpha(\varphi) := \sup\limits_{\substack{x,y\in E\\ x\neq y}} \dfrac{|f(x)-f(y)|}{d(x,y)^\alpha}$ et $|\cdot|_\infty$ est la norme de la convergence uniforme .

Soit $(\mu_x)_{x\in E}$ une collection de mesures de probabilité sur $\mathbb{L}\text{ip}(E,E)$ et l'on considère la chaîne de Markov $(X_n)_{n\geq 0}$ sur E dont le noyau de transition P est donné pour toute fonction borélienne bornée $\varphi : E \to \mathbb{C}$ et tout $x \in E$, par

$$P\varphi(x) = \int_{\mathbb{L}\text{ip}(E,E)} \varphi(T(x))\mu_x(\mathrm{d}T).$$

Nous citons le résultat suivant donné dans (Ladjimi et Peigné [3], Proposition 2.2).

Theorem 2.1. *[Ladjimi and Peigné [3]] Supposons qu'il existe $\alpha \in (0,1]$ such that*

H1. $r := \sup\limits_{\substack{x,y\in E\\ x\neq y}} \displaystyle\int_{\mathbb{L}\text{ip}(E,E)} \Big(\frac{d(T(x),T(y))}{d(x,y)}\Big)^\alpha \mu_x(\mathrm{d}T) < 1,$

H2. $R_\alpha := \sup\limits_{\substack{x,y\in E\\ x\neq y}} \dfrac{|\mu_x-\mu_y|}{d(x,y)^\alpha} < +\infty,$

H3. *Il existe $\delta > 0$ et une mesure de probabilité μ sur E telle que*

$$\forall x \in E \qquad \mu_x \geq \delta\mu$$

H4. *Il existe $x_0 \in E$ et une suite $(\xi_n)_{n\geq 0}$ des fonctions continues de semi-groupe T_μ engendré par le support de μ tels que*

$$\forall x \in E \quad \lim_{n\to+\infty} \xi_n(x) = x_0.$$

Il existe alors sur E une unique mesure de probabilité P-invariante ν. De plus, il existe des constantes $\kappa > 0$ et $\rho \in]0,1[$ telles que, pour toute fonction $\varphi \in \mathscr{H}_\alpha(E)$, on ait

$$\forall x \in E \quad |P^n\varphi(x) - \nu(\varphi)| \leq \kappa\rho^n. \tag{2.1}$$

3 Application à la chaîne de Diaconis-Freedman sur $[0,1]$

Nous appliquons les résultats précédents pour étudier le comportement asymptotique d'un exemple de chaînes de Markov itératives, Il s'agit de la chaîne de Diaconis- Freedman sur $[0,1]$ décrivant les mouvements d'une particule entre deux points attractifs.

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Estimation of functional regression with missing data at random

BENCHIHA Abbassia*

Laboratory of Statistics and Stochastic Processes, University of Djillali Liabes,BP 89,Sidi Bel-Abbès,22000,Algeria
E-mail: benchiha.abbassia@yahoo.fr

Abstract
In this paper, we consider the problem of the co-variability analysis between a functional variable X and a scalar response variable Y which is not totally observed. We use the local linear approach to model this relationship by constructing a local linear estimator of the regression operator when missing data occur in the response variable. Asymptotic results, in term of the pointwise almost complete consistencies, is established for the constructed estimator.
Motivated by its interaction with other applied fields, the functional data analysis (FDA) became a major topic of research in recent years. A key references on this topic are the monographs of Ramsay and Silverman (2005), Bosq (2000), Ferraty and Vieu (2006), Cuevas (2014), Zhang (2014) and Hsing and Eubank (2015). In this paper, we are interested by studying the local linear estimate of the regression operator when the regressor is of functional kind and the response variable presents some missing observations.

Notice that the local polynomial approach has various advantages over the kernel method, namely in the bias term (see, Fan and Gijbels 1996, for an extensive discussion on the comparison between both these methods). The local linear modeling has been recently introduced in the FDA by Bàillo and Grané (2009). They proved the L^2-consistency of the local linear estimate of the regression function when the explanatory variable takes values in a Hilbert space. Barrientos et al. (2010) have considered an alternative and simplified version which can be used for more general functional covariates. Chouaf and Laksaci (2012) as far as they are concerned have studied a spatial version of the estimate proposed by Barrientos et al. (2010). They established the almost complete consistency of this estimate when the observations are spatially dependent. We return to Demongeot et al. (2014) for the functional local linear estimate of the conditional cumulative distribution function. All these works are only concerned with the complete data case. Our main purpose, in this paper, is to study the case when the response variable has missing data at random (MAR).

1 Introduction

Consider n independent pairs of random variables (X_i, Y_i) for $i = 1, \ldots, n$ that we assume drawn from the pair(X, Y). The latter is valued in $\mathcal{F} \times \mathbb{R}$, where $\mathcal{F}$ is a semi-metric space and d denotes a semi-metric.

Key Words and Phrases: *Almost complete convergence, Local linear method, Missing data, Uniform consistency, Regression operator*

Usually, the local polynomial estimate is constructed by approximating, locally, the nonparametric model by a polynomial function. In functional statistics, when the data are complete, Barrientos et al. (2010) have proposed to approximate the regression operator $R(x) = \mathbb{E}[Y|X = x]$ in a neighborhood of x by

$$R(Z) = a + b\beta(z, x), \text{ where } z \text{ is in a neighborhood of } x.$$

The scalars a and b are estimated by

$$\min_{(a,b)\in\mathbb{R}^2} \sum_{i=1}^{n} (Y_i - a - b\beta(X_i, x))^2 K(h^{-1}\varrho(x, X_i)) \tag{1.1}$$

where K is a kernel and $h = h_{K,n}$ is a sequence of positive real numbers, $\beta(.,.)$ is a known function from $\mathcal{F}^2$ into $\mathbb{R}$ such that, $\forall \xi \in \mathcal{F}$, $\beta(\xi, \xi) = 0$ and $\varrho(.,.)$ is a function of $\mathcal{F}\times\mathcal{F}$ such that $d(.,.) = |\varrho(.,.)|$. In what follows, we consider the case where the response variables are MAR data. In this case, we introduce a Bernoulli random variable δ such that $\delta = 1$ if Y is observed and $\delta = 0$ otherwise. The MAR consideration implies that the two variables Y and δ are independent conditionally to a given X, in the sense that

$$\mathbb{P}(\delta = 1|X, Y) = \mathbb{P}(\delta = 1|X) = P(X)$$

where $P(\cdot)$ is an unknown functional operator which is called the conditional probability of observing the response variable Y given the explanatory variable X.

2 Main results

Theorem 2.1. *Under hypotheses, we have* $|\widehat{R}(x) - R(x)| = O\left(h^{\kappa}\right) + O\left(\sqrt{\frac{\log n}{n\ \ \phi_x(h)}}\right)$, *a.co.*

Lemma 2.1. *Under the hypotheses of Theorem 2.1, we have that:*
$\left|\widehat{R}_D(x) - \mathbb{E}[\widehat{R}_D(x)]\right| = O_{a.co.}\left(\sqrt{\frac{\log n}{n\phi_x(h)}}\right)$ *and* $|\widehat{R}_N(x) - \mathbb{E}[\widehat{R}_N(x)]| = O_{a.co.}\left(\sqrt{\frac{\log n}{n\phi_x(h)}}\right)$.

Corollary 2.1. *Under the hypotheses of Lemma 2.1, there exists a positive real C such that* $\sum_{n=1}^{\infty} \mathbb{P}\left(\widehat{R}_D(x) < C\right) < \infty$.

Lemma 2.2. *Under hypotheses, we have:* $\left|\widehat{B}(x)\right| = O(h^{\kappa})$.

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Weak law of large numbers for dependent random variables with infinite means

Ismahen Bernou*

Mailing address: Laboratoire de Statistique et Modélisations aléatoires, Abou Bekr Belkaid, Tlemcen, Algeria
E-mail: ismahen825@gmail.com

Abstract
We provide necessary and sufficient conditions for the convergence in probability of weighted averages of random variables with infinite means. Our results extend and improve the corresponding theorems obtained in the independent setup by Adler (2012) and Nakata (2016).

1 Introduction

Consider a sequence $\mathcal{X} = \{X_n, n \geq 1\}$ of real-valued random variables (rv's) defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and satisfying

$$\mathbb{P}(|X_j| > x) \asymp x^{-\alpha} \quad \text{for } j \geq 1 \text{ and some } \ 0 < \alpha \leq 1 \tag{1.1}$$

and

$$\limsup_{x\to\infty} \sup_{j\geq 1} x^{\alpha}\mathbb{P}(|X_j| > x) < \infty, \quad \text{for some } \ 0 < \alpha \leq 1 \tag{1.2}$$

2 Main result

Theorem 2.1. *[2] Let $0 < \alpha \leq 1$ and consider two sequences of positive constants $\tilde{a} = \{a_n,\ n \geq 1\}$ and $\tilde{b} = \{b_n,\ n \geq 1\}$ such that $\sum_{j=1}^{n} a_j^{\alpha} = o(b_n^{\alpha})$. If $\mathcal{X} = \{X_n, n \geq 1\}$ is a sequence of rv's satisfying* (1.1), (1.2) *and a Rosenthal-type maximal inequality, then*

$$\frac{1}{b_n} \max_{1\leq k\leq n} \left| \sum_{j=1}^{k} a_j (X_j - c_{nj}) \right| \xrightarrow{\mathbb{P}} 0 \quad as \quad n \to \infty,$$

for a suitable sequence $\{c_{nj},\ 1 \leq j \leq n\}$.

Key Words and Phrases: *Weak laws, weighted exact laws, Rosenthal–type maximal inequalities.*

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Heteroscedastisity testing for functional data

Aicha Henien Djelloul Saiah *

Department of Mathematics, ENS, Mostaganem, Algeria
E-mail: aichahenien@yahoo.com

Jacques Demongeot

University Grenoble Alps, laboratory AGEIS EA 7407, France
E-mail: jacques.demongeot@yahoo.fr

Ali Laksaci

Department of Mathematics, College of Science, King Khalid University, Abha, Saudi Arabia
E-mail: alilak@yahoo.fr

Mustapha Rachdi

University Grenoble Alps, laboratory AGEIS EA 7407, France
E-mail: mustapha.rachdi@univ-grenoble-alpes.fr

Abstract
We present in this work a consistent nonparametric test for heteroscedasticity when the data are of functional kind. To construct our test statistic we based on the empirical version of the moment condition by evaluating the difference between the conditional variance and unconditional variance. By adding some standard assumptions, our test statistic has a asymptotic normal distribution under the homoscedasticity's hypothesis, we also established that this test can detect local alternatives distinct from the null. It worth to noting that, as well as the kernel estimation, some tools have been used here as the small ball probabilities $P(X \in d(x, X))$ where it appears in the asymptotic developments where $X \in F$ and d is a semi-metric, in addition to the degenerate and non-degenerate U-statistic theories. Finally, to testing our results some simulated data examples are presented .

1 Introduction

The prediction of scalar response given explanatory functional random variable is an important subject in the modern statistics. The regression operator is the most preferred model in this prediction problem. However, this model is not efficiency in the heteroscedasticity case, it requires the homoscedasticity of the data which cannot be guaranteed a priory. In this paper we shall construct a test statistic to detect the heteroscedasticity for functional data.

Key Words and Phrases: *Functional data analysis, Nonparametric test, Functional nonparametric statistics, Kernel estimate, U-statistic.*

2 Main results

- Under H_0 : $V[\epsilon|X] = \sigma^2$

Theorem 2.1. *When (H1)-(H4) and (??)-(??) hold we have*

$$n\sqrt{\phi(h)}\,W_n \xrightarrow{\mathcal{D}} \mathcal{N}(0, s^2) \text{ as } n \to \infty$$

where $s^2 = 2\left(K^2(1) - \int_0^1 (K^2(s))'\tau(s)ds\right)[f(X)V^2[\epsilon_2^2|X]]$.
Moreover,

$$T_n = n\sqrt{\phi(h)}\,\frac{W_n}{\widehat{s}} \xrightarrow{\mathcal{D}} \mathcal{N}(0,1) \text{ as } n \to \infty$$

where

$$\widehat{s}^2 = \frac{1}{n(n-1)\phi(h)}\sum_{i=1}^{n}\sum_{j\neq i,=1}^{n} K\left(\frac{d(X_i,X_j)}{h}\right)(\widehat{\epsilon_j}^2 - \widehat{\sigma}^2)^2(\widehat{\epsilon_i}^2 - \widehat{\sigma}^2)^2.$$

- Under H_1 : $V[\epsilon|X] \neq \sigma^2$

Theorem 2.2. *When (H1)-(H4) and (??)-(??) hold we have*

$$\frac{T_n}{n\sqrt{\phi(h)}} \longrightarrow [(V[\epsilon|X] - \sigma^2)^2 f(X)]/s_1, \quad \text{In probability}$$

where $s_1^2 = \frac{\left(K^2(1) - \int_0^1 (K^2(s))'\tau(s)ds\right)}{\left(K(1) - \int_0^1 (K(s))'\tau(s)ds\right)}[(V[\epsilon^2|X] + (V[\epsilon|X] - \sigma^2)^2)f(X)]$.

- Under H_{1n} : $V[\epsilon|x] - \sigma^2 = \delta_n g(x)$

Corollary 2.1. *Given (H1)-(H4) and (??)-(??), we have, under* H_{1n} *with* $\delta_n = n^{-1/2}\phi^{-1/4}(h)$

$$T_n \xrightarrow{\mathcal{D}} \mathcal{N}(\mu, 1) \text{ as } n \to \infty$$

where $\mu = \left(K(1) - \int_0^1 (K(s))'\tau(s)ds\right)[g^2(X)f(X)]/s$.

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Simulation de l'estimateur à noyau de la fonction de régression relative pour des données incomplètes et dépendantes

HAMRANI Farida*

Département Probabilités et statistiques, Université des Sciences et Technologie Houari Boumediene, Alger, Algérie
E-mail: farida.h1989@gmail.com

Abstract
Nous nous interessons dans ce travail de simulation à calculer l'estimateur à noyau de la fonction régression relative pour un modèle tronqué à gauche (RLT), lorsque les données présentent une forme de dépendance dite association, afin de tester la perfermance des résultats théoriques retrouvés pour ce type de données.

1 Introduction

Soit $\{(X_i, Y_i); i = 1, \cdots, N\}$ une suite strictement stationnaire de vecteurs aléatoires définie dans le même espace de probabilté $(\Omega, F, \mathbb{P})$ valeurs dans $\mathbb{R}^d \times \mathbb{R}$, ayant la même loi que (X, Y). Si X et Y ne sont pas indépendants et X est observable, il est raisonnable de prédire Y à partir des informations apportées par X. La relation entre X et Y est exprimée par le modèle de régression suivant

$$Y = m(X) + \epsilon$$

où ϵ est une variable alatoire réelle centrée et indpendante de X et m une fonction de $\mathbb{R}^d$ à valeurs dans $\mathbb{R}$. La régression relative est le cas où la fonction m est solution du problème d'optimisation suivant :

$$\text{for} \quad Y > 0, \quad \min_m \mathbb{E}\left[\left(\frac{Y - m(\mathbf{x})}{Y}\right)^2 |\mathbf{X} = x\right]. \tag{1.1}$$

La solution de ce problème peut facilement être explicitée; en utilisant la dérivée par rapport à m de la fonction donnée en (1.1), on trouve comme solution:

$$m(x) = \frac{\mathbb{E}[Y^{-1}|\mathbf{X} = x]}{\mathbb{E}[Y^{-2}|\mathbf{X} = x]}. \tag{1.2}$$

Dans certains modèles de survie, il arrive que la variable d'intrêt Y ne soit pas complètement observable. Nous nous intéressons ici au modèle de troncature à gauche, dans lequel, nous n'observons Y_i que si $Y_i \geq T_i$, où $\{T_i, i = 1, \cdots, N\}$ désigne une suite de variables de troncature de même loi que T. Il est clair que nous disposons alors d'un

Key Words and Phrases: *Association, modèle tronqué à gauche, Régression relative, Simulation.*

échantillon observé $\{(X_i, Y_i, T_i); i = 1, \cdots, n\}$ de taille $n \leq N$. On se propose d'estimer la fonction de régression relative $m(x)$ donnée en (1.2) sur la base de cet échantillon, en utilisant un estimateur non paramétrique à noyau. En pratique, Le fait de supposer que les données sont toujours indépendantes est peu réaliste, c'est pour cela depuis quelques années plusieurs auteurs ont concentrés leurs études sur les données dépendantes.
Dans notre travail, nous allons présenter premiérement l'estimateur à noyau de la fonction de régression relative pour un modèle tronqué à gauche, ainsi les résultats sur sa convergence dans le cas des données faiblement dépendantes (associées). Puis on effectue des simulations afin de vrifier sa qualit et de confronter les rsultats pratiques ceux attendus par la thorie.

2 Présentation de l'estimateur

En suivant les mêmes étapes que dans Ould Saïd and Lemdani (2006), on définit l'estimateur à noyau $\hat{m}(\mathbf{x})$ de la fonction de régression relative $\hat{m}(\mathbf{x})$ donnée en (1.2) par

$$\hat{m}(\mathbf{x}) =: \frac{\dfrac{\alpha_n}{nh_n^d}\displaystyle\sum_{i=1}^{n}\frac{Y_i^{-1}}{G_n(Y_i)}K_d\left(\frac{\mathbf{x}-\mathbf{X}_i}{h_n}\right)}{\dfrac{\alpha_n}{nh_n^d}\displaystyle\sum_{i=1}^{n}\frac{Y_i^{-2}}{G_n(Y_i)}K_d\left(\frac{\mathbf{x}-\mathbf{X}_i}{h_n}\right)}.$$

avec

- $K_d : \mathbb{R}^d \to \mathbb{R}$ est la fonction à noyau,
- h_n est appelée fenêtre qui est une suite de réels tendant vers 0 quand $n \to \infty$,
- G_n est l'estimateur de Lynden-Bell (1971) de fonction de répartition G de la variable aléatoire de troncature T,
- α_n est l'estimateur de $\alpha := \mathbb{P}(Y \geq T)$ proposé par He and Yang (1998).

On illustre à travers des simulations le comportement de cet estimateur pour des tailles d'échantillon et des taux de troncature différents sous l'hypothèse que la suite $\{(\mathbf{X}_i, Y_i);\ i = 1, \ldots, N\}$ est associée.

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Nonparametric estimation of the cumulative distribution function in the functional data by using local linear method

Oussama Bouanani*

University of Abdlhamid Ibn Badis Mostaganem, Algeria,
E-mail: oussama.bouanani@univ-mosta.dz

Abstract
We consider the estimation of a cumulative distribution function based on the local linear method with a scalar response variable conditioned by a random variable taking values in a semimetric space. The principal aim of this work is to establish the asymptotic normality of our estimator when the observations are independent identically distributed

1 Assumptions and notations

In what follows we denote by x (resp. y) a fixed point in $\mathfrak{F}$ (resp. $\mathcal{R}$), $\mathcal{N}_x$ (resp. $\mathcal{N}_y$) a fixed neighborhood of x (resp. of y) and $\phi_x(r_1, r_2) = \mathbb{P}(r_2 \leq \delta(X, x) \leq r_1)$. Then, in order to establish our results the following assumptions will be needed.

(H1) For any $r > 0$, $\phi_x(r) := \phi_x(-r, r) > 0$, and there exists a function $\chi_x(\cdot)$ such that, for all $t \in [-1, 1]$, $\lim\limits_{h_K \to 0} \dfrac{\phi_x(-h_K, th_K)}{\phi_x(h_K)} = \chi_x(t)$.

(H2) The conditional distribution function F^x satisfies that there exist some positive constants b_1 and b_2, such that: for all $(x_1, x_2, y_1, y_2) \in \mathcal{N}_x \times \mathcal{N}_x \times \mathcal{N}_y \times \mathcal{N}_y$:

$$|F^{x_1}(y_1) - F^{x_2}(y_2)| \leq C \left(|\delta(x_1, x_2)|^{b_1} + |y_1 - y_2|^{b_2} \right),$$

where C is a positive constant depending on x.

(H3) The functions $\delta(.,.)$ and $\beta(.,.)$ are such that:

(i) for all $z \in \mathfrak{F}, |\delta(x, z)| = d(x, z)$ and $C_1 |\delta(x, z)| \leq |\beta(x, z)| \leq C_2 |\delta(x, z)|$ where C_1 and $C_2 > 0$.

(ii) $\sup\limits_{u \in B(x,r)} |\beta(u, x) - \delta(x, u)| = o(r)$, where $B(x, r) = \{z \in \mathfrak{F} / |\delta(z, x)| \leq r\}$ denotes the closed-ball centered at x and of radius r.

(H4) The kernel K is a positive function which is supported within $[-1, 1]$, differentiable and for which the first derivative K' satisfies:

$$K^2(1) - \int_{-1}^{1} (K^2(u))' \chi_x(u) du > 0.$$

Key Words and Phrases: *Functional data, Local linear estimate, cumulative distribution, non parametric estimation, Asymptotic normality*

(H5) The kernel H is a differentiable function and its first derivative H' is symmetric and such that: $\int |t|^{b_2} H'(t)dt < \infty$.

(H6) The bandwidths are such that: there exists a positive integer n_0 for which

$$-\frac{1}{\phi_x(h_K)} \int_{-1}^{1} \phi_x(zh_K, h_K) \frac{d}{dz} \left(z^2 K(z)\right) dz > C_3 > 0 \text{ for } n > n_0.$$

$$\lim_{n\to\infty} h_K = \lim_{n\to\infty} h_H = 0 \text{ and } \lim_{n\to\infty} \frac{\log n}{n\phi_x(h_K)} = 0,$$

and

$$h_K \int_{B(x,h_K)} \beta(u,x) dP(u) = o\left(\int_{B(x,h_K)} \beta^2(u,x)\, dP(u) \right),$$

where $dP(u)$ is the cumulative distribution of X.

2 Main results

Under assumptions (H1)-(H6), we obtain

$$\sqrt{n\phi_x(h_K)}(\widehat{F}^x(y) - F^x(y) - B_n(x,y)) \xrightarrow{D} N(0, V_{HK}(x,y)),$$

where

$$V_{HK}(x,y) = \frac{M_2}{M_1^2} F^x(y)(1 - F^x(y)), \tag{2.1}$$

and

$$B_n(x,y) = \frac{\mathbb{E}(\widehat{F}_N^x(y))}{\mathbb{E}(\widehat{F}_D^x)} - F^x(y), \tag{2.2}$$

with $\xrightarrow{D}$ denoting the convergence in distribution

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Modeling heterogeneity with time-dependent covariates in reliability analysis

Samia TEGHRI[1]

Mailing address: Mathematics Department, University Badji Mokhtar Annaba, Annaba, Algeria
E-mail: samia.teghri@gmail.com

Abstract
Failure data are frequently collected from many distinct units under various operational circumstances, which could lead to data heterogeneity. The observed covariate's values and their effects on the item's reliability are usually known. The impact of unidentified or missing factors is called unobserved covariates, which is the source of heterogeneity. Most of the time, observed covariates are considered constant over time. However, in practice, there are many situations where the covariates are time-dependent and change over time. A lot of studies in reliability analysis have neglected the impact of both unobserved factors and time-dependent covariates. This will result in inadequate models as well as incorrect estimations and decisions. In this paper, frailty models implemented with time-dependent covariates will be applied to model heterogeneity.

1 Introduction

The term heterogeneity refers to variability that may be caused by unknown risk factors. Observed covariates are factors whose impacts are known, and are collected with the failure data. They might be time-dependent or time-independent. Unobserved covariates have unknown effects, so, the failure database does not contain information on associated operating hours or time of failure. The research showed that failure data analysis frequently eliminates unobserved covariates. However, if unobserved covariates were neglected, the estimation of the impact of covariates would differ significantly. The most popular statistical method for evaluating the impact of covariates on the reliability performance of an item is the Cox regression model, which includes the proportional hazard model (PH) and its extension. However, the Cox model assumes that populations are homogenous. The frailty model is an extension of the Cox model. The unobserved covariates are usually described by frailty models, and the heterogeneity is referred to as frailty. An item's failure rate is calculated by multiplying the hazard rate by two positive functions: an observed covariate function and an unobserved (frailty) distribution. Generally, the gamma distribution, with a mean equal to one and finite variance, is the most commonly used function for frailty distributions. This paper's main objective is to capture unknown factors with time-dependent covariates in failure data analysis using frailty models.

Key Words and Phrases: *Frailty models, Heterogeneity, Reliability, Time-dependent covariates.*

2 Main results

2.1 Univariate frailty model

Let $Z > 0$ represent an unobserved random variable that indicates the item's frailty. Note the frailty $Z = z_i$ and the time-independent covariates x_i, then the hazard function for the i^{th} item is :

$$\lambda(t_i/z_i, x_i) = z_i\lambda_0(t_i)exp(\sum_{i=1}^{n} x_i\beta), \quad i = 1, 2, \ldots, n, \tag{2.1}$$

Where β is a vector for regression coefficients associated with the vector of time-independent covariates x_i, $\lambda_0(t_i)$ is a baseline hazard function, which is assumed to be common for all items. Furthermore, the frailty z_i acts multiplicatively on the baseline hazard function. Thus, items with z_i >1 are more frail and may fail faster. In contrast, items that have $z_i < 1$ are less frail and more reliable. In eq(2.1), all covariates are assumed to be time-independent. However, in many situations, the covariates are time-dependent. For instance, a covariate used to describe crack evolution may change over the period of an item's operational life. Hence, it needs to be modeled as a time-dependent covariate. Taking into consideration the presence of p_1 time-independent covariates and p_2 time-dependent covariates, an item's hazard function is defined as :

$$\lambda(t_i/z_i, x_i, x_j(t)) = z_i\lambda_0(t_i)exp(\sum_{i=1}^{p_1} x_i\beta + \sum_{j=1}^{p_2} x_j(t)\delta) \tag{2.2}$$

where $x_j(t)$ is a vector of time-dependent covariates and δ is the associated regression coefficient.
The reliability function of the i^{th} item can be obtained as follows:

$$R(t_i/z_i, x_i, x_j(t)) = exp\left(-z_i\Lambda_0(t)exp(\sum_{i=1}^{p_1} x_i\beta)\int_o^t exp(\sum_{j=1}^{p_2} x_j(u)\delta)du\right)$$

where $\Lambda_0(t) = \int_o^t \lambda_0(u)du$.

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Prediction of nonparametric regression under long-range dependance data.

HAMEL Elhadj*

Laboratory of mathematics and Applications
Hassiba Benbouali University - Chlef -Algeria
E-mail: ha2.hamel@gmail.com

Abstract
Let $\{Z_i, i \in N\}$ be a strictly stationary real valued time series. We predict Z_{d+p} with $\{p \geq 1\}$ from $\{Z_1, ..., Z_d\}$ by a robust nonparametric method under long range dependance (long memory)data. The predictor is defined by regression kernel estimates from the conditional law of $Y = \varphi(Z_{d+p})$ on $X = (Z_1, ..., Z_d)$ in $R \times R^d$. This result allows us to study the convergence and the asymptotic normality of kernel regression estimators under suitable conditions of long memory data.

1 Introduction

Let $X_t(t \in \mathbb{Z})$ denote a weakly stationary stochastic process with autocovariance function $cov(X_t, X_{t+k}) = \gamma_X(k) = E(X_t X_{t+k})$. Then X_t exhibits (linear) Long memory or long-range dependence, if $\sum_{k=-\infty}^{+\infty} \gamma_X(k) = +\infty$, and Short memory if $\gamma_X(k) < 0$.
The importance of long memory can be seen in various applications, for instance in finance, internet modeling, hydrology, linguistics...., and other areas (see beran, 1993;Park et al., 2011; Leonenko and Olenko, 2014; Lwang, 2015; Samorodnitsky, 2007, 2016, and the references therein).
One of the most important problems in time series analysis is prediction of future observations. Namely, given the observed series $Z_1, Z_2, ..., Z_n$, the aim is to predict the unobserved value Z_{n+p}, for some integer $p \geq 1$.The classical approach to this problem is to find some nonparametric estimate of the regression function (1.1).

$$m(x) = \mathbb{E}(Y/X = x). \tag{1.1}$$

2 Main results

In the basis of $(X_i, Y_i) = (Z_i, ..., Z_{i+d+1})$,$i = 1, ..., n$. Let us consider a univariate time series $Z_1, Z_2, ...Z_n$ be a strictly stationary sequence of long memory dependance random variables, and let φ be a real-valued function defined on $\mathbb{R}$. The prediction problem in a discrete parameter time series given by :

Key Words and Phrases: *regression function, Kernel estimate, Asymptotic normality ,prediction, long range dependance*

$$X_i = (Z_i, \ldots, Z_{i+d}), Y_i = \varphi(Z_{i+d+1}), i \geq 1, d \geq 1 \tag{2.1}$$

The regression function is defined by

$$m(x) = \frac{f_n(x,y)}{f_n(x)} = \mathbb{E}[\varphi(Y_1) \mid X_1 = x] = \mathbb{E}[\varphi(Z_{d+1}) \mid Z_1, \ldots, Z_d], x \in \mathbb{R}^d \tag{2.2}$$

Theorem 2.1. *Under asymptions A1 - A7 and for $x \in \mathbb{R}^d$, we have*

$$\sqrt{nh^d}\,[m_n(x) - m(x)] \xrightarrow[n\longrightarrow\infty]{d} N\left(0, \sigma^2(x)\right), x \in \mathbb{R}. \tag{2.3}$$

where $\sigma^2(x) = \sigma_\varphi^2(x) \int_{\mathbb{R}^d} \frac{K^2(t)dt}{f(x)}, (f(x) > 0)$, and $\sigma_\varphi^2(x) = \mathbb{E}\left\{[\varphi(Y_1) - m(x)]^2 \mid X_1 = x\right\}$ $\xrightarrow{d}$ means the convergence in distribution.

The proof of the Theorem 2.1 is based on the following results:

Lemma 2.1. *1.*

$$\frac{1}{\sqrt{nh^d}} \sum_{j=1}^{n} [m(X_j) - m(x)]\, K\left(\frac{x - X_j}{h}\right) \xrightarrow[n\longrightarrow\infty]{P} 0 \tag{2.4}$$

2.

$$\begin{aligned} &\frac{1}{\sqrt{nh^d}} \sum_{j=1}^{n} [\varphi(Y_j) - m(X_j)]\, K\left(\frac{x - X_j}{h}\right) \xrightarrow{d}_{n\longrightarrow\infty} \mathcal{N}\left(0, \tau^2(x)\right), \\ &\tau^2(x) = f(x)\sigma^2(x) = \sigma_\varphi^2(x) \int_{\mathbb{R}^d} K^2(t)dt \end{aligned} \tag{2.5}$$

3. $f_n(x) \longrightarrow^p f(x)$, *as* $n \longrightarrow \infty$.

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PART H

Dynamical systems and chaos

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Study in Time scale the Neural Networks with Time-Varying and Distributed Delays

ibtissem Daira*

Department of Mathematic and Informatics,Souk Ahras University,Souk Ahras,Algeria
E-mail: ibtissem.daira@gmail.com

Abstract
In this work, we study in time scale the positivity and periodicity solutions of neural networks with Time-Varying and Distributed Delays by the aid of the fixed point theory on cones

1 Introduction

In this paper, mainly motivated by the content , we are interested in the following delay dynamical system in terms of analysis of the qualitative properties of periodic solutions :

$$\begin{aligned} u_i^{\Delta}(t) &= r_i(t)\, u_i(t) + \sum_{j=1}^{n} a_{ij}(t)\, h_j(u_j(t)) + \sum_{j=1}^{n} b_{ij}(t)\, f_j(u_j(t-\delta_{ij})) \\ &\quad + \sum_{j=1}^{n} c_{ij}(t) \int_{-\infty}^{t} D_{ij}(t,s)\, g_j(u_j(s))\, \Delta s, \text{ for } i = 1,2,\dots,n \text{ ,} t \in \mathbb{T} \end{aligned}$$

where $u(t) = [u_1(t), u_2(t), .., u_n(t)]^T \in \mathbb{R}^n$, when $r_i \in \mathcal{R}^+(\mathbb{T}, \mathbb{R}^+)$ and $a_{ij}, b_{ij}, c_{ij}, \delta_{ij} \in C_{rd}(\mathbb{T}, \mathbb{R}^+)$ are all ω-periodic functions with respect to t,and for $i, j = 1, 2, \dots, n$

2 Main results

Throughout this paper, we will assume the functions h_j, f_j, g_j are negative and uniformly continuous in u.
We will first make some preparations and list a few preliminary results. In order to use Theorem 2.1 to prove the existence of periodic solution of our system , we shall consider $(C_\omega, \|.\|) = (X, \|.\|)$.
Throughout the next steps, For $(t,s) \in \mathbb{R}^2, 1 \le i \le n$, we define

$$G_i(t,s) = \frac{e_{\ominus r_i}(\sigma(s), t)}{e_{\ominus r_i}(\omega, 0) - 1} \tag{2.1}$$

and assume
For all $(t,s) \in \mathbb{R}$

$$G_i(t,s)\, h_j(u) \ge 0,\ G_i(t,s) f_j(u) \ge 0,\ G_i(t,s)\, g_j(u) \ge 0. \tag{2.2}$$

Key Words and Phrases: *Fixed point theorem · Positive periodic solutions · Variable delays. time scale*

$$A_i := \frac{e_{\ominus r_i}(\omega, 0)}{1 - e_{\ominus r_i}(\omega, 0)} \leq |G_i(t,s)| \leq \frac{1}{1 - e_{\ominus r_i}(\omega, 0)} =: B_i, i = 1, 2, \ldots, n. \tag{2.3}$$

let

$$A = \min_{1 \leq i \leq n} \{A_i\}, \ B = \min_{1 \leq i \leq n} \{B_i\} \tag{2.4}$$

and

$$\sigma = \min_{1 \leq i \leq n} \{e_{\ominus r_i}(\omega, 0), \ i = 1, 2, \ldots, n\},$$

Define K as a cone in C_ω by

$$\begin{aligned} k &= \Big\{u(.) = (u_1(.), u_2(.), \ldots, u_n(.))^T \in C_\omega : \\ &\quad u_i(t) \geq 0, u_i(t) \geq \sigma |u_i|_0, t \in [0, \omega]\}, \end{aligned} \tag{2.5}$$

and we easily verify that K is a cone in C_ω.
We define an operator $T : C_\omega \to C_\omega$ as follows:

$$(Tu)(t) := [(T_1u_1)(t), (T_2u_2)(t), \ldots, (T_nu_n)(t)]^T, \tag{2.6}$$

where

$$\begin{aligned} (T_iu_i)(t) = \int_t^{t+\omega} G_i(t,s) \Bigg[\sum_{j=1}^n a_{ij}(s) h_j(u_j(s)) + \sum_{i=1}^n b_{ij}(s) f_j(u_j(s - \delta_{ij}(s))) \\ + \sum_{j=1}^n C_{ij}(s) \int_{-\infty}^s D_{ij}(s,z) g_j(u_j(z)) \Delta z\Bigg] \Delta s i = 1, 2, \ldots, n \end{aligned} \tag{2.7}$$

To make use of the fixed point theorem in cones, firstly, we need to introduce some lemmas and assumptions. Let us start by obtaining an equivalent formulation for our problem

Lemma 2.1. *The function $u(.)$ is an ω-periodic solution if and only if $u(.)$ is an ω-periodic solution of the following equation*

$$(Tu)(t) := u(t). \tag{2.8}$$

Lemma 2.2. *The $T : K \to K$ is well defined and the $T : K \to K$ is completely continuous*

$$\overline{a}_{ij} = \frac{1}{\omega}\int_0^\omega a_{ij}(s) ds \geq 0, \ \overline{b}_{ij} = \frac{1}{\omega}\int_0^\omega a_{ij}(s) ds \geq 0, \ \overline{c}_{ij} = \frac{1}{\omega}\int_0^\omega a_{ij}(s) ds \geq 0,$$

for $i, j = 1, ..., n$,

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Stability of fractional difference neural networks with Nonsingular and Nonlocal Kernels

Amel Hioual*, **Adel Ouannas**, **Taki Eddine Oussaeif**

Mailing address: Department of Mathematics and Computer Science, University of Larbi Ben M'hidi, Oum El Bouaghi , Algeria

E-mail: amel.hioual@univ-oeb.dz, ouannas.adel@univ-oeb.dz, taki_maths@live.fr

Abstract

Stability of fractional difference neural networks with discrete Mittag-Leffler kernels is investigated in this study. Using a novel modified Gronwall inequality and the method of steps, finite-time stability conditions of fractional difference neural networks with discrete Mittag-Leffler kernel are induced. Finally, an example is provided to demonstrate the validity of the key findings.

1 Introduction

Fractional calculus is a fast - growing mathematical field. It is gaining popularity among scientists because to its numerous applications in domains such as science, biology, and engineering. Along with the advancement of theories on fractional differential equations, fractional difference equations have been examined more thoroughly. Abdeljawad and Baleanu [1] have suggested a novel description of the nabla discrete left Caputo(ABC) type fractional difference using a discrete Mittag-Leffler kernel.

Fractional-order discrete-time neural networks represent a class of discrete systems described by fractional-order difference operators. Even though the stability of fractional-order discrete-time neural networks is a prerequisite for their successful applications [2], very few papers have been published on this topic. For example, [3] investigated Mittag–Leffler stability of a network model based on the nabla Caputo h-discrete operator also a class of variable fractional-order discrete-time neural network was introduced and its asymptotic stability discussed. [4] presented a variable-order fractional discrete neural network model and its Ulam–Hyers stability was studied.

2 Main results

The main results of this study are as follows: First, we propose the following fractional-order discrete-time neural network

$$ {}_{a}^{ABC}\nabla^{\alpha}x(t) = -Ax(t) + Bg(t, x(t)) + I. \tag{2.1} $$

Key Words and Phrases: *Caputo AB nabla discret difference operator, Discrete-time fractional order neural networks, Modified Gronwall inequality, Finite time stability*

Where ${}_a^{ABC}\nabla^\alpha$ is the Caputo AB nabla discret difference operator with order α, $0<\alpha<1$, $x(t)=(x_1(t),x_2(t),...,x_n(t))^T\in\mathbb{R}^n$ is the state vector, $A=diag(a_1,a_2,...,a_n)\in\mathbb{R}^{n*n}$ is the self-feedback connection weight with $a_i>0$, $B=(b_{ij})_{n*n}\in\mathbb{R}^{n*n}$ is the connection weight matrix, $g(x(t))=(g_1(t,x(t)),g_2(t,x(t)),...,g_n(t,x(t)))^T : C(\mathbb{N}_{a+1}\to\mathbb{R}^n)$ is the acctivation function, $I=(I_1,...,I_n)^T$ the vector of external inputs.
Then, we present Theorem 2.1 and its proof to ensure the finite time stability of the neural network (2.1)

Theorem 2.1. *Suppose that $g(t,x(t))$ correspends to a continous function and satisfies the lipschtitz conditon with respect to x. Furthermore, Assume that $\|\Phi\|<\delta$ and δ,ϵ are positive numbers. Then, system (2.1) is finite-time stable w.r.t δ,ϵ,T, $\delta<\epsilon$, if the following conditions are satisfied*

$$\frac{B(\alpha)w(t)}{B(\alpha)-(1-\alpha)w(t)}E_{\overline{\alpha}}\left(\frac{\alpha\beta}{B(\alpha)-\beta(1-\alpha)},t-a\right)<\epsilon,\quad 0<\frac{\beta\alpha}{B(\alpha)-(1-\alpha)\beta}<1, \tag{2.2}$$

where $$w(t)=\|\Phi\|+M\left(\frac{1-\alpha}{B(\alpha)}+\frac{\alpha}{B(\alpha)\Gamma(\alpha+1)}(t-a)^{\overline{\alpha}}\right),\quad M=\|I\|,$$
$$\beta=\max_{i=1,...,n}\left\{a_i+\sum_{i=1}^n|b_{ij}|l_j\right\}.$$

Finally, to illustrate the validity of the major conclusions we provide an example for different initial conditions x_0 and with the following parameters:

$$A=\begin{pmatrix}0.25 & 0\\ 0 & 0.2\end{pmatrix},\quad B=\begin{pmatrix}-0.1 & 0.05\\ 0.15 & -0.1\end{pmatrix},\quad I=\begin{pmatrix}0\\0\end{pmatrix} \tag{2.3}$$

Also, using the following numerical solution we provide numerical simulations

$$\begin{cases} x(i)=x(0)+\dfrac{1-\alpha}{B(\alpha)}[-Ax(i)+B\tanh(x(i))+I]\\ \qquad +\dfrac{\alpha}{B(\alpha)\Gamma(\alpha)}\sum_{j=1}^{i}\dfrac{\Gamma(i-j+v)}{\Gamma(i-j+1)}(-Ax(j)+B\tanh(x(j))+I)\end{cases} \tag{2.4}$$

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Moreau's Sweeping Process with Upper-Semi Continuous Right-Hand

Loubna Boulkemh*

Department of Mathematics, Jijel University, Jijel, Algeria
E-mail: l.boulkemh@gmail.com

Doria Affane

Department of Mathematics, Jijel University, Jijel, Algeria
E-mail: affanedoria@yahoo.fr

Abstract
In this work, in the setting in a finite dimensional space, we present an existence of solution for the first order sweeping process:

$$(\mathcal{SP}) \quad \begin{cases} \dot{u}(t) \in -N_{C\left(t,u(t)\right)}\big(u(t)\big) + G\big(t,u(t)\big), & \text{a.e. } t \in [T_0,T]; \\ u(t) \in C\big(t,u(t)\big), & \text{for all } t \in [T_0,T]; u(T_0) = u_0 \in C(T_0,u_0), \end{cases}$$

where $N_{C\left(t,u(t)\right)}(u(t))$ denoted the normal cone to the equi-uniformly-subsmooth moving set $C\big(t,u(t)\big)$ at $u(t)$ and the perturbation G is a set-valued mapping with nonempty closed convex values, upper semi-continuous unnecessarily bounded.

1 Introduction

The perturbed state-dependent sweeping process is an evolution differential inclusion governed by the normal cone to a mobile set depending on both time and state variables of the form $(\mathcal{SP})$, where G is a set-valued or single-valued mapping playing the role of a perturbation to the problem, that is an external force applied on the system. This type of problems was initiated by J. J. Moreau in the 1970's (see [4]) and extensively studied by himself when the sets $C(t)$ are assumed to be convex and $G \equiv \{0\}$. Generalization of the sweeping process have been the object of many studies, see for example [1, 2, 3] and the references therein.

Key Words and Phrases: *Differential inclusion, Sweeping process, Subsmooth sets, Normal cone, Perturbation.*

2 Main results

Let $C : [T_0, T] \times \mathbb{R}^n \rightharpoonup \mathbb{R}^n$ be a set-valued mapping with nonempty closed values such that:

($\mathcal{H}_1$) for all $(t, x) \in [T_0, T] \times \mathbb{R}^n$, the sets $C(t, x)$ are equi-uniformly-subsmooth;

($\mathcal{H}_2$) there are two constants $L_1 \geq 0$, $L_2 \in [0, 1[$ such that, for all $s, t \in [T_0, T]$, and any $x_1, x_2, y \in \mathbb{R}^n$

$$\left|d_{C(t,x_1)}(y) - d_{C(s,x_2)}(y)\right| \leq L_1|t - s| + L_2\left\|x_1 - x_2\right\|.$$

And let $G : [T_0, T] \times \mathbb{R}^n \rightharpoonup \mathbb{R}^n$ be a set-valued mapping with nonempty closed convex values, upper semi-continuous such that:

($\mathcal{H}_3$) for some real $\alpha \geq 0$,

$$d\big(0, G(t, x)\big) \leq \alpha\big(1 + \|x\|\big),$$

for all $(t, x) \in [T_0, T] \times \mathbb{R}^n$.

Then, for every $u_0 \in C\big(T_0, u_0\big)$, the problem ($\mathcal{SP}$) admits an absolutely continuous solution $u : [T_0, T] \to \mathbb{R}^n$.

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Perturbation analysis in a free boundary problem arising in tumor growth model

Ahlem ABDELOUAHAB*
Department of Mathematics, University of Tlemcen, B.P. 119, Tlemcen 13000, Algeria
E-mail: *ahlemabdelouahab98@gmail.com*

Sabri BENSID
Department of Mathematics, University of Tlemcen, B.P. 119, Tlemcen 13000, Algeria
E-mail: *edp_sabri@yahoo.fr*

Abstract
We study the existence and multiplicity of solutions of the following free boundary problem

$$(P)\begin{cases} \Delta u &= \lambda(\varepsilon + (1-\varepsilon)H(u-\mu)) \qquad \text{in } \Omega(t) \\ u &= \overline{u}_\infty \qquad \text{on } \partial\Omega(t) \end{cases}$$

where $\Omega(t) \subset \mathbb{R}^3$ a regular domain at $t > 0$, ε, $\overline{u}_\infty$, λ, μ are a positive parameters and H is the Heaviside step function.
The problem (P) has two free boundaries: the outer boundary of $\Omega(t)$ and the inner boundary whose evolution is implicit generated by the discontinuous nonlinearity H. The problem (P) arise in tumor growth models as well as in other contexts such as climatology. First, we show the existence and multiplicity of radial solutions of problem (P) where $\Omega(t)$ is a spherical domain. Moreover, the bifurcation diagrams are giving. Secondly, using the perturbation technic combining to local methods, we prove the existence of solutions and characterize the free boundaries of problem (P) near the corresponding radial solutions.

1 Introduction

We are concerned to study the radial symmetric growth of problem (P). More precisely, taking the radial part of Laplace operator in three dimension and from the principle of conservation of volume, we obtain

$$\begin{cases} \frac{1}{r^2}\frac{\partial}{\partial r}\big(r^2\frac{\partial u}{\partial r}\big) &= \lambda\big(\varepsilon + (1-\varepsilon)H(u-\mu)\big) \qquad 0 < r < R(t),\ \ t > 0 \\ u(R(t),t) &= u_\infty,\ \frac{\partial u}{\partial r}(0,t) = 0 \qquad \text{for } \ t > 0 \\ R^2(t)\frac{dR(t)}{dt} &= \int_0^{R(t)} S(u)r^2dr - \int_0^{R(t)} N(u)r^2dr \\ R(0) &= R_0. \end{cases} \tag{1.1}$$

where $\mu > 0$, $\varepsilon > 0$, $S(u)$ and $N(u)$ describe the proliferation and the mortality rates of tumor cells. They are given by

$$S(u) = \lambda f(u), \qquad N(u) = \eta > 0.$$

In this work, we will give the existence, multiplicity and diagram of bifurcation of solutions of problem (1.1) with some properties of their free boundaries by using a perturbation method

2 Main results

The first theorem concern the existence and multiplicity of stationary solutions of problem (1.1)

Key Words and Phrases: *Discontinuous nonlinearity, free boundary, perturbation, tumor growth.*

Theorem 2.1. *For $\varepsilon > 0$, there exists two crucial values of the parameter λ such that :*

1. *If $\lambda < \lambda_1$, then there exists a unique solution $u_{\lambda,\mu}$ without free boundary.*
2. *For $\varepsilon \in (\frac{3}{2}, +\infty)$:*
   - *(i) If $\lambda = \lambda_2$, then there exists a unique solution $u^*_{\lambda_2,\mu}$ giving rise to a free boundary given by $r_{\lambda_2} = \frac{2\varepsilon-3}{3(\varepsilon-1)}R$*
   - *(ii) If $\lambda \in (\lambda_2, \lambda_1]$ we have two distinct positive solutions $\overline{u}_{\lambda,\mu}$ and $\underline{u}_{\lambda,\mu}$ with theirs corresponding free boundaries $\overline{r}_\lambda$ and $\underline{r}_\lambda$*
   - *(iii) If $\lambda \in (\lambda_1, +\infty)$, then there exists a unique positive solution $u_{\lambda,\mu}$*
3. *For $\varepsilon \in (0, 9/8)$, if $\lambda \geq \lambda_1$ there exists one solution $u_{\lambda,\mu}$ and one free boundary*
4. *For $\varepsilon \in (9/8, 3/2)$, we have*
   - *(i) If $\lambda \in (\lambda_1, \lambda_2]$, there exists a unique solution and one free boundary $r_{\lambda,1}$.*
   - *(ii) If $\lambda \in (\lambda_2, +\infty)$, then there exists a unique solution and one free boundary $r_{\lambda,2}$.*

Theorem 2.2. *For any $R_0 > 0$ and for $0 < \varepsilon < 1$, the problem (1.1) has a unique global solution $(u(r,t), R(t))$ for $t > 0$.*

Next, the result concerning the asymptotic behavior of transient solution $(u(r,t), R(t))$, we have

Theorem 2.3. *For any initial value $R_0 > 0$ and for $0 < \varepsilon < 1$, we have*

1. *If $\eta > \lambda$, then $\lim\limits_{t\to+\infty} R(t) = 0$*
2. *If $\eta \leq \lambda$, then $\lim\limits_{t\to+\infty} R(t) = R_s, \quad \lim\limits_{t\to+\infty} u(r,t) = u_{\lambda,\mu}$*

where $(u_{\lambda,\mu}, R_s)$, $(u(r,t), R(t))$ are the stationary and the global solution of the problem (1.1).

Theorem 2.4. *The problem*

$$\begin{cases} \Delta u = \lambda(\varepsilon + (1-\varepsilon)\chi_{\Omega_\beta \setminus \Omega_{\beta,\psi}}) & \text{in } \Omega_\beta = B(0, R+\beta(\theta)) \\ u = \overline{u}_\infty & \text{on } \partial\Omega_\beta \end{cases} \tag{2.1}$$

has a unique solution $u_{\lambda,\beta} \in C^{1,\alpha}(\overline{\Omega_\beta}, \mathbb{R})$ with $\Omega_{\beta,\psi} = \{(r,\theta) \in \Omega_\beta, \quad r < \psi(\theta)\}$ and $\alpha = 1 - \frac{3}{p}$. Moreover, if $u_{\lambda,\beta}(\psi(\theta), \theta) = \mu$ and $\overline{u}_\infty > \mu$ then $u_{\lambda,\beta}$ is solution of (P).

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Limit Cycle of The Discontinous Piecewise Differential System Formed by Nilpotent Saddles and Linear Center Separated by a Straight line

Imane Benabdallah*

Mailing address: Department of Mathematics, University Mohamed El Bachir El Ibrahimi of Bordj Bou Arreridj 34000, El Anasser, Algeria
E-mail: imane.benabdallah@univ-bba.dz

Rebiha Benterki

Mailing address:Department of Mathematics, University Mohamed El Bachir El Ibrahimi of Bordj Bou Arreridj 34000, El Anasser, Algeria
E-mail: r.benterki@univ-bba.dz

Abstract
The solution of the second part of the sixteenth Hilbert's problem for discontinuous piecewise differential systems have deserved the attention of many researchers. It is a question of finding the upper bound and the possible configuration of the limit cycles for a planar polynomial differential system of degree n. Here we are interested in solving the second part of the sixteenth Hilbert's problem for the discontinuous piecewise differential systems separated by straight line and formed by linear center and an arbitrary differential cubic Hamiltonian system with nilpotent saddles.

1 Introduction

Any *planar* polynomial differential system takes the form $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$, where $P(x, y)$ and $Q(x, y)$ are polynomial functions, the *degree* of this system is the maximum degree of these polynomials.
The study of the existence and determination of the upper bound of the maximum number of *limit cycles* of planar polynomial differential systems is an attractive research topic that, until know, is still an unsolved problem in the qualitative theory of differential systems. As one of the 23 problems presented at the international congress of mathematicians in Paris in 1900, this problem is known as the second part of the sixteenth Hilbert problem. For more information, read, for example [4, 5].
This paper deals with piecewise discontinuous differential systems of the form

$$(\dot{x}, \dot{y}) = F(x, y) = \begin{cases} F^-(x, y) = \Big(F_1^-(x, y), F_2^-(x, y)\Big)^T & \mathbf{y} \in \Sigma^-, \\ F^+(x, y) = \Big(F_1^+(x, y), F_2^+(x, y)\Big)^T & \mathbf{y} \in \Sigma^+, \end{cases} \tag{1.1}$$

Key Words and Phrases: *Discontinous piecewise differential system, limit cycles, Straight line.*

such that the separation line of the plane is $\Sigma = \{(x,y) : x = 0\}$ and

$$\Sigma^- = \{(x,y) : x \leq 0\}, \quad \Sigma^+ = \{(x,y) : x \geq 0\}.$$

In this paper we shall work with discontinuous piecewise differential systems in $\mathbb{R}^2$, and the definition of these differential systems on the separation line of their two pieces in $\mathbb{R}^2$ follow the rules of Filippov.
The second part of the famous sixteenth Hilbert problem consists in finding an upper bound for the maximum number of limit cycles that the polynomial differential systems in the plane of a given degree can have, see [5]. In the last years many authors have been involved in solving the extension of this problem to some classes of discontinuous piecewise differential systems.
In the literature we find many papers interested in studying piecewise differential linear systems separated by either a straight line or an algebraic curve, such as a conic or a reducible or irreducible cubic curve, see for instance [1].
The main goal of this paper is to solve the extension of the second part of the sixteenth Hilbert problem to the class of discontinuous piecewise differential systems formed by linear center and an arbitrary nilpotent saddles separated by the straight line $x = 0$.

2 Main results

Theorem 2.1. *The maximum number of crossing limit cycles of the discontinuous piecewise differential systems separated by the straight line $x = 0$, and formed by linear center and a Hamiltonian nilpotent saddles $(\mathcal{C}_i)$ for $i \in \{1,2,3,4,5,6\}$ after an arbitrary affine change of variables is at most one. For all these classes, there are systems exhibiting exactly one limit cycles.*

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Sur la stabilité des systèmes d'espaces d'états à deux dimension

SALMI Souad*
BOUAGADA Djillali
Département de Mathématiques et Informatique, Equipe ACSY-Laboratoire de Mathématiques Pure et Application, Université Abdelhamid Ibn Badis Mostaganem, BP 227/118, Université de Mostaganem, 27000 Mostaganem, Algérie
E-mail: souad.salmi.etu@univ-mosta.dz*
djillali.bouagada@univ-mosta.dz

Abstract
Dans ce travail nous nous intéressons a une nouvelle classe des systèmes bidimensionnels d'ordre fractionnaire. Le rayon de stabilité du système perturbé est décrit selon la norme $\mathcal{H}_\infty$. Des conditions suffisantes pour assurer la marge de stabilité du système en boucle fermée sont offertes en termes d'inégalités matricielles linéaires.

1 Introduction

Une attention particulière a été focalisée récemment sur les équation différentiels d'ordre fractionnaire pour modéliser de nombreux procèdes pratiques comme leurs homologues les systèmes a deux dimensions linéaires d'ordre entier. Notons que ces derniers se rencontrent dans l'étude des systèmes inter-connectés, les réseaux électriques, la robotique, plus généralement les structures mécaniques. Les systèmes bidimensionnels (2D) ont attiré beaucoup d'intérêt au cours des dernières décennies en raison de leur présence dans de nombreux problèmes pratiques. Ces systèmes (2D) sont caractérisés par la propagation de deux variables indépendantes dans deux directions différentes, telles que le temps et la distance ou la hauteur et la largeur. Considérant les exigences de robustesse et de performance des systèmes en boucle fermée. Des recherches récentes ont montré que la faible perturbation des coefficients du contrôleur rendrait le système en boucle fermée instable ou fragile par rapport à des incertitudes qui ne peuvent être ignorées.
La marge de stabilité est utilisée comme mesure de l'ampleur des perturbations du modèle ou des incertitudes qui peuvent être tolérées avant qu'un système ne perde sa stabilité. Dans ce cadre, nous établissons des conditions sur la localisation des valeurs propres tout en se basant sur l'analyse de la stabilisée du système perturbé.

2 Main results

Les systèmes stables sont vulnérables aux perturbation, donc l'objectif principale de ce travail est de trouver des conditions suffisantes pour garantir la préservation de la stabilité de ces type du systèmes.

Key Words and Phrases: *Systèmes fractionnaires, systèmes bidimensionnels, stabilité, inégalités matricielles linéaires.*

SYSTÈME EN BOUCLE OUVERT

Considérons le système linéaire fractionnaire décrit par la forme suivante :

$$\begin{cases} \begin{bmatrix} \lambda_1^\alpha I_n & 0 \\ 0 & \lambda_2^\alpha I_n \end{bmatrix} x(t_1,t_2) = \begin{bmatrix} A_1 & I_n \\ A_0 & 0 \end{bmatrix} x(t_1,t_2) + \begin{bmatrix} 0 \\ B \end{bmatrix} u(t_1,t_2) \\ \\ y(t_1,t_2) = \begin{bmatrix} C & 0 \end{bmatrix} x(t_1,t_2) + Du(t_1,t_2), \end{cases} \tag{2.1}$$

où,

- ⋆ $0 < \alpha < 1, A_0, A_1 \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ et $D \in \mathbb{R}^{p\times m}$ sont des matrices données.
- ⋆ $x(t_1,t_2) = \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix}$, avec $x^h(t_1,t_2)$ et $x^v(t_1,t_2)$ sont respectivement les vecteurs d'états horizontal et vertical dans $\mathbb{R}^n$ pour tout $t_1, t_2 \geq 0$.
- ⋆ $u(t_1,t_2) \in \mathbb{R}^m$ et $y(t_1,t_2) \in \mathbb{R}^p$ sont les vecteurs d'entrées et de sorties.
- ⋆ λ_1^α et λ_2^α sont les opérateurs différentiels de Laplace s_1^α, s_2^α si (2.1) est un système à temps continu et les opérateurs à retard z_1^α, z_2^α si (2.1) est à temps discret.

SYSTÈME EN BOUCLE FERMÉ

Concéderons le système (2.1) et on pause $u = \Delta y$, on obtient

$$\begin{bmatrix} \lambda_1^\alpha I_n & 0 \\ 0 & \lambda_2^\alpha I_n \end{bmatrix} x(t_1,t_2) = \begin{bmatrix} A_1 & I_n \\ A(\Delta) & 0 \end{bmatrix} x(t_1,t_2), \tag{2.2}$$

Où

$$A(\Delta) = A_0 + B(I_m - \Delta D)^{-1}\Delta C, \tag{2.3}$$

qui découle de la relation $(I_m-\Delta D)^{-1}\Delta = \Delta(I_p-D\Delta)^{-1}$, ceci est vérifié par $\Delta(I_p-D\Delta) = (I_m - \Delta D)\Delta$.

Theorem 2.1. *Supposons que le système en boucle ouverte* (2.1) *est strictement stable. Alors le système en boucle fermée* (2.2) *est strictement stable si et seulement si* $\Delta \in \mathbb{R}^{m\times p}$ *vérifier*

$$\|\Delta\|_2 < \mu_\star^{-1}, \tag{2.4}$$

où,

$$\mu_\star := \|G(.,.)\|_\infty := \sup_{(\lambda_1^\alpha,\lambda_2^\alpha)\in\partial\Gamma_1\times\partial\Gamma_2} \|G(\lambda_1^\alpha,\lambda_2^\alpha)\|_2\,, \tag{2.5}$$

$$G(\lambda_1^\alpha,\lambda_2^\alpha) = \begin{bmatrix} C & 0 & -D \end{bmatrix} \begin{bmatrix} -\begin{pmatrix} \lambda_1^\alpha E - A_1 & -I_n \\ -A_0 & \lambda_2^\alpha I_n \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ B \end{pmatrix} \\ \\ I_m \end{bmatrix}. \tag{2.6}$$

Le calcul de $\mu_\star$ revient à construire des conditions sur le calcul de la borne supérieur μ de $\mu_\star$.

Pour les systèmes à temps continu ; $\mu > \mu_\star \geq 0$, si et seulement si

$$\left[\begin{array}{cc|c|c} -X_1A_1 - A_1^\top X_1 & -X_1 - A_0^\top X_2 & 0 & C^\top \\ -X_2A_0 - X_1 & 0 & -X_2B & 0 \\ \hline 0 & -B^\top X_2 & \mu I_m & D^\top \\ \hline C & 0 & D & \mu I_p \end{array}\right] \succ 0, \tag{2.7}$$

où,

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}, \; X = X^\top.$$

Pour le cas d'un système à temps discret ; $\mu > \mu_\star \geq 0$, si et seulement si

$$\left[\begin{array}{cc|c|c} X_1 - A_1^\top X_1A_1 - A_0^\top X_2A_0 & -A_1^\top X_1 & -A_1^\top X_2B & C^\top \\ -X_1A_1 & X_2 - X_1 & 0 & 0 \\ \hline -B^\top X_2A_0 & 0 & \mu I_m - B^\top X_2B & D^\top \\ \hline C & 0 & D & \mu I_p \end{array}\right] \succ 0, \tag{2.8}$$

avec,

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix}, \; X = X^\top.$$

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One-dimensional dynamic systems with Allee effect

Chabane Bedjguelel*

Mailing address: Departement of mathematics, University of Bejaia, Bejaia, Algeria
E-mail: chabane.bedjguelel@univ-bejaia.dz

Hacene-Gharout

Mailing address: Departement of mathematics, University of Bejaia, Bejaia, Algeria
E-mail: hacene.gharout@univ-bejaia.dz

Abstract In this work, we present a new one-dimensional model that depends on 4 parameters. A complete analysis of these singularities has been established. Three types of dynamics (extinction, bistability, and essential extinction) are observed in this model.

Introduction

One-dimensional dynamical systems given by the iteration of a continuous function over an interval are the most suitable mathematical representation of the life history of organisms that reproduce only once a year over a very short season. Thus, the interannual dynamics are determined by a difference equation of order 1 $x_n + 1 = f(x_n)$, where x_n is the n-th generation population.The function f will generally be what a biologist calls "density-dependent" and what mathematicians call non-linear. One of the important phenomena that has attracted the attention of many researchers and that can be observed in these models is the Allee effect. A biological phenomenon characterized by a positive correlation between the density of a population and its per capita growth rate at low densities, which can cause critical population thresholds below which the population is not able to grow [1]. Several biological and environmental factors are responsible for this Allee effect, such as facilitating reproduction, difficulty in finding mates, predation,... etc.

Recently, the study of the phenomenon of the Allee effect, associated with the behavior of extinction in biology has been an important subject of study for several authors in different fields of research and applications see, for example, [2, 3, 4]. The model that will be the subject of our work is:

$$x_{n+1} = f(x_n),\ avec\ f(x) = rx^{\alpha}e^{-qx^{p}}, \tag{0.1}$$

where r, α, p, q are positive real parameters.

The main objective of this work is to study and compare the analysis of singularities in the model in the case where the Allee effect is present and in the case where it is absent. We will limit ourselves to the study of the parameter space:

$$R_0 = \left\{(r, \alpha, p, q) \in \mathbf{R^4} :\ with\ r, \alpha, p, q > 0 \ \text{ and } \ \alpha \geq 1\right\}.$$

Key Words and Phrases: *Dynamical system, Allee effect, stability, Extinction*

1 Main results

Definition 1.1. [4] Let $f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ function defined by equation 0.1 in R_0. The essential extinction region, denoted by R_{ess}, is defined as follows:

$$R_{ess} = \left\{(r, \alpha, p, q) \in R_0 : f^2(c) < x_f\right\},$$

where x_f is the first fixed point and $c = \sqrt[p]{\frac{\alpha}{pq}}$ is the critical point of f.

In this zone, we have f admits at least three fixed points and this indicates that almost all initial densities will lead to extinction see Figure 1.

Proposition 1.1. *Let $f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ defined by equation0.1 in R_0 . Consider the values:*

$$A = \max\{f^{-1}(x_f^*)\}, \ B = \min\{f^{-1}(x_f^*)\},$$

such that $I =]A, B[\subset]x_f^*, x_f[$. If $(r, \alpha, p, q) \in R_{ess}$, then $\lim_{n\to+\infty} f^n(x) = 0$ for all $x \in I$.

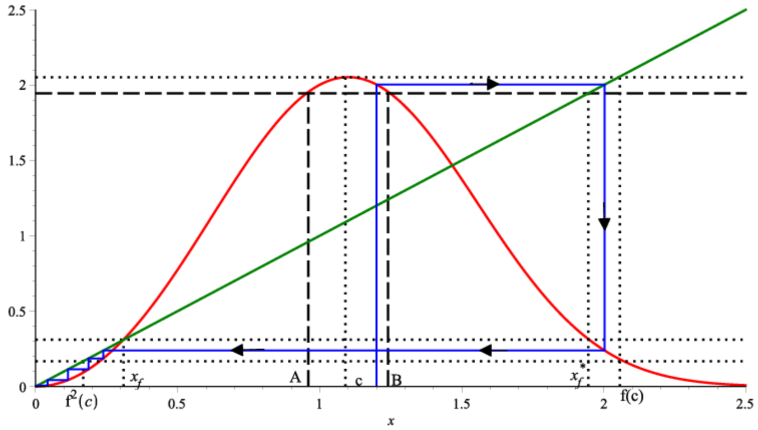

Figure 1: The graph of f when $f^2(c) < x_f$ show that for any initial point ready in the interval $]A, B[$ its orbit approaches the fixed point zero.

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Synchronization and Anti-synchronization between a Classe of Fractional-Order and Integer-Order Chaotic Systems

Messaouda Labid*
Department of Mathematics, University Center of Mila
Mila 43000, Algeria
E-mail: mes.laabid@centre-univ-mila.dz

Nasr-eddine Hamri
Laboratory of Mathematics and their interactions
Department of Science and Technology, University Center of Mila, Mila 43000, Algeria
E-mail: n.hamri@centre-univ-mila.dz

Abstract
This paper investigates the phenomenon of chaos synchronization and anti-synchronization between the fractional-order lesser date moth and the integer-order chaotic systems. Based on the Lyapunov stability theory and numerical differentiation, Nonlinear feedback control is the method used to achieve the synchronization and anti-synchronization between the fractional-order and the integer-order chaotic systems. Numerical examples are implemented to illustrate and validate the results.

1 Introduction

Chaos is a very interesting nonlinear phenomenon that has been intensively studied over the past two decades. The chaos theory is found to be useful in many areas such as data encryption , financial systems , biology and biomedical engineering [2], etc. The synchronization between the fractional-order chaotic system and the integer-order chaotic system is thoroughly a new domain and began to attract much attention in recent years because of its potential applications in secure communication and cryptography. Then the idea of the synchronization is to use the output of the master (drive) system to control the slave (response) system so that the output of the slave system tracks asymptotically the output of the master system. In the past twenty years, various types of synchronization have been proposed and investigated, e.g., complete synchronization, lag synchronization, phase synchronization, project synchronization, generalized synchronization, etc. As a special case of generalized synchronization, anti-synchronization is achieved when the sum of the states of master and slave systems converges to zero asymptotically with time. In this research work,we apply nonlinear control theory to synchronize and anti-synchronize two chaotic systems when an fractional- order system is chosen as the drive system and a integer-order system serves as the response system, we demonstrate the technique capability on the synchronization and anti-synchronization between fractional-order lesser date moth chaotic system and integer-order chaotic system.

Key Words and Phrases: *Chaos, Synchronization, Nonlinear control, Fractional-order chaotic system, Integer-order chaotic system.*

2 Problem Formulation for Fractional-Order and Integer-Order Chaotic System

Consider the following fractional-order chaotic system as a drive (master) system

$$D^{\alpha}x_1 = Ax_1 + g(x_1), \tag{2.1}$$

where $x_1 \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ is the linear part, $g(x_1)$ is a continuous nonlinear function, and D^{α} is the Caputo fractional derivative.
Also, the response system (slave) can be described as

$$\dot{x}_2 = Ax_2 + g(x_2) + u(t), \tag{2.2}$$

where $x_2 \in \mathbb{R}^n$ is the state vector, $A \in \mathbb{R}^{n\times n}$ is the linear part, and $g(x_2)$ is a continuous nonlinear function and $u(t) \in \mathbb{R}^n$ is the control.
Define the synchronous errors as $e = x_2 - x_1$.
Our aim is to determine the controller $u(t) \in \mathbb{R}^n$ such that the drive system and response system are synchronized (i.e., $\lim\limits_{t\to\infty} \|e(t)\| = 0$).
The synchronisation error system between the driving system (2.1) and the response system (2.2) can be expressed as

$$\dot{e} = \dot{x}_2 - \dot{x}_1,$$

where $\dot{x}_2$ is obtained from the response system (2.2), while no exact expressions of $\dot{x}_1$ can be obtained from the driving system (2.1). Therefore, the numerical differentiation method is used to obtain $\dot{x}_1$. According to the definition of derivative, the derivative is approximately expressed using the difference quotient as

$$g'(a) \approx \frac{g(a+h) - g(a)}{h} \tag{2.3}$$

$$g'(a) \approx \frac{g(a) - g(a-h)}{h}, \tag{2.4}$$

where $(h > 0)$ is a small increment.

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Full state hybrid projective synchronization between Lorenz Stefenlo system and Liu's system in the commensurate case

AMIRA Rami*

Mailing address: Laboratory of Mathematics, Informatics and Systems (LAMIS),Department of Mathematics and informatics, University of Larbi Tebessi, Tebessa, Algeria
E-mail: rami.amira@univ-tebessa.dz

HANNACHI Fareh

Mailing address:Department of Managment Sciences, University of Larbi Tebessi, Tebessa, Algeria
E-mail: fareh.hannachi@univ-tebessa.dz

Abstract

The aim of this work is to investigate the full state hybrid projective synchronization (FSHPS) between the fractional Lorenz stefenlo system and the fractional Liu's system. firstly, 0-1 test and Lyapunov exponent are depicted to prove that the underlying fractional systems have chaotic behavior. In addition, four Controllers are given to synchronize Lorenz-Stefenlo system and Liu's system. Numerical simulations are implemented to see the effectiveness of the proposed controllers in synchronization.

1 Introduction

Synchronization is a phenomenon that have many application in nature. The fractional chaotic systems have applications in different fields such as ecologie, medecine, cryptosystems, encryption. In this research, we highlight the FSHPS synchronization between two systems of order four, were the lorenz stefenlo system drive the Liu's system. Using Matlab, a numerical simulation are presented to show the effectiveness of the theoretical results.

2 Main results

Theorem 2.1. *The FSHPS occurs between the master system (3.1) and the slave system (3.2) under the control law defined by:*

$$V = -R - Ce \tag{2.1}$$

where $C = (c_{ij}) \in M_4(\mathbb{R})$ is feedback gain matrix selected in such way $B - C$ is a negative definite matrix.

Key Words and Phrases: *Lyapunov Exponents, 0-1 test, FSHPS, Synchronization.*

3 Synchronization between the fractional order Lorenz-Stefenlo system and the fractional Liu's system using FSHPS

In this section, the synchronization betwwen the fractional order Lorenz-Stefenlo system and the fractional Liu's system using FSHPS is made using a suitable controllers. We assume that the fractional Lorenz-stefenlo system derives the Liu's system.
We concider the master system given by:

$$\begin{cases} {}_0^C D_t^{q_1} x(t) = \sigma(y-x) + sv, \\ {}_0^C D_t^{q_2} y(t) = rx - y - xz, \\ {}_0^C D_t^{q_3} z(t) = xy - bz, \\ {}_0^C D_t^{q_3} z(t) = -x - \sigma v. \end{cases} \tag{3.1}$$

And the slave system given by:

$$\begin{cases} {}_0^C D_t^{q_1} x(t) = -ax - ey^2 + v_1, \\ {}_0^C D_t^{q_2} y(t) = by - kxz + v_2, \\ {}_0^C D_t^{q_3} z(t) = xy - bz + v_3, \\ {}_0^C D_t^{q_3} z(t) = -cz + mxy + v_4. \end{cases} \tag{3.2}$$

After applying the algorithm of full state hybrid projective syncronization we obtain the error system as follows:

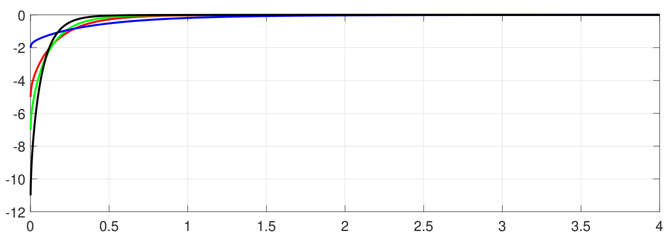

Figure 1: The evolution of the error functions $e_1(t), e_2(t), e_3(t), e_4(t)$

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On periodic solutions of a neutral delay population model with harvesting term

Lynda Mezghiche*

Department of Mathematics, University of 20 August 1955, Skikda, Algeria
E-mail: linomezg3@gmail.com

Rabah Khemis

Department of Mathematics, University of 20 August 1955, Skikda, Algeria
E-mail: kbra28@yahoo.fr

Abstract
This work presents a population model described by a neutral differential equation with harvesting term. The method used in our work is to convert our equation into an integral equation. Next, we apply Krasnoselskii's fixed point theorem and contraction mapping principle as well as some useful properties of a the Green's functions to create some sufficient criteria that ensure the existence and uniqueness of positive periodic solutions. Our new findings add to those of some earlier research.

1 Introduction

In this work, we consider the following class of first-order neutral functional differential equations with delay terms:

$$[y(t) - cy(t-\tau(t))]' = -Q(t)\,y(t) + f\left(t, y(t), y^{[2]}(t)\right) - H(t, y(t-\tau(t))), \qquad (1.1)$$

where $y^{[2]}(t) = (y \circ y)(t) = y(y(t))$ is the second iterate of the function y, $c \in]0,1[$, $Q(t), \tau(t) : \mathbb{R} \to]0,\infty[$ are $T-$periodic continuous fonctions and $f : \mathbb{R}^3 \to]0,\infty[$, $H : \mathbb{R}^2 \to]0,\infty[$ are $T-$periodic continuous functions.

2 Main results

For $m > 0$ and $L, M \geq 0$, let

$$P_T = \{y \in \mathcal{C}(\mathbb{R},\mathbb{R}),\ y(t+T) = y(t)\},$$

equipped with the norm, and

$$P_T(L,m,M) = \{y \in P_T,\ m \leq y \leq M,\\ |y(t_2) - y(t_1)| \leq L\,|t_2 - t_1|,\ \forall t_1, t_2 \in [0,T]\}.$$

Key Words and Phrases: *Periodic solution, population model, fixed point, Green's function*

Then $(P_T, \|\cdot\|)$ is a Banach space and $P_T(L, m, M)$ is a closed convex and bounded subset of P_T. In this work, we will assume that:
$(\mathbf{H}_1)$ There exists $f_0 > 0$ such that:

$$(t, y_1, y_2) \geq f_0,\ \forall t \in [0, T],\ \forall y_1, y_2 \in \mathbb{R}. \tag{2.1}$$

$(\mathbf{H}_2)$ The functions $f(t, y_1, y_2)$ and $H(t, y_1)$ are globally Lipschitz in y_1, y_2, i.e. there exist positive constants k_1, k_2 and l,such that:

$$|f(t, y_1, y_2) - f(t, z_1, z_2)| \leq k_1 \|y_1 - z_1\| + k_2 \|y_2 - z_2\|, \tag{2.2}$$

and

$$|H(t, y_1) - H(t, z_1)| \leq l \|y_1 - z_1\|. \tag{2.3}$$

$(\mathbf{H}_3)$ The following estimates are satisfied:

$$\eta_2 T(\Lambda M + f_1) + cM \leq M, \tag{2.4}$$

$$\eta_1 T f_0 - \eta_2 T(lM + H_1) - cT\eta_2 Q_1 M + cm \geq m, \tag{2.5}$$

and

$$\begin{aligned} \eta_2 (2 + Q_1 T)((\Lambda M + f_1) + (lM + H_1) + cMQ_1) \\ + L(1 + L)c \leq L. \end{aligned} \tag{2.6}$$

Theorem 2.1. *Suppose that conditions* (2.1) – (2.6) *hold, then equation* (1.1) *has at last one solution* $y \in P_T(L, m, M)$.

Theorem 2.2. *Besides the assumptions of Theorem* 2.1. *If*

$$\eta_2 T(\Lambda + l + cQ_1) + c < 1, \tag{2.7}$$

then equation (1.1) *has a unique solution* $y \in P_T(L, m, M)$.

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Etude d'un Système Chaotique d'Ordre Fractionnaire

Chettouh Besma*
Department of mathématics,University Mohamed Khaidher, Biskra, Algeria
E-mail: chettouhbesma0707@gmail.com

Menacer Tidjani
Department of mathématics,University of Mohamed Khaidher, Biskra, Algeria
E-mail: tidjanimenacer@yahoo.fr

Abstract
In this work we chose the fractional-order jerk system to stady the stability by using the fractional Routh-Hurwitz conditions. These conditions have also been used to control the chaos of the proposed systems towards their equilibrium. It has been shown that the fractional-order systems are controlled at their equilibrium point un-like those of fractional orde.

1 Introduction

The fractional calculus is more than 300 years old with the first written note dated to 1695 . Several physical phenomena can be described more accurately by fractional differential equations rather than integer-order models.However, during the last twenty years or so, fractional calculus starts to attract increasing attentions of physicists and engineers from an application point of view. It was found that many systems in interdisciplinary fields could be elegantly described with the help of fractional derivatives. Chaos theory is a very important field of application of fractional calculus. Chaos is a very interesting nonlinear phenomenon that has been widely studied over the past decades . The tool of chaos theory can be found in many fields such as data encryption,biomedical engineering and financial systems In this work we choose "The Jerk System" to apply the theories of stability and control using the generalized Routh-Hurwitz criterion to fractional order.

2 Main results

The results obtained in this work show the effect of the fractional order on the control, which proves the effectiveness of the method applied to distinguish the fractional case and that of the integer-order case and to underline the importance of the control of the fractional systems, those systems that have proven to be more accurate than its whole order counterparts.

Key Words and Phrases: *Fractional order, Dynamical system, stability, Routh-Hurwitz criterion , control, choaos*

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On a rational nonautonomous three dimensional system of difference equations

Hamida Hamioud[1], **Nouressadat Touafek**[1], **Imane Dekkar**[1], **Yasin Yazlik**[2]
[1] *Department of Mathematics, Mohamed Seddik Ben Yahia University, Jijel, Algeria*
[2] *Department of Mathematics, Nevsehir Haci Bektas Veli University, Nevsehir, Turkey*
E-mail: hmd.hamioud@univ-jijel.dz, ntouafek@gmail.com, imane_dek@hotmail.fr, yyazlik@gmail.com

Abstract
In this talk, we investigate the behavior of the solutions of the following rational nonautonomous three dimensional system of difference equations of fourth-order defined by

$$x_{n+1} = \frac{p_n + y_n}{p_n + y_{n-3}},\ y_{n+1} = \frac{q_n + z_n}{q_n + z_{n-3}},\ z_{n+1} = \frac{r_n + x_n}{r_n + x_{n-3}},\ n = 0, 1, 2, \ldots,$$

where $\{p_n\}, \{q_n\}, \{r_n\}$ are 3-periodic sequences of positive numbers, and the initial values x_i, y_i, $z_i \in [0, \infty)$, for $i = -3, -2, -1, 0$.

1 Introduction

In the present work, we study the global behavior of the following nonautonomous three dimensional fourth-order rational system of difference equations defined by

$$x_{n+1} = \frac{p_n + y_n}{p_n + y_{n-3}}, \quad y_{n+1} = \frac{q_n + z_n}{q_n + z_{n-3}}, \quad z_{n+1} = \frac{r_n + x_n}{r_n + x_{n-3}}, \quad n = 0, 1, 2, \ldots, \tag{1.1}$$

where $\{p_n\}$, $\{q_n\}$, $\{r_n\}$ are 3-periodic sequences of positive numbers, and the initial values x_i, y_i, $z_i \in [0, \infty)$, for $i = -3, -2, -1, 0$.

Let

$$p_n = \begin{cases} \alpha_1, & if \ \ n = 3k \\ \alpha_2, & if \ \ n = 3k+1 \\ \alpha_3, & if \ \ n = 3k+2 \end{cases},\ q_n = \begin{cases} \beta_1, & if \ \ n = 3k \\ \beta_2, & if \ \ n = 3k+1 \\ \beta_3, & if \ \ n = 3k+2 \end{cases},\ r_n = \begin{cases} \gamma_1, & if \ \ n = 3k \\ \gamma_2, & if \ \ n = 3k+1 \\ \gamma_3, & if \ \ n = 3k+2 \end{cases},$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3$ are positive numbers such that

$$\alpha_i \neq \alpha_j,\ \beta_i \neq \beta_j,\ \gamma_i \neq \gamma_j,\ i \neq j,\ i, j = 1, 2, 3.$$

The System (1.1) has a unique equilibrium point $(\overline{x}, \overline{y}, \overline{z}) = (1, 1, 1)$. In the following, we will study the stability and the attractivity of this unique equilibrium point, we will establish also some results on the non existence of periodic solutions. To confirm our results, several numerical examples will be provided.

Key Words and Phrases: *Fourth-order rational system of difference equations, Stability, Periodicity.*

2 Main results

In the next theorem, we summarise the behavior of the solutions of the system (1.1).

Theorem 2.1. *Consider the system (1.1). Then, the following statements holds:*

a) *Every solution* $\{(x_n, y_n, z_n)\}_{n\geq -3}$ *of System (1.1) is bounded and persists.*

b) *The unique equilibrium point of the system (1.1) is locally asymptotically stable if and only if* $(\alpha_3+1)(\beta_2+1)(\gamma_1+1) > 2+\sqrt{5}$, $(\alpha_2+1)(\beta_1+1)(\gamma_3+1) > 2+\sqrt{5}$ *and* $(\alpha_1+1)(\beta_3+1)(\gamma_2+1) > 2+\sqrt{5}$.

c) *Assume that* $(\alpha_3+1)(\beta_2+1)(\gamma_1+1) < 2+\sqrt{5}$ *or* $(\alpha_2+1)(\beta_1+1)(\gamma_3+1) < 2+\sqrt{5}$ *or* $(\alpha_1+1)(\beta_3+1)(\gamma_2+1) < 2+\sqrt{5}$. *Then the unique equilibrium point of System (1.1) is unstable.*

d) *Assume that* $\alpha_1\beta_3\gamma_2 > 8$, $\alpha_2\beta_1\gamma_3 > 8$ *and* $\alpha_3\beta_2\gamma_1 > 8$. *Then the unique equilibrium point of System (1.1) is globally asymptotically stable.*

e) *Assume that* α_i, β_i, γ_i, $(i = 1, 2, 3) \in [1, +\infty)$ *such that*

   – *At least one of* α_1, β_3, $\gamma_2 \in (1, +\infty)$.
   – *At least one of* α_2, β_1, $\gamma_3 \in (1, +\infty)$.
   – *At least one of* α_3, β_2, $\gamma_1 \in (1, +\infty)$.

   Then the unique positive equilibrium of System (1.1) is globally asymptotically stable.

Now, we will show the non existence of periodic solutions.

Theorem 2.2. *System (1.1) has neither prime period-three nor prime period-six solutions.*

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Study of a Differential-Algebraic System With Hybrid Functional Response

M.C. Benkara Mostefa*

Mailing address: Department of Mathematics, University of Mentouri Constantine1, Constantine, Algeria
E-mail: che.benkara@gmail.com

N.E. Hamri

Mailing address: Mathematics and Their Interactions Laboratory (Melilab), University Center of Mila, Mila, Algeria
E-mail: n.hamri@centre-univ-mila.dz

Abstract
Examining the dynamics of a bio-economic predator-prey system that only employs harvesting and a hybrid Holling II and Bedding-DeAngelis response function is the aim of this work. The system has an algebraic equation since it generates financial revenue. We offer a thorough mathematical study of the suggested model to explain some of the key findings. The model's solutions boundedness and positivity were looked into. The coexistence equilibrium of the bio-economic system has been extensively studied, and qualitative theories of dynamical systems (such local stability and Hopf bifurcation) have been used to describe the behaviors of the model concerning them. The acquired results serve as a helpful starting point for understanding the role of economic revenue v.

1 Introduction

There is a lot of interest in understanding and designing bio-economic models for biodiversity for the long-term benefit of humanity. Researchers are working to develop certain potentially beneficial results in order to ensure the ecosystem's long-term viability and prosperity.
The investigation of such dynamics has been the focus of a lot of research. The dynamical behavior of a type of predator-prey ecosystem was explored using numerous differential equations and an algebraic equation [1, 3]. They discovered important discoveries like interior equilibrium stability, Hopf bifurcation, limit cycle, singularity driven bifurcation and control, and so on. However, only the prey population is harvested in all of the models studied. With the assumption that the isolated predator species has a natural mortality, the interaction between predator and prey was explored using several functional response such as Holling I, Holling II, Holling III [2], and Beddington-DeAngelis [4] .
As far as we know, the dynamical study of a predator-prey model withe hybrid Holling II

Key Words and Phrases: *Predator-prey system, Equilibrium point, Stability, Algebraic differential equations, Hopf bifurcation.*

and Beddington-DeAngelis response where both prey and predator increase logistically, has not been previously investigated. As a result, in this study, we analyze this sort of model and discuss its dynamical behaviors, such as stability and Hopf bifurcation. Furthermore, we want to discover some theoretical principles for the management and control of renewable resources.

2 Main results

We demonstrate that a positive equilibrium point is locally asymptotically stable when the profit v is less than a given critical value v^*. Our research makes it obvious. According to our research, economic income has the power to balance the system and ensure the existence of all species.

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Dynamic Analysis of New One-Dimensional Chaotic Map with Fractional-Order

Seyf El Islam Bouzeraa*

Mailing address: University Center of Abdelhafi d Boussouf, Mila, Algeria.
E-mail: s.bouzeraa@centre-univ-mila.dz

Rabah Bououden

Mailing address: University Center of Abdelhafi d Boussouf, Mila, Algeria.
E-mail: rabouden@yahoo.fr

Abstract
In this work, a fractional one-dimensional map is proposed using the caputo-like delta difference. The proposed fractional map's dynamical behaviour is numerically investigated. The bifurcation diagram and the Lyapunov exponent are used to illustrate the effect of fractional-order on the fractional map.

1 Introduction

Because fractional calculus can accurately explain many more and a lot of real problems, fractional systems have recently received considerable and growing attention from researchers. Despite the widespread interest in and numerous results presented in discrete fractional chaotic systems, indentifying chaos in fractional maps remains an open topic. In this work, we will propose and analyze a one-dimentional fractional chaotic map.
In the next section, we introduce the basic concepts of the discrete fractional calculus.

2 Preliminaries

Definition 2.1. [3] Let $u : \mathbb{N}_a \longrightarrow \mathbb{R}$, and $v > 0$ by given. Then the fractional sum of v order is defined by

$$\Delta_a^{-v} u(t) = \frac{1}{\Gamma(v)} \sum_{s=a}^{t-v} (t - \sigma(s))^{(v-1)} u(s),\ t \in \mathbb{N}_{a+v}, \tag{2.1}$$

where a is the starting point, $\sigma(s) = s + 1$, $\mathbb{N}_a = \{a, a + 1, a + 2....\}$, $t^{(v)}$ is the flling fractional function defnied as

$$t^{(v)} = \frac{\Gamma(t+1)}{\Gamma(t+1-v)}. \tag{2.2}$$

Key Words and Phrases: *Fractional-order, chaotic map, bifurcation, Lyapunov exponent*

Definition 2.2. [1] For $v > 0$, $v \notin \mathbb{N}$ and $u(t)$ defined on $\mathbb{N}_a$, the Caputo-like delta difference is defined by :

$$^c\Delta_a^v x(t) = \Delta_a^{-(m-v)} \Delta^m x(t) \tag{2.3}$$

$$= \frac{1}{\Gamma(m-v)} \sum_{s=a}^{t-(m-v)} (t-\sigma(s))^{(m-v-1)} \Delta_s^m x(s), \tag{2.4}$$

where $t \in \mathbb{N}_{a+m-v}$, $m = [v]+1.$, $[v]$ means the integer order difference with starting point 0 and a, respectively.

Theorem 2.1. *[2] For the delta fractional difference equation.*

$$\begin{cases} ^c\Delta_a^v u(t) = f(t+v-1, u(t+v-1)) \\ \Delta^k u(a) = u_k, m = [v]+1, k = 0, \ldots, m-1, \end{cases}$$

the equivalent discrete integral equation can be obtained as

$$u(t) = u_0(t) + \frac{1}{\Gamma(v)} \sum_{s=a+m-v}^{t-v} (t-\sigma(s))^{(v-1)} \times f(s+v-1,\ u(s+v-1)) \tag{2.5}$$

$$,\ t \in \mathbb{N}_{a+m},$$

where the initial iteration $u_0(t)$ reads:

$$u_0(t) = \sum_{k=0}^{m-1} \frac{(t-a)^{(k)}}{k!} \Delta^k u(a) \tag{2.6}$$

3 Main results

The proposed one-dimensionl map can be defined as follows:

$$x_{n+1} = \frac{r x_n (1-x_n)}{(1+x_n)^2}, \tag{3.1}$$

Where r is the control parameter, the map (3.1) represents chaos behavior when r values range between 7 and 8 as shown in figure (1).
We can easily calculate the first-order difference for map (3.1) as follows

$$\Delta x_n = x_{n+1} - x_n = \frac{r x_n (1-x_n)}{(1+x_n)^2} - x_n, \tag{3.2}$$

we obtained a fractional map by replacing the first difference operator Δ with the Caputo difference operator $^C\Delta_a^v$ in equation (3.2).

$$^C\Delta_a^v x(t) = \frac{r x(t+v-1)(1-x(t+v-1))}{(1+x(t+v-1))^2} - x(t+v-1), \tag{3.3}$$

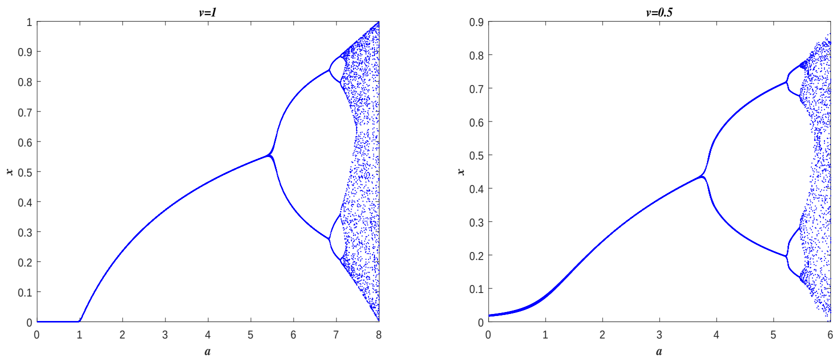

Figure 1: Bifurcation diagrams of fractional map (3.1) for $v = 1$ and $v = 0.5$.

From Theorem (2.1), we can obtain equivalent discrete integral of Eq (3.3)

$$x(t) = x(a) + \frac{v}{\Gamma(v)} \sum_{s=1-v}^{t-v} \frac{\Gamma(t-s+v)}{\Gamma(t-s+1)} \left(\frac{rx(s+v-1)(1-x(s+v-1))}{(1+x(s+v-1))^2} - x(s+v-1)\right). \tag{3.4}$$

For $a = 0$ and $j = s + v$, the numerical solution of map (3.3) is given as

$$x_n = x_0 + \frac{v}{\Gamma(v)} \sum_{j=1}^{n} \frac{\Gamma(n-j+v)}{\Gamma(n-j+1)} \left(\frac{rx(j-1)(1-x(j-1))}{1+x(j-1)} - x(j-1)\right). \tag{3.5}$$

using the numerical formula (3.5), and using r as a bifurcation parameter, we can plot the bifurcation diagram of fractional map (3.1) for fractional order $v = 0.5$ shown in figure (1),

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The Limit Cycles in a Family of Planar Piecewise Linear Differential Systems Separated by Two Circles

Baymout Louiza*
Mailing address: Mathematics Department, Mohamed El Bachir El Ibrahimi University of Bordj Bou Arreridj, Algeria
E-mail: louiza.baymout@univ-bba.dz

Benterki Rebiha
Mailing address: Mathematics Department, Mohamed El Bachir El Ibrahimi University of Bordj Bou Arreridj, Algeria
E-mail: r_benterki@yahoo.fr

Abstract
The importance of studying piecewise linear differential systems has grown in recent years, due to their applications. Like we can see the appearance of this kind of system in modeling many natural phenomena, as in physics, biology, economics, etc. It is well known, that the limit cycles play a main role in the study of qualitative theory of piecewise differential systems. In most of the published papers that studied the limit cycles of piecewise differential systems formed by linear systems consider only two pieces. In this paper we investigate the maximum number of limit cycles for a family of piecewise linear differential systems formed by linear Hamiltonian differential systems without equilibria, where the separation curve which is twi concentric circles splits the plane into three pieces and that made a big difference.

1 Introduction

Around the end of the nineteenth century, Poincaré's works provide the first examination limit cycles. Then it was realized that many natural phenomena modeled by limit cycles, such as the Sel'kov model of glycolysis, the Belousov Zhavotinskii model, or the model of Van der Pol, etc.

Nowadays, discontinuous piecewise differential systems generate the most of scientific research. These kinds of systems are used by several disciplines, such as economics, ecology, engineering, and epidemiology, to model and describe the behavior of several real-world phenomena. Due to their extensive use in applications as we can see in electrical circuit design and control theory, piecewise discontinuous differential systems are of great interest, and among the most important problems to study for these systems is to find the upper bound number of limit cycles and their possible configurations in the plane.

These systems have been the subject of numerous articles that have been published, some of which include in [4, 5] and the references they contain. The simplest classes of discontinuous piecewise linear differential systems in the plane are those separated by one

Key Words and Phrases: *Hamiltonian systems, limit cycles, discontinuous piecewise differential systems, circle*

straight line, and it remains an open problem to know if three is the maximum number of limit cycles that this class of systems can have.
Regarding the separation curve, it's important to ask: If the separation curve is not a straight line, what happens to the number of limit cycles? How does the maximum number of limit cycles changes depending on how many zones the separation curve creates?
In this work, the main objective is to study the number of limit cycles, that can exhibit the special family of planar discontinuous piecewise linear differential systems formed by linear Hamiltonian differential systems without equilibria and separated by two concentric circles.

2 Main results

Our main results are given in the following Theorem.

Theorem 2.1. *For the discontinuous piecewise differential systems separated by two concentric circles and formed by linear Hamiltonian systems without equilibria, the maximum number of limit cycles is at most three limit cycles.*

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A Coupled Caputo–Hadamard Fractional Differential System with Multipoint Boundary Conditions

Samir Aibout*

Laboratory of Mathematics, University of Saïda–Dr. Moulay Tahar, P.O. Box 138, EN-Nasr, 20000 Saïda, Algeria
E-mail: adamaibout1982@yahoo.com

Saïd Abbass

Laboratory of Mathematics, University of Saïda–Dr. Moulay Tahar, P.O. Box 138, EN-Nasr, 20000 Saïda, Algeria
E-mail: abbasmsaid@yahoo.fr

Mouffak Benchohra

Laboratory of Mathematics, Djillali Liabes University of Sidi Bel-Abbès, P.O. Box 89, Sidi Bel-Abbès 22000, Algeria
E-mail: benchohra@yahoo.com

Martin Bohner

Department of Mathematics and Statistics, Missouri S and T, Rolla, Missouri 65401, USA
E-mail: bohner@mst.edu

Abstract

This paper deals with existence of solutions for a coupled system of Caputo–Hadamard fractional differential equations with multipoint boundary conditions in Banach spaces. Some applications are made of some fixed point theorems on Banach spaces. An illustrative example is presented in the last section.

Key Words and Phrases: *Fractional differential equation, coupled system, mixed Hadamard integral of fractional order, Caputo–Hadamard fractional derivative, solution, fixed point.*
AMS (MOS) Subject Classifications: 26A33.

1 Introduction

They use the technique of measure of weak non compactness and the fixed point theory to discuss the existence of weak solutions. In this article, we discuss the existence and uniqueness of solutions for a coupled system of Caputo–Hadamard fractional differential equations of the form

$$\begin{cases} (\alpha_1 u)(t) = f_1(t, u(t), v(t)) \\ (\alpha_2 v)(t) = f_2(t, u(t), v(t)) \end{cases} \quad ; \ t \in I := [1, T], \tag{1.1}$$

with the multipoint boundary conditions

$$\begin{cases} a_1 u(1) - b_1 u'(1) = d_1 u(\xi_1) \\ a_2 u(T) + b_2 u'(T) = d_2 u(\xi_2) \\ a_3 v(1) - b_3 v'(1) = d_3 v(\xi_3) \\ a_4 v(T) + b_4 v'(T) = d_4 v(\xi_4), \end{cases} \tag{1.2}$$

where $T > 1$, $a_i, b_i, d_i \in \mathbb{R}$, $\xi_i \in (1, T)$, $i = 1, 2, 3, 4$, are given continuous functions, ^{m} for $m \in$ is the Euclidean Banach space with a suitable norm $\|\cdot\|$, α_j is the Caputo–Hadamard fractional derivative of order α_j, $j = 1, 2$.

2 Main results

In this section, we are concerned with existence and uniqueness results of the coupled system (**??**)–(**??**).

Definition 2.1. By a solution of the problem (**??**)–(**??**), we mean coupled continuous functions $(u, v) \in C \times C$ satisfying the boundary conditions (**??**) as well as the equations (**??**) on I.

The following hypotheses will be used in the sequel.

1 The functions f_i, $i = 1, 2$, satisfy the generalized Lipschitz condition

$$\|f_i(t, u_1, v_1) - f_i(t, u_2, v_2)\| \leq \frac{1}{G_i^*}(\phi_i(\|u_1 - u_2\|) + \psi_i(\|v_1 - v_2\|))$$

for $t \in I$ and $u_i, v_i \in^m$, where ϕ_i, ψ_i, $i = 1, 2$, are comparison functions.

2 There exist continuous functions $h_i, p_i, q_i : I \to_+$, $i = 1, 2$, such that

$$\|f_i(t, u, v)\| \leq h_i(t) + p_i(t)\|u\| + q_i(t)\|v\| fort \in Iandu, v \in^m .$$

Set

Theorem 2.1. *Assume 1. Then* (**??**)–(**??**) *has a unique solution.*

Now, we prove an existence result by using Schauder fixed point theorem. Set

$$h_i^* := \sup h(t) \qquad p_i^* := \sup p(t) \qquad q_i^* := \sup q(t) \qquad i = 1, 2$$

Theorem 2.2. *Assume 2. If*

$$G_1^* h_1^* + G_2^* h_2^* < 1,$$

then the coupled system (**??**)–(**??**) *has at least one solution defined on* I.

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On a Relaxed Problem with Control Constraints

Abdi Nadjiba *
Mailing address: LMPA Laboratory, Department of Mathematics, Mohammed Seddik Benyahia University, Jijel, Algeria
E-mail: nadjibaabdi@gmail.com.

Soumia Saïdi
Mailing address: LMPA Laboratory, Department of Mathematics, Mohammed Seddik Benyahia University, Jijel, Algeria
E-mail: soumiasaidi44@gmail.com.

Abstract
In a real Hilbert space, we deal with a relaxed problem on the solutions of a control system with maximal monotone operators. Under suitable assumptions, we answer the question of existence of optimal solutions to the relaxed problem. We will show that the minimization problem of an integral functional has a minimizing sequence which converges to an optimal solution.

1 Introduction

Let $I := [0, T]$ be an interval, H a real Hilbert space and X a separable reflexive Banach space. Consider the control system

$$\begin{cases} -\dot{x}(t) \in A_t x(t) + B(t, x(t))u(t) + F(t, x(t)) & \text{a.e. } t \in I, \\ x(0) = x_0 \in D(A_0), \end{cases} \tag{1.1}$$

with the control mixed constraint

$$u(t) \in U(t, x(t)), \tag{1.2}$$

where $A_t : D(A) \subset H \rightrightarrows H$ is a maximal monotone operator, $B : I \times H \to \mathcal{L}(X, H)$, where $\mathcal{L}(X, H)$ is the space of continuous linear operators and $F : I \times H \to H$ is a nonlinear mapping.
Consider the following problem (P)

$$\inf \int_I h(t, x(t), u(t))dt, \tag{1.3}$$

subject to the solutions of the control system (1.1) with the constraint (1.2).
We set

$$h_u(t, x, u) = \begin{cases} h(t, x, u) & u \in U(t, x) \\ +\infty, & \text{otherwise.} \end{cases}$$

Now, we consider the relaxed problem

$$\inf \int_0^1 h_U^{**}(t, x(t), u(t))dt, \tag{1.4}$$

on the solutions of the control system (1.1) with the constraint

$$u(t) \in \overline{co} U(t, x(t)). \tag{1.5}$$

Key Words and Phrases: *Optimal solution, control system, minimization problem.*

2 Main results

Under suitable assumptions, we show that the problem (1.4) has an optimal solution $(x_*(\cdot), u_*(\cdot))$ and for any optimal solution there exists a minimizing sequence $(z_n(\cdot), v_n(\cdot))$ of problem (1.3) that converges to an optimal solution.

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Evolution Problems with Maximal Monotone Operators and Perturbations

Fatima Fennour*

Mailing address: LMPA Laboratory, Department of Mathematics, Mohammed Seddik Ben Yahia University, Jijel, Algeria
E-mail: fennourfatima38@gmail.com

Soumia Saïdi

Mailing address: LMPA Laboratory, Department of Mathematics, Mohammed Seddik Ben Yahia University, Jijel, Algeria
E-mail: soumiasaidi44@gmail.com

Abstract
In this work, we study the existence of absolutely continuous solutions for a differential inclusion governed by time and state dependent maximal monotone operators with perturbation. The perturbation acts as external forces. The result is obtained in the context of Hilbert spaces.

1 Introduction

Differential inclusions with maximal monotone operators have interested many authors see eg., [1], [2], [3], [4], [5], and the references therein. Such evolution problems find applications in various areas such as optimization algorithms, dynamical systems, and partial differential equations. We aim to contribute on this topic.

2 Main results

Let $I := [0, 1]$ be an interval of $\mathbb{R}$ and let H be a Hilbert space. Assume that $A(t, x) : D(A(t, x)) \subset H \rightrightarrows H$ is a maximal monotone operator for any $(t, x) \in I \times H$.
In the present work, we propose a new existence result to the first-order differential inclusion governed by time and state dependent maximal monotone operators of the form

$$\begin{cases} -\dot{u}(t) \in A(t, u(t))u(t) + F(t, u(t)) & \text{a.e. } t \in I \\ u(0) = u_0, \end{cases}$$

where F is a non-convex perturbation.
Following some ideas from [1], and a generalized fixed point theorem, we prove that for any $u_0 \in D(A(0, u_0))$ the considered problem has an absolutely continuous solution $u(\cdot)$.

Key Words and Phrases: *Differential inclusion, maximal monotone operator, Hilbert spaces, non-convex perturbation.*

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Global Chaos Combination Synchronization of Identical or Different Fractional-Order Chaotic systems

Soumia Bensimessaoud*

Department of mathematics and computer sciences
Abdelhafid Boussouf University Center, Mila. Algeria
E-mail: soumiabensimessaoud@gmail.com

Smail Kaouache

Laboratory of Mathematics and their interactions
Abdelhafid Boussouf University Center, Mila. Algeria
E-mail: s.kaouache@centre-univ-mila.dz

Abstract
Chaos combination synchronization is one of the main features of chaos applied in practical engineering. Thus, chaos synchronizations have been active research topics. In this paper we invistigate the chaos combination synchronization of identical or different fractional-order chaotic systems with unknoun parameter by using adaptive control. Numerical simulation results further demonstrate that the proposed method is effective and robust.

1 Some Basic about fractional derivative

Definition 1.1. Let $n-1 < p \leq n$, $n \in \mathbb{N}$; the Caputo fractional derivative of order p of function y [1] is defined as

$$^{c}D^{p}y(t) = \frac{1}{\Gamma(n-p)} \int_0^t (t-\xi)^{n-p-1} y^{(n)}(\xi) d\xi \tag{1.1}$$

Theorem 1.1. *If there exist a positive Lyapunov function V such that $D^pV(x(t)) < 0$, for all $t \geq t_0$ then the trivial solution of the equation $D^px(t) = F(x(t))$ is asymptotically stable.*

Lemma 1.1. *[2] Let $x(t) \in \mathbb{R}$ be a derivable function, then for all $t > t_0$,*

$$\frac{1}{2}D^{\alpha}x^2(t) \leq x(t)^t D^{\alpha}x(t), \quad \alpha \in (0,1). \tag{1.2}$$

Keywords: *Fractional derivative, Chaos Synchronization, Adaptive controller.*

2 General Method of Global Chaos Combination Syncronization

Consider the two drive systems in the form

$$D^p x = f(x) + F(x)\alpha, \tag{2.1}$$

$$D^p y = g(y) + G(y)\beta, \tag{2.2}$$

where $p \in (0,1)$ is the fractional order, $x = (x_1, x_2, ..., x_n)^T$, $y = (y_1, y_2, ..., y_n)^T \in \mathbb{R}^n$ are the state vectors of the drive systems, $f, g : \mathbb{R}^n \to \mathbb{R}^n$ are vector functions without uncertain parameter, $F, G \in \mathbb{R}^{n\times n}$ are matrix functions, $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)^T \in \mathbb{R}^n$, $\beta = (\beta_1, \beta_2, ..., \beta_n)^T \in \mathbb{R}^n$ are the uncertain parameter vectors of the drive systems.
On the other hand, the response system is assumed by

$$D^p z = h(z) + H(z)\gamma + u, \tag{2.3}$$

where $z = (z_1, z_2, ..., z_n)^T \in \mathbb{R}^n$ is the state vector of the response system, $h : \mathbb{R}^n \to \mathbb{R}^n$ is vector function without uncertain parameter, $H \in \mathbb{R}^{n\times n}$ is matrix function, $\gamma = (\gamma_1, \gamma_2, ..., \gamma_n)^T \in \mathbb{R}^n$ is the uncertain parameter vectors of the response system and $u = (u_1, u_2, ..., u_n)^T \in \mathbb{R}^n$ is the control input vector, which is used to realize synchronization of the systems.
Let $e = x + y - z$ be the combination synchronization error. Our goal is to design an appropriate controller u such that the response system (2.3) can synchronize the drive systems (2.1) -(2.2) globally and asymptotically.

Theorem 2.1. *If the nonlinear controller u is taken as*

$$u = f(x) + F(x)\hat{\alpha} + g(y) + G(y)\hat{\beta} - h(z) - H(z)\hat{\gamma} + ke, \tag{2.4}$$

where the control amplitude $k = diag(k_1, k_2, ..., k_n)$ is positive constant matrix, and the adaptive laws of parameters are taken as

$$D^p\hat{\alpha} = [F(x)]^T e,\ D^p\hat{\beta} = [G(y)]^T e,\ D^p\gamma = -[H(z)]^T e. \tag{2.5}$$

where $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ are estimations of the unknown parameters α, β and γ, respectively, then the response system (2.3) can synchronize the drive systems (2.1) -(2.2) globally and asymptotically.

Démonstration. □

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Mathematical study of a non-local epidemiological model

HATHOUT Fatima Zohra*

Departement of Mathematics, University Aboubekr Belkaid, Tlemcen, Algeria
E-mail: fatima.zohra.hathout@gmail.com

TOUAOULA Mohammed Tarik

Departement of Mathematics, University Aboubekr Belkaid, Tlemcen, Algeria
E-mail: tarik.touaoula@univ-tlemcen.dz

Abstract
In this research, we consider the influence of protection measures on the spread of infectious diseases in an age-structured population. Protection strategy can take different forms as isolation, treatment, or renewable vaccine; to mathematically represent it, we include a new compartment p standing for protected individuals, in a classical age structured si model. Global analysis of the proposed model is made by the introduction of total trajectories and a suitable Lyapunov functional. We give a particular importance to the protection strategy and many numerical simulations are provided to illustrate our theoretical results.

1 Introduction

The world witnessed a huge increase in the number of newly discovered diseases, we mention as example HIV, bovine-babesiosis, COVID-19, ebola, avian flu, which makes a necessity to increase the scientific contributions for stopping the spread of these contagion diseases and protecting our community from the outcome of this spread. There are many ways for limiting the spread of these new infectious diseases by making vaccines and treatments. When a new vaccine/treatment is developed, it is very tough to make it available to the whole population due to its expensiveness and scarcity. For some epidemics and in the absence of treatments or vaccine, some governments use other measures for limiting the spread of diseases, as imposing the use of protection materials or isolation in some extreme situations. Giving a particular importance to loosing protection for some individuals (vaccine needing to be updated, treatment needing to be repeated, isolation not

Key Words and Phrases: *Age structured SIR model; Lyapunov functional; uniform persistence; Total trajectory; α and ω limit sets.*

carefully respected) we formulate our model in the following structure:

$$\begin{cases} \dfrac{\partial s(t,a)}{\partial t} + \dfrac{\partial s(t,a)}{\partial a} = -(\gamma_s(a) + h(a))s(t,a) - \beta(a)s(t,a)J(t), & t > 0, \\ \dfrac{\partial p(t,a)}{\partial t} + \dfrac{\partial p(t,a)}{\partial a} = -(\gamma_p(a) + k(a))p(t,a), & t > 0, \quad a > 0, \\ \dfrac{\partial i(t,a)}{\partial t} + \dfrac{\partial i(t,a)}{\partial a} = -(\gamma_i(a) + \mu(a))i(t,a), & t > 0, \quad a > 0, \\ s(t,0) = A + (1-\alpha)\displaystyle\int_0^\infty k(a)p(t,a)da, & t > 0, \\ p(t,0) = \displaystyle\int_0^\infty h(a)s(t,a)da + \alpha \int_0^\infty k(a)p(t,a)da, & t > 0, \\ i(t,0) = J(t)\displaystyle\int_0^\infty \beta(a)s(t,a)da, & t > 0, \\ J(t) = \displaystyle\int_0^\infty \theta(a)i(t,a)da, \end{cases} \tag{1.1}$$

with initial conditions:

$$s(0,.) = \bar{s}(.) \in L^1(\mathbb{R}^+,\mathbb{R}^+), \quad p(0,\cdot) = \bar{p}(\cdot) \in L^1(\mathbb{R}^+,\mathbb{R}^+), \quad i(0,.) = \bar{i}(\cdot) \in L^1(\mathbb{R}^+,\mathbb{R}^+). \tag{1.2}$$

2 Main results

Theorem 2.1. *System* (1.1) *has always the trivial equilibrium expressed as* $E_0 = (s_0(a), p_0(a), 0)$,

Theorem 2.2. *If* $\mathcal{R}_0 > 1$ *then system* (1.1) *has a unique endemic equilibrium, which is globally stable.*

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L'étude du modèle SIQI avec diffusion

Guezzen Chérifa*
Mailing address: Département de mathématiques, Université de Tlemcen, Tlemcen, Algériée
E-mail: cherifaguezzen@gmail.com

Touaoula Tarik Mohammed
Mailing address: Département de mathématiques, Université de Tlemcen, Tlemcen, Algérie
E-mail: *touaoula_tarik@yahoo.fr*

Abstract
Dans ce travail, nous avons étudié la dynamique globale du modèle SIQI de difusion avec une fonction d'incidence générale. D'abord nous avons commencé par établir l'existance et la bornitude des solutions de notre système. Ensuite, nous avons montré que la solution semi-flux est ultimement bornée. Après on a remarqué que le semi-flux n'est pas compact. Pour cette raison, nous avons utilisé la condition de k-contraction pour démontrer que notre semi-flux est asymptotiquement smooth. Par conséquent on a confirmé l'existence d'un attracteur compact global. En outre, nous avons identifié le taux de reproduction de base R_0 en utilisant la procédure standard de l'opérateur de la prochaine génération. Puis nous avons montré que le R_0 est un paramètre seuil pour la stabilité du modèle. En effet, l'équilibre sans maladie est globalement asymptotiquement stable lorsque $R_0 \leq 1$, tandis que le phénomène de persistance uniforme se produisent lorsque $R_0 > 1$. Finalement, nous avons étudié la stabilité locale et globale de l'équilibre endémique dans le cas où tous les paramètres sont constants.

1 Introduction

La quarantaine joue un rôle important pour aider à quantifier des stratégies de lutte contre une épidémie. Mais pour certains maladies cette méthode ne marche pas c'est à dire on peut avoir une rechute. C'est pour cela on a proposé d'étudier un modèle de diffusion où les individus isolés peuvent retourner vers le compartiment des infectés. On a le modèle suivant:

$$\begin{cases} \frac{\partial S}{\partial t} = d_1 \Delta S + A(x) - \beta(x) f(S,I) - \mu(x) S,\ t>0,\ x \in \Omega \\ \frac{\partial I}{\partial t} = d_2 \Delta I + \beta(x) f(S,I) - (\mu(x)+\theta(x)) I + d(x) Q,\ t>0, x \in \Omega \\ \frac{\partial Q}{\partial t} = \theta(x) I - (\mu(x)+d(x)) Q,\ t>0,\ x\in\Omega \\ \frac{\partial S}{\partial \eta} = 0, \frac{\partial I}{\partial \eta} = 0,\ x \in \partial\Omega \end{cases} \tag{1.1}$$

avec

$$S(x,0) = S_0(x) \geq 0,\ I(x,0) = I_0(x) \geq 0,\ Q(x,0) = Q_0(x) \geq 0, \tag{1.2}$$

Où $S(t,x)$, $I(t,x)$ and $Q(t,x)$ représentent les individus sains, infectés et isolés (resp). d_1 et d_2 sont les coefficients de diffusion de S et I(resp). μ représente le taux de mortalité et β est le taux de transmission.d représente le taux rechute.A est le taux de naissance.

Key Words and Phrases: *Hétérogénéité spatiale, Fonctions de Lyapunov, Taux de reproduction de base, Valeur propre principale*

2 Main results

On a démontré la convergence de notre système vers l'équilibre sans maladie lorsque $R_0 \leq 1$ en utilisant une fonction de lyapunov .
Le système est uniformément persistant lorsque $R_0 < 1$.
L'existence d'une valeur propre principale lorsque $R_0 > 1$.

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Existence result for a non-convex truncated differential inclusion

Imene Mecemma*

Mailing address: Department of Mathematics, University of Jijel, Jijel, Algerian
E-mail: mecemma.imene@yahoo.com

Sabrina Lounis

Mailing address: Department of Mathematics, University of Jijel, Jijel, Algerian
E-mail: lounis_18sabrina@yahoo.fr

Mustapha Fateh Yarou

Mailing address: Department of Mathematics, University of Jijel, Jijel, Algerian
E-mail: mfyarou@yahoo.com

Abstract
In our paper, by using a discretization approach we discuss the existence result for a class of second order nonconvex sweeping process depending jointly on time, state and velocity, with an unbounded perturbation in an infinite dimensional Hilbert space by proposing a semi-implicit discretization scheme based on Moreau's catching-up algorithm.

1 Introduction

The sweeping process is a particular differential inclusion governed by a normal cone to a moving set. This type of problem plays an important role in elastoplasticity and dynamics. The naming of "sweeping process" is due to the fact that $u(t)$ is swept by $C(t)$. In the present paper, we are mainly interested to study the following new variant of the sweeping process

$$(\mathcal{D})\begin{cases}\ddot{u}(t) \in -N_{C(t,u(t),\dot{u}(t))}(\dot{u}(t)) - F(t,u(t),\dot{u}(t)), \quad \text{a.e.} \quad t \in [0,T];\\ u(0) = u_0, \dot{u}(0) = v_0 \in C(0,u_0,v_0),\end{cases}$$

where $N_{C(t,u(t),\dot{u}(t))}(\dot{u}(t))$ stands for the Clark normal cone to the closed set $C(t,u(t),\dot{u}(t))$ at a point $\dot{u}(t)$, C is an unbounded and uniformly subsmooth set depending jointly on time, state and velocity, and $F : [0,T] \times \mathbb{R}^d \times \mathbb{R}^d \rightrightarrows \mathbb{R}^d$ is a perturbation, that is external forces applied on the system. The second order sweeping process is subject to numerous research works. In [1], C. Castaing, for the first time studied this problem where C depends on the state, then various extensions have been obtained by many authors. For instance: adding external forces applied on the system known as perturbations, or reducing the convexity on the moving set. For more details, we refer the reader to [2, 3, 4].

Key Words and Phrases: *Moreau's sweeping process, differential inclusion, subsmooth set, truncated, perturbation*

Recently, in the infinite-dimensional space H, [4] provided a new result to the problem $(\mathcal{D})$ where the moving set depends on both time and state under the assumption that the moving set is closed convex and has some Lipschitz variation. In our work, we are generalizing this result in several directions by considering the non-convex case, that is, for uniformly subsmooth sets, and under the assumptions that the set-valued map $C(\cdot,\cdot,\cdot)$ depending on time, state and velocity, is Lipschitz and is controlled through the truncated Hausdorff-Pompeiu distance. Furthermore, we consider an unbounded set-valued perturbation for which only the element of minimum norm satisfies a linear growth condition.

2 Main results

The aim of this section is to study the existence of solution for the problem $(\mathcal{D})$ where the moving set is subsmooth and depends jointly on time, state and velocity. Let $C : [0,T] \times H \times H \rightrightarrows H$ be a set-valued mapping with nonempty closed values and $F : [0,T] \times H \times \rightrightarrows H$ with nonempty closed convex values. Assume the following assumptions:

$(\mathcal{A}_F)$ F is scalary upper semicontinuous, that is, for each $y \in H$, the function $(t, u, v) \mapsto \sigma(y, F(t, u, v))$ is upper semicontinuous, and for all $(t, u, v) \in [0,T] \times H \times H$ and some real $\beta > 0$

$$d_{F(t,u,v)}(0) \leq \beta(1 + \|u\| + \|v\|).$$

$(\mathcal{A}_{C_1})$ For all $(t, u, v) \in [0,T] \times H \times H$, the sets $C(t, u, v)$ are equi-uniformly subsmooth.

$(\mathcal{A}_{C_2})$ There exist a real $L \in]0,1[$ and an extended real $\rho \geq \big(\|v_0\| + \|u_0\| + 2T\frac{L+2\beta}{1-L}\big)e^{T\frac{2\beta+1}{1-L}}$ such that for every $t, s \in [0, T]$ and all $u, u', v, v' \in H$

$$\mathcal{H}_\rho(C(t, u, v), C(s, u', v')) \leq L\Big(|t-s| + \|u-v\| + \|u'-v'\|\Big).$$

$(\mathcal{A}_{C_3})$ For any bounded subsets A and B of H the set $C(t, A, B)$ is ball-compact, i.e., the intersection of $C([0,T] \times A \times B)$ with any closed ball of H is compact.

Now, we are able to give our existence theorem.

Theorem 2.1. *Suppose that assumptions $(\mathcal{A}_F)$, $(\mathcal{A}_{C_1})$, $(\mathcal{A}_{C_2})$ and $(\mathcal{A}_{C_3})$ are satisfied, then for every $(u_0, v_0) \in H$ with $v_0 \in C(0, u_0, v_0)$ there exist at least a $W^{1,2}_H([0, T])$ solution $u(\cdot)$ of problem $(\mathcal{D})$.*

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SYNCHRONISATION ROBUSTE COMPOSÉE DE QUATRE SYSTÈMES CHAOTIQUES IDENTIQUES D'ORDRE FRACTIONNAIRE

Smail Kaouache*
Département de mathématiques et informatique, Centre universitaire Abdelhafid Boussouf, Mila
E-mail: s.kaouache@centre-univ-mila.dz

Résumé
Dans cette communication, nous étudions le problème de synchronisation robuste composée de quatre systèmes chaotiques identiques d'ordre fractionnaire en utilisant la technique de contrôle continu. Une analyse basée sur la théorie de la stabilité des systèmes d'ordre fractionnaire est effectuée pour conclure sur la stabilité pratique aussi bien que sur la convergence asymptotique des erreurs de synchronisation associées. Enfin, des simulations numériques sont illustrés pour vérifier la capacité de schéma de synchronisation proposée.

1 Formulation du problème

Dans cette section, on va présenter le concept de synchronisation combinée entre quatre systèmes chaotiques identiques d'ordre fractionnaire ainsi que la convergence asymptotique des erreurs de synchronisation associées. Le modèle mathématiques des systèmes étudiés est donné par

$$D^{\alpha}x = Ax + F(x), \tag{1.1}$$

$$D^{\alpha}y = Ay + G(y), \tag{1.2}$$

$$D^{\alpha}z = Az + H(z) + u, \tag{1.3}$$

$$D^{\alpha}k = Ak + W(k) + v, \tag{1.4}$$

où D^{α} représente l'opérateur différentiel de Caputo, α est l'ordre de dérivée fractionnaire compris entre 0 et 1, x, y sont les variables d'état des deux systèmes émetteurs (1.1)-(1.2), z et k sont les variables d'état des deux systèmes récepteurs (1.3)-(1.4), $A \in \mathbb{R}^{n\times n}$ est la partie linéaire du système chaotique considéré. $F, G, H, W : \mathbb{R}^n \to \mathbb{R}^n$ sont des parties non linéaires de ce système et $u, v \in \mathbb{R}^n$ sont des vecteurs des contrôles actifs à déterminer.

La définition de la synchronisation qui je vais étudie est donnée par :

Définition 1.1. Les deux systèmes émetteur (1.1)-(1.2) et les deux systèmes récepteurs (1.3)-(1.4) se synchronisent lorsqu'il existe des contrôles u et v tel que l'erreur de synchronisation :

$$e(t) = x(t) + y(t) - (z(t) + k(t)),$$

vérifié la condition

$$\lim_{t\to\infty} ||e(t)|| = 0.$$

Mots Clés et Phrases: *Synchronisation, Chaos, Systèmes fractionnaires, Stabilité.*

Pour quantifier notre objectif, le système erreur est défini par la relation suivante

$$e = (x + y) - (z + k) \tag{1.5}$$

La dynamique de l'erreur de synchronisation devient alors

$$D^{\alpha}e = Ae + F(x) + G(y) - H(z) - W(k) - u - v. \tag{1.6}$$

Afin de démontrer la stabilité du système (1.6), on suppose que les hypothèses suivantes sont satisfaites
Hypothèses : Supposons que les contrôles u et v sont structurés de la façons suivante

$$u + v = Me + F(x) + G(y) - H(z) - W(k), \tag{1.7}$$

où M est la matrice de contrôle vérifiant

$$|\arg(\lambda_i\ (A + M))| > \alpha\frac{\pi}{2}, \tag{1.8}$$

avec $\arg(\lambda_i\ (A+M))$, $i = 1, 2, ..., n$, sont des arguments des valeurs propres λ_i de $A+M$. Nous avons donc le résultat suivant

2 Résultat théorique

Théorème 2.1. *Sous les hypothèses présentées ci dessus, l'erreur de synchronisation (1.6) converge vers zéro, et la synchronisation proposées est atteint.*

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Existence, Stability, and Numerical Solutions for a New Fractional Chaotic Jerk Model

Meriem Mansouria BELHAMITI[1]*

LMPA, Faculty SEI, UMAB Mostaganem, Algeria
E-mail: meriembelhamiti@gmail.com

Abstract
We provide a new Jerk equation model with fractional derivatives. We begin by presenting an existence and uniqueness result for the problem, by means of the contraction mapping principle. The existence of at least one solution is then established using the Schaefer fixed point theorem. Ulam-type stabilities are also investigated. Two concrete examples are also addressed. Lastly, a Caputo derivative approximation is proposed, and certain chaotic behaviors are properly handled using the Runge Kutta 4th order method.

1 Introduction

Fractional differential equations have evolved as a valuable tool for modeling a variety of phenomena in several areas, including viscoelasticity, fluid flow, electrical circuits, and so on.
In this talk, we will look at several applications of dynamical systems exhibiting chaotic behavior, we attempt to discover an accurate fractional representation, especially for a simple Jerk circuit that allows us to explore some entropy, some papers dealing with Jerk equations and systems for chaotic behaviours can be found in [1, 2, 3].
So, let us consider the following fractional differential problem:

$$D^{\alpha}(D^2 + \lambda^2 D^{\alpha})y(t) = f(t, y(t), D^{\alpha}y(t)), \quad t \in [0, T], \qquad T > 0, \tag{1.1}$$

subject to a nonlocal and integral boundary condition and initial conditions as follows:

$$y(0) = 0, \qquad D^{1-\alpha}D^{\alpha}y(0) = 0, \qquad y(T) = \beta J^{\gamma}y(\eta), \quad 0 < \eta \le T, \tag{1.2}$$

where D^{α} Caputo fractional derivatives order $\alpha \in [0, 1]$, J^{γ} is a Riemann-Liouville fractional integral order $\gamma \in [0, \infty[$, $\lambda \in \mathbb{R}_+$ and $\beta \in \mathbb{R}$.
It is crucial to highlight that Eq. (1.1) is broad enough to address many problems that occur in applied mathematics, as an example, it includes Gottlieb's typical Jerk equation [1]. It also contains models described by Sportt [2] and Munmuangsaen et al. [3].
The integrated nonlocal condition is an adequate physical measurement to describe processes in various positions, while the integral boundary condition is a smart approach to explaining vascular hemodynamics, so we carry out the same process for charges in electrical circuits.

Key Words and Phrases: *Jerk equation, Caputo derivative, Fixed-point theory, Ulam stability, Runge-Kutta method.*

2 Main results

The structure of this speech is as follows: First of all we review some pertinent concepts, notations, and auxiliary results regarding fractional derivatives. In the second part, we prove the existence and uniqueness by the Banach contraction principle, and by means of the Schaefer fixed point theorem, we establish the existence of at least one solution for (1.1)-(1.2). We also discuss the Ulam-Hyers-Rassias stabilities.

Throughout the last part, we will attempt to develop a numerical approach to Caputo derivative inspired by existing approaches [4]. Furthermore, in order to investigate the dynamic behavior of the problem a new fractional equivalent system will also be suggested. In the context of this reduction, we intend to demonstrate the coexistence of attractors sensitive to initial conditions as well as fractional order in phase space, we then review three 3D applications that are capable of generating attractors as **(Oscillateur De Van Der PolDuffing Jerk, Jerk Oscillateur & FPGA , Cubiques Jerk)**, see [5]. Numerical simulations of our results will be performed using the Caputo approach and the fourth-order Runge-Kutta method.

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The Homoclinic pattern for a discrete Schrodinger equation

Allal Mehazzem*

University Center Abdelhafidh Boussouf, Mila, Algeria
E-mail: a.mehazzem@centre-univ-mila.dz

Abstract
We discuss the existence of homocline orbits by numerical simulation and a formal study of a discrete nonlinear Schr odinger equation. According to the initial data, homoclinic structures are known as sources of sensitivities which, in the presence of small perturbations, can change between various qualitatively different solutions. Spectral analysis is used to demonstrate the existence of homoclinic solutions. Thus, we show the existence of homoclinic solutions by using the symmetrical properties of reversible systems for local solutions.

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Key Words and Phrases: *discrete Schrodinger equation, Homoclinic orbits, Instability*

Offset boosting, bifurcation and circuit simulation of a new multistable 4D hyperchaotic system

Khaled Benkouider
Automatic Department, Jijel University, Jijel, Algeria
E-mail: benkouider.khaled@univ-jijel.dz

Abstract
This paper reports the finding of a new 4D dynamical system which experiences hyperchaos, multistability, and offset boosting phenomenon.

1 Introduction

The 4D hyperchaotic systems with two positive Lyapunov exponents can generate very complex and random signals [1]. Moreover, multistability phenomenon allows the system to behave with a high degree of flexibility [2]. The offset boosting control is extremely essential to transform a bipolar signal to a unipolar signal or vice versa [3]. In this paper, we propose a new 4D system which has the above-mentioned special features.

2 Model, dynamical analysis, and circuit desing

The new 4D hyperchaotic system is defined by the algebraic equations that follows:

$$\dot{x} = ax - yz + w, \quad \dot{y} = xz - by, \quad \dot{z} = xy - cz, \quad \dot{w} = -y + d \tag{2.1}$$

System (2.1) generates hyperchaotic attractors as shown in Figure 1. The corresponding LEs are: $LE_1 = 3.036$, $LE_2 = 0.010$, $LE_3 = 0$, $LE_4 = -50.032$ and $D_{KY} = 3.061$. System (2.1) has two unstable equilibrium points: $E_1 = [c/b\sqrt{bd^2/c}, d, \sqrt{bd^2/c}, (1 - ac/d)\sqrt{bd^2/c}]$ and $E_2 = [-c/b\sqrt{bd^2/c}, d, -\sqrt{bd^2/c}, (ac/d - 1)/d\sqrt{bd^2/c}]$. It belongs to the self-excited family.

2.1 Bifurcation analysis and multistability

When $7 < c < 8$, system (2.1) exhibits periodic behavior . When $8 < c < 9$, system (2.1) exhibits quasi-periodic behavior. When $9 < c < 10$, system (2.1) exhibits chaotic behavior. When $10 < c < 15$, system (2.1) exhibits hyperchaotic behavior. In addition, system (2.1) can generate coexistence of: one chaotic and one periodic attractors (Figure 3(a)), two coexisting chaotic attractors (Figure 3(b)), and two coexisting hyperchaotic attractors (Figure 3(c)).

2.2 Offset boosting control

System (2.1)'s first differential equation can be rewritten as follows: $\dot{x} = ax - yz + (w + k)$, where k is a controller for offset boosting. From Figure 4, we can see the boosted attractor of System (2.1). This special property has a wide range of applications.

Key Words and Phrases: *hyperchaos, dynamical systems, bifurcation, Lyapunov exponents, offset boosting.*

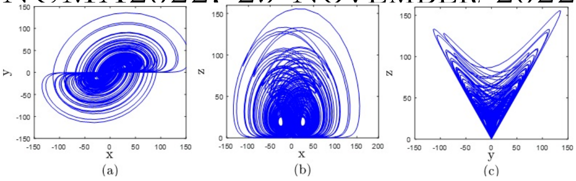

Figure 1: Hyperchaotic attractors of the new system (2.1).

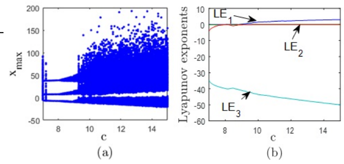

Figure 2: (a) Bifurcation diagram and (b) LEs spectrum of system (2.1) vs. c.

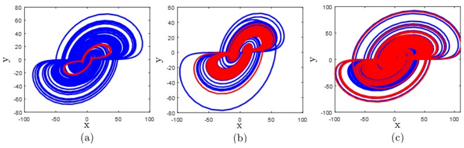

Figure 3: the coexisting attractors.

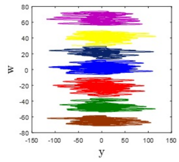

Figure 4: $y - w$ boosted attractor.

2.3 Electronic circuit simulation

The analogue circuit of system (2.1) is implemented using Multisim as shown in Figure 5. Results confirm the physical feasibility of our proposed mathematical model.

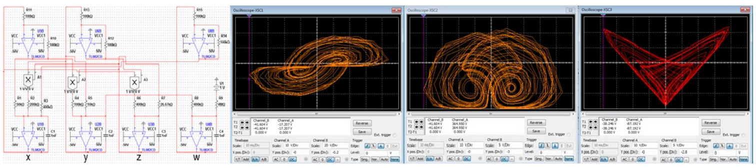

Figure 5: Schematic and Multisim plots of the equivalent electronic circuit.

3 Conclusion

This work introduced a new hyperchaotic system with special features, such as: 1- It has two positive LEs. 2- It can generate different coexisting attractors. 3- It is offset boosting controllable. 4- Physically feasible. The above-mentioned properties making the new system (2.1) more useful for chaos-based applications than ordinary reported systems.

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Analyzing Tuberculosis dynamics in Algeria using a VSLIT model

Bouchra Chennaf*

Department of Mathematics and Computer Sciences, University Centre Abdelhafid Boussouf, Mila, Algeria
E-mail: bouchra.chennaf@centre-univ-mila.dz

Mohammed Salah Abdelouahab

Department of Mathematics and Computer Sciences, University Centre Abdelhafid Boussouf, Mila, Algeria
E-mail: m.abdelouahab@centre-univ-mila.dz

Abstract
Despite low TB mortality rates in many countries such as China, Europe and the United States, other countries such as India are still struggling to control the epidemic. The aim of this study is to analyze tuberculosis (TB) dynamics in Algeria and investigate the vaccinations and treatment effects of the disease breaks. For this purpose, a VSLIT mathematical model is considered. First, a qualitative study is conducted to discuss the stability property of both disease-free and endemic equilibria, then by means of the least square method and using the TB-reported data of Algeria from 1990 to 2020 the model parameters are estimated. Finally, some numerical simulations are performed to confirm the theoretical results.

1 Introduction

In Algeria, as in many countries, tuberculosis still remains a serious problem. Therefore, scientists and governments have tried to keep epidemics under control [1]. The coronavirus disease outbreak (COVID-19) has recently brought attention back to the importance of epidemic research and the creation of mathematical models to understand the behavior of epidemics [2]. The main problem is to determine the model parameters that describe the behaviour in a realistic way. Which is important to calculate the basic reproduction number, $\mathcal{R}_0$.
In this study, TB disease in Algeria has been analyzed by using a VSLIT Compartmental epidemiological model.

Key Words and Phrases: *Tuberculosis model, epidemic, vaccination, parameter estimation*

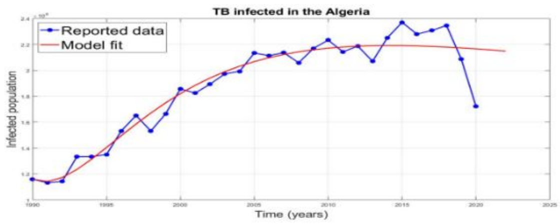

Figure 1: Data fitting of the number of TB cases in Algeria

2 Main results

The population is divided into five classes: vaccinated V, susceptible S, latent(exposed) L, infected(TB active) I, and under treatment T. Based on this strategy we propose the following VSLIT system to describe the dynamics of TB infection in Algeria.

$$\begin{cases} \dot{V}(t) = \Lambda - kV(t), \\ \dot{S}(t) = kV(t) - \beta S(t)I(t)/N - \mu S(t), \\ \dot{L}(t) = \beta S(t)I(t)/N - (\epsilon + \mu)L(t) + (1-\alpha)\delta T(t), \\ \dot{I}(t) = \epsilon L(t) + \alpha\delta T(t) - (\gamma + \mu + \sigma)I(t), \\ \dot{T}(t) = \gamma I(t) - (\mu + \delta + \eta)T(t). \end{cases} \tag{2.1}$$

We obtained the best fitted parameter values by minimizing the error between actual TB incidence data taken from the WHO Global Tuberculosis Report [3], from 1990 to 2020, and the solution of the proposed model (2.1). The objective function used in this parameter estimation is given by

$$\psi = \sum_{i=1}^{n} (I_{t_i} - I^*_{t_i})^2$$

where $I^*_{t_i}$ denotes the actual TB infected case and I_{t_i} are the corresponding model solution at time t_i, n is the number of available actual data. The behaviour of infected individuals for VSLIT model is plotted in Figure 1 together with the reported data. As can be seen, the obtained predictions for VSLIT model are in good agreement with the reported data.

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Existence and Ulam Stability for Katugampola Random Fractional Differential In Banach Spaces

Bekada Fouzia*
University Dr Moulay Tahar, Saida, Algeria
E-mail: bekadafouzia@gmail.com

Abstract
This paper deals with some existence of random solutions and the stability of Ulam for for a class of Katugampola random fractional differential equations in Banach spaces. A random fixed point theorem is used for the existence of random solutions, and we prove that our problem is generalized Ulam-Hyers-Rassias stable. An illustrative example is presented in the last section.

1 Introduction

Fractional calculus plays a very important role in several fields such as physics, chemical technology, economics, biology.
In 2011, Katugampola introduced a derivative that is a generalization of the Riemann-Liouville fractional operators and the Fractional integral of Hadamard in a single form.
In this work we investigate the following class of Katugampola random fractional differential equation

$$({}^\rho D_0^\varsigma x)(\xi, w) = f(\xi, x(\xi, w), w);\ \xi \in I = [0, T],\ w \in \Omega, \tag{1.1}$$

with the terminal condition

$$x(T, w) = x_T(w);\ w \in \Omega, \tag{1.2}$$

where $x_T : \Omega \to E$ is a measurable function, $\varsigma \in (0, 1],\ T > 0,\ f : I \times E \times \Omega \to E,\ {}^\rho D_0^\varsigma$ is the Katugampola operator of order ς, and Ω is the sample space in a probability space, and $(E, \|\cdot\|)$ is a Banach space.

Definition 1.1. [2] The problem (1.1)-(1.2) is generalized Ulam-Hyers-Rassias stable with respect to Φ if there exists $c_{f,\phi} > 0$ such that for each solution $x(\cdot, w) \in C_{\varsigma,\rho}(I)$ of the inequality

$$\|({}^\rho D_0^r x)(\xi, w) - f(\xi, u(\xi, w), w)\| \leq \Phi(\xi, w);\ for\ \xi \in I,\ \ and\ w \in \Omega. \tag{1.3}$$

there exists $y(\cdot, w) \in C_{\varsigma,\rho}(I)$ satisfies (1.1)-(1.2) with

$$\|\xi^{\rho(1-\varsigma)} x(\xi, w) - \xi^{\rho(1-\varsigma)} y(\xi, w)\| \leq c_{f,\phi}\phi(\xi, w);\ \xi \in I;\ w \in \Omega.$$

Theorem 1.1. *[4] Let X be a nonempty, closed convex bounded subset of the separable Banach space E and let $G : \Omega \times X \to X$ be a compact and continuous random operator. Then the random equation $G(w)u = u$ has a random solution.*

Key Words and Phrases: *Katugampola fractional integral; Katugampola fractional derivative; random solution; Banach space; Ulam stability*

2 Main results

Definition 2.1. By a random solution of problem (1.1)-(1.2), we mean a measurable function $x(w,\cdot) \in C_{\varsigma,\rho}(I)$ such that

$$x(t,w) = \frac{\rho^{1-\varsigma}}{\Gamma(\varsigma)} \int_0^t \frac{s^{\rho-1}}{(t^\rho - s^\rho)^{1-\varsigma}} f(t,x,w)ds - C(w)t^{\rho(\varsigma-1)}, \tag{2.1}$$

where

$$C(w) = \frac{1}{T^{\rho(r-1)}} \left(\frac{\rho^{1-\varsigma}}{\Gamma(\varsigma)} \int_0^T \frac{s^{\rho-1}}{(T^\rho - s^\rho)^{1-\varsigma}} f(T,u,w)ds - u_T(w) \right).$$

Theorem 2.1. *If (H_1) and (H_2) hold, and*

$$\frac{\rho^{-\varsigma}T^\rho}{\Gamma(1+\varsigma)} l_2^*(w) < 1, \tag{2.2}$$

then there exists a random solution for (1.1)-(1.2).

Theorem 2.2. *If (H_1), (H_3), (H_4) and*

$$\frac{\rho^{-\varsigma}T^\rho}{\Gamma(1+\varsigma)} \Phi^*(w)q^*(w) < 1, \tag{2.3}$$

hold. Then the problem (1.1)-(1.2) has random solutions defined on I, and it is generalized Ulam-Hyers-Rassias stable.

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Positivity and Stability of Conformable Descriptor Linear Continuous-time Systems

Amina Faraoun*

Department of Mathematics and Computer Science, ACSY Team-Laboratory of Pure and Applied Mathematics, Abdelhamid Ibn Badis University Mostaganem, P.O.Box 227/118 University of Mostaganem, 27000 Mostaganem, Algeria
E-mail: amina.faraoun.etu@univ-mosta.dz

Abstract
The aim of this work is the application of the Shuffle algorithm of a new descriptor continuous-time linear systems which is based on the conformable derivative operator, due to these interesting properties such as: quotient, product, chain rules, Rolle's theorem, and mean-value theorem. A new conditions of the positivity and stability have established and a numerical examples will be presented.

1 Introduction

The descriptor (singular) linear systems have been greatly appeared in different area such as robotics, physics, chemic, electrical circuit $\cdots$. In control theory, the variables of this systems are positive in nature this makes the concept of positivity important and have received considerable interest of many scientists, the main property of these systems is that if the initial state is positive (or at least non-negative), then the state trajectory is entirely non-negative.
Recently, The shuffle algorithm is apperad in Luenberger, (1978), this procedure transform the descriptor systems into a standard form, that simplifies the analysis of dynamic properties of the system under consideration. The standard linear continuous-time system with conformable derivative in unidimensional (1D) has received much attention in the last two years [1, 2]. This new derivative has been proposed by *Khalil et al.* [3] and took part on several areas as engineering, finances, biology, medicine, physics and applied mathematics. The most advantages of this derivative is that it preserves the properties of the usual exact derivatives. More than that, conformable derivative does not contain any integral terms, that make it much more easier to apply on the fractional differential equations. The positivity and stability of Conformable standard continuous-time linear systems are derived in [4], but there is no result in singular case, i.e when E is no invertible matrix.
In this context, we will be applied this algorithm on the Conformable descriptor continuous-time linear systems to checking the positivity and the stability of this positive system.

Key Words and Phrases: *Descriptor systems, Conformable derivative, Shuffle algorithm, Positivity, Stability.*

2 Main results

This section is devoted to present our main results. For this purpose, we will consider the following continuous-times linear systems

$$E\mathbf{T}^{\alpha}x(t) = Ax(t) + Bu(t), \tag{2.1}$$

$$y(t) = Cx(t) + Du(t), \tag{2.2}$$

where $\mathbf{T}^{\alpha}$ presents the conformable derivative operator of order α with $0 < \alpha \leq 1$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are, respectively, the state, the control, and the output of the system. $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$ with $\det E = 0$ and $\det(Es - A) \neq 0$ for some $s \in \mathbb{C}$. The boundary condition of the system is given by $x(0) = x_0$, then the system of equations (2.1) and (2.2) has a unique solution. Performing the steps in the shuffle algorithm and applying some operations on the system of equations (2.1) and (2.2) we obtain the following theorem

Theorem 2.1. *Let q be an integer number with $q > 1$, then the Conformable descriptor system of equations* (2.1) *and* (2.2) *have the following standard form*

$$\mathbf{T}^{\alpha}x(t) = \begin{bmatrix} E_q \\ A_{q+2} \end{bmatrix}^{-1} \left[\begin{bmatrix} A_{q+1} \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} B_{q+1} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -B_{q+2} \end{bmatrix} \mathbf{T}^{\alpha}u(t) + \cdots + \begin{bmatrix} 0 \\ H_q \end{bmatrix} \mathbf{T}^{q\alpha}u(t) \right].$$

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